Department of Economics University of Maryland

Economics 325
Intermediate Macroeconomic Analysis
Midterm Exam
Suggested Solutions
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Fall 2008

NAME: ______________________________

The Exam has a total of five (5) problems and pages numbered one (1) through eleven (11). Each problem's total number of points is shown below. Your solutions should consist of some appropriate combination of mathematical analysis, graphical analysis, logical analysis, and economic intuition, but in no case do solutions need to be exceptionally long. Your solutions should get straight to the point – **solutions with irrelevant discussions and derivations will be penalized.** You are to answer all questions in the spaces provided.

You may use one page (double-sided) of notes. You may **not** use a calculator.

Problem 1	**/ 25**
Problem 2	**/ 25**
Problem 3	**/ 11**
Problem 4	**/ 15**
Problem 5	**/ 24**
TOTAL	**/ 100**

Problem 1: Consumption and Savings in the Two-Period Economy (25 points). Consider a two-period economy (with no government), in which the representative consumer has no control over his income. The lifetime utility function of the representative consumer is $u\left(c_1, c_2\right) = \ln c_1 + \ln c_2$, where $\ln$ stands for the natural logarithm. We will work here in purely real terms: suppose the consumer's **present discounted value of ALL lifetime REAL income is 26.** Also suppose the consumer begins period 1 with zero net assets.

a. **(17 points)** Set up the lifetime Lagrangian formulation of the consumer's problem, in order to answer the following: i) is it possible to numerically compute the consumer's optimal choice of consumption in period 1? If so, compute it; if not, explain why not. ii) is it possible to numerically compute the consumer's optimal choice of consumption in period 2? If so, compute it; if not, explain why not. iii) is it possible to numerically compute the consumer's real asset position at the end of period 1? If so, compute it; if not, explain why not.

Solution: We know that with zero initial assets, the LBC of the consumer is

$$c_1 + \frac{c_2}{1+r_1} = y_1 + \frac{y_2}{1+r_1},$$

where the notation is standard from class. The Lagrangian is thus

$$u(c_1, c_2) + \lambda\left[y_1 + \frac{y_2}{1+r_1} - c_1 - \frac{c_2}{1+r_1}\right],$$

where λ of course is the Lagrange multiplier (note there's only one multiplier since this is the lifetime formulation of the problem not the sequential formulation of the problem). The first-order conditions with respect to c_1 and c_2 (which are the objects of choice) are, as usual:

$$u_1(c_1, c_2) - \lambda = 0$$

$$u_2(c_1, c_2) - \frac{\lambda}{1+r_1} = 0$$

(And of course the FOC with respect to the multiplier just gives back the LBC.) Also as usual, these FOCs can be combined to give the consumption-savings optimality condition, $\frac{u_1(c_1, c_2)}{u_2(c_1, c_2)} = 1 + r_1$. With the given utility function, the marginal utility functions are $u_1 = 1/c_1$ and $u_2 = 1/c_2$, so the consumption-savings optimality condition in this case becomes $c_2/c_1 = 1 + r_1$. This can be rearranged to give $c_2 = (1+r_1)c_1$, which we can then insert in the LBC to give $c_1 + c_1 = y_1 + \frac{y_2}{1+r_1}$ (no, that's not a typo, it's $c_1 + c_1$ after the substitution…).

In this problem, you are given neither y_1 nor y_2. Instead, what you are given is $y_1 + \frac{y_2}{1+r_1} = 26$.

Thus, we have that the optimal quantity of period-1 consumption is $c_1^* = 13$ (which solves part i). We can **not** compute c_2^*, however, because we are not given the interest rate r_1 (which you would need in order to use the expression $c_2 = (1+r_1)c_1$ computed above. (This solves part ii). To compute the asset position at the end of period 1, we would need to compute $y_1 - c_1^*$, but since we don't know y_1, we cannot compute this either (which solves part iii).

Problem 1 continued

b. **(8 points)** To demonstrate how important the concept of the real interest rate is in macroeconomics, an interpretation of it (in addition to the couple of different interpretations we have already discussed in class) is that it reflects the rate of consumption growth between two consecutive periods. Using the consumption-savings optimality condition for the given utility function, **briefly** describe/discuss **(rambling essays will not be rewarded)** whether the real interest rate is **positively related to or negatively related to the rate of consumption growth between period one and period two.** For your reference, the definition of the rate of consumption growth rate between period 1 and period 2 is $\frac{c_2}{c_1}-1$ (completely analogous to how we defined in class the rate of growth of prices between period 1 and period 2). (**Note:** No mathematics are especially required for this problem; also note this part can be fully completed even if you were unable to get all the way through part a).

Solution:

The familiar consumption-savings optimality condition is $\frac{u_1}{u_2}=1+r$. As we just saw above, for the given utility function, this becomes $\frac{1/c_1}{1/c_2}=1+r$, or, rewriting,

$$\frac{c_2}{c_1}=1+r .$$

The left-hand-side of this expression obviously measures the consumption growth rate between period 1 and period 2. That is, if c_1 = 100 and c_2 = 103, clearly the consumption growth rate is 3 percent between period 1 and period 2. Which would mean that r = 0.03. If the real interest rate were instead larger, clearly the left-hand-side, c_2/c_1, would be larger as well. Thus, **the higher is the real interest rate, the higher is the consumption growth rate between periods – the real interest rate and the consumption growth rate are positively related to each other.**

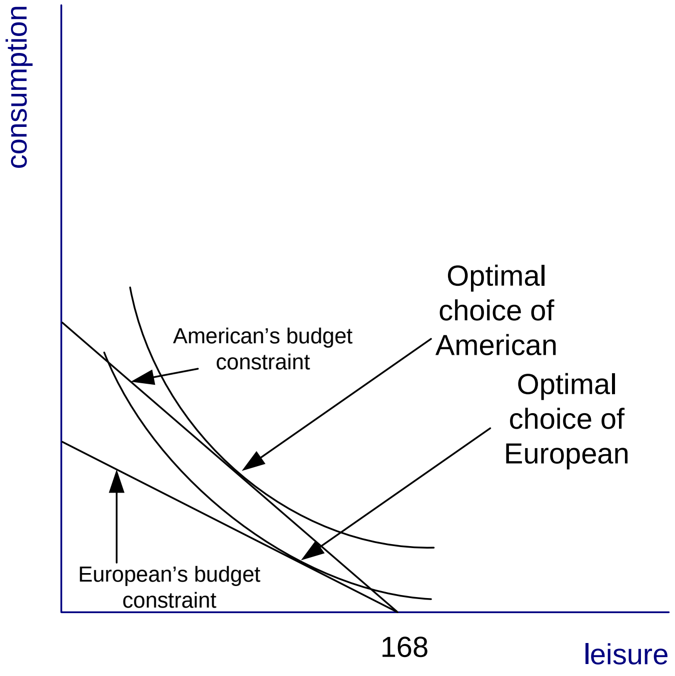
This is thus yet another way to think about the real interest rate. The two other ways we discussed in class of thinking intuitively about the real interest rate is that it measures the price of current (period-1) consumption in terms of future (period-2) consumption; and as reflecting the fundamental degree of (human) impatience of individuals in the economy. All of these various (and ultimately inter-related) ways of thinking about the real interest underline its fundamental importance in macroeconomic theory.

Note that simply arguing/explaining here that a rise in the real interest rate leads to a fall in period-1 consumption does not address the question – the question is about the **rate of change of consumption between period 1 and period 2,** not about the **level** of consumption in period 1 by itself.

Problem 2: European and U.S. Consumption-Leisure Choices (25 points). Europeans work fewer hours than Americans. There are likely very many possible reasons for this, and indeed in reality this fact arises from a combination of many reasons. In this question, you will consider two reasons using the simple (one-period) consumption-leisure model.

a. **(13 points)** Suppose that both the utility functions and pre-tax real wages W/P of American and European individuals are identical. However, the labor income tax rate in Europe is higher than in America. In a **single** carefully-labeled indifference-curve/budget constraint diagram (with consumption on the vertical axis and leisure on the horizontal axis), show how it can be the case that Europeans work fewer hours than Americans. Provide any explanation of your diagram that is needed.

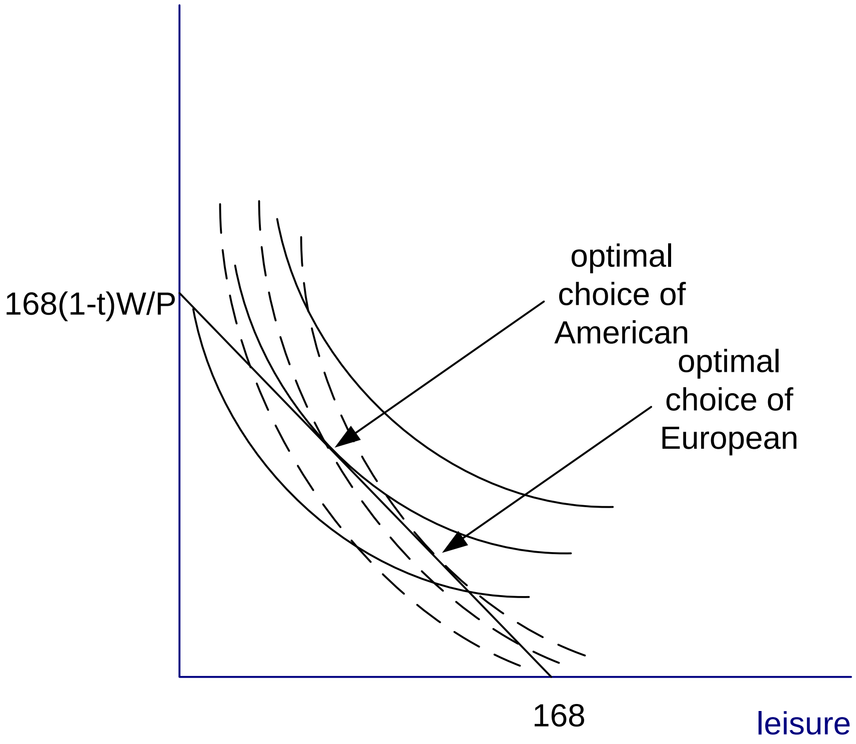
Solution: If Europeans work fewer hours than Americans, then Europeans have more leisure time than Americans, simply because (in our weekly model) $n+l=168$. Europeans and Americans have identical utility functions, which means that their indifference maps are identical. This means that the difference in hours worked must arise completely from differences in their budget constraints. With a higher labor income tax in Europe, the budget constraint of the European consumer is less steep than the budget constraint of the American, as the diagram below shows (because the slope of the budget constraint is $(1-t)W/P$, and you are given that W/P is the same in the two countries). The diagram shows that the European optimally chooses more leisure (hence less labor) and less consumption than the American. Here, the difference between Europeans and Americans is solely in the relative prices (embodied by the slope of the budget constraint) they face. (For full credit here, you had to somehow make clear that the indifference maps of the representative European and the representative American are identical.)



Problem 2 continued.

b. **(12 points)** Suppose that both the pre-tax real wages W/P and the labor tax rates imposed on American and European individuals are identical. However, the utility function $u^{AMER}(c,l)$ of Americans differs from that of Europeans $u^{EUR}(c,l)$. In a **single** carefully-labeled indifference-curve/budget constraint diagram (with consumption on the vertical axis and leisure on the horizontal axis), show how it can be the case that Europeans work fewer hours than Americans. Provide any explanation of your diagram that is needed.

Solution: In this case, the budget constraints of the European consumer and American consumer are identical, so the difference in hours worked must arise completely from differences in their utility functions. Graphically, this means that the two types of consumers have different indifference maps (i.e., a different set of indifference curves). In the diagram below, the budget line is the common budget line of the European and the American. The solid indifference curves are the American's, while the dashed indifference curves are the European's. With steeper indifference curves, the European's optimal choice along the same budget line must occur at a point that features more leisure (hence less labor) and less consumption than the American's optimal choice. Here, the difference between Europeans and Americans is solely in their preferences.



Problem 3: Government Budgets and Government Asset Positions (11 points). Just as we can analyze the economic behavior of consumers over many time periods, we can analyze the economic behavior of the government over many time periods. **Suppose that at the beginning of period *t*, the government has zero net assets**. Also assume that the real interest rate is **always *r* = 0.** The following table describes the **real** quantities of government spending and **real** tax revenue the government collects starting in period *t* and for several periods thereafter.

Period	**Real government expenditure (*g*) during the period**	**Real tax collections during the period**	**Quantity of net government assets at the END of the period**
t	10	12	2
***t*+1**	8	14	8
***t*+2**	15	10	3
***t*+3**	10	10	3
***t*+4**	8	12	7

a. **(7 points)** Complete the last column of the table based on the information given. **Briefly** explain the logic behind how you calculate these values.

Solution: If this were the two-period model, we could compute the government asset position at the end of period, say, one, by rearranging the period-1 government budget constraint: $b_1 = t_1 - g_1 + b_0$ -- in this expression we have used the assumption that $r = 0$. Furthermore (and again with $r = 0$), we can compute the government asset position at the end of period two as: $b_2 = t_2 - g_2 + b_1$ (In the simple two-period model, we assumed $b_2 = 0$, but if we want to extend past two periods, we of course would not make this assumption.) Directly extending this logic to an infinite-period setting, then, the government's asset position at the **end** of any particular period *t* is given by: $b_t = t_t - g_t + b_{t-1}$. Successively applying this rule then gives rise to the net asset positions presented in the table above.

b. **(4 points)** Suppose instead the government ran a balanced budget every period (i.e., every period it collected in taxes exactly the amount of its expenditures that period). In this balanced-budget scenario, what would be the government's net assets at the end of period *t*+4? **Briefly** explain/justify.

Solution: A balanced budget means *g* equals tax collections every period. If this were true in the above table, and applying the logic of part a above, the government net assets at the end of **every** period would always be zero; thus at the end of period *t*+4 they are zero as well.

Problem 4: A Contraction in Credit Availability (15 points). The graph below shows our usual two-period indifference-curve/budget constraint diagram, with period-1 consumption plotted on the horizontal axis, period-2 consumption plotted on the vertical axis, and the downward-sloping line representing, as always, the consumer's LBC. Throughout all of the analysis here, assume that ***r* = 0 always.** Furthermore, there is no government, hence never any taxes.

Suppose that the representative consumer has lifetime utility function $u(c_1, c_2) = \ln c_1 + \ln c_2$, and that the **real** income of the consumer in period 1 and period 2 is $y_1 = 12$ and $y_2 = 8$. Finally, suppose that the initial amount of net assets the consumer has is $a_0 = 0$. **EVERY** consumer in the economy is described by this utility function and these values of y_1, y_2, and a_0.

a. **(6 points)** If there are no problems in credit markets whatsoever (so that consumers can borrow or save as much or as little as they want), compute the numerical value of the optimal quantity of period-1 consumption. (**Note:** if you can solve this problem without setting up a Lagrangian, you are free to do so as long as you explain your logic.)

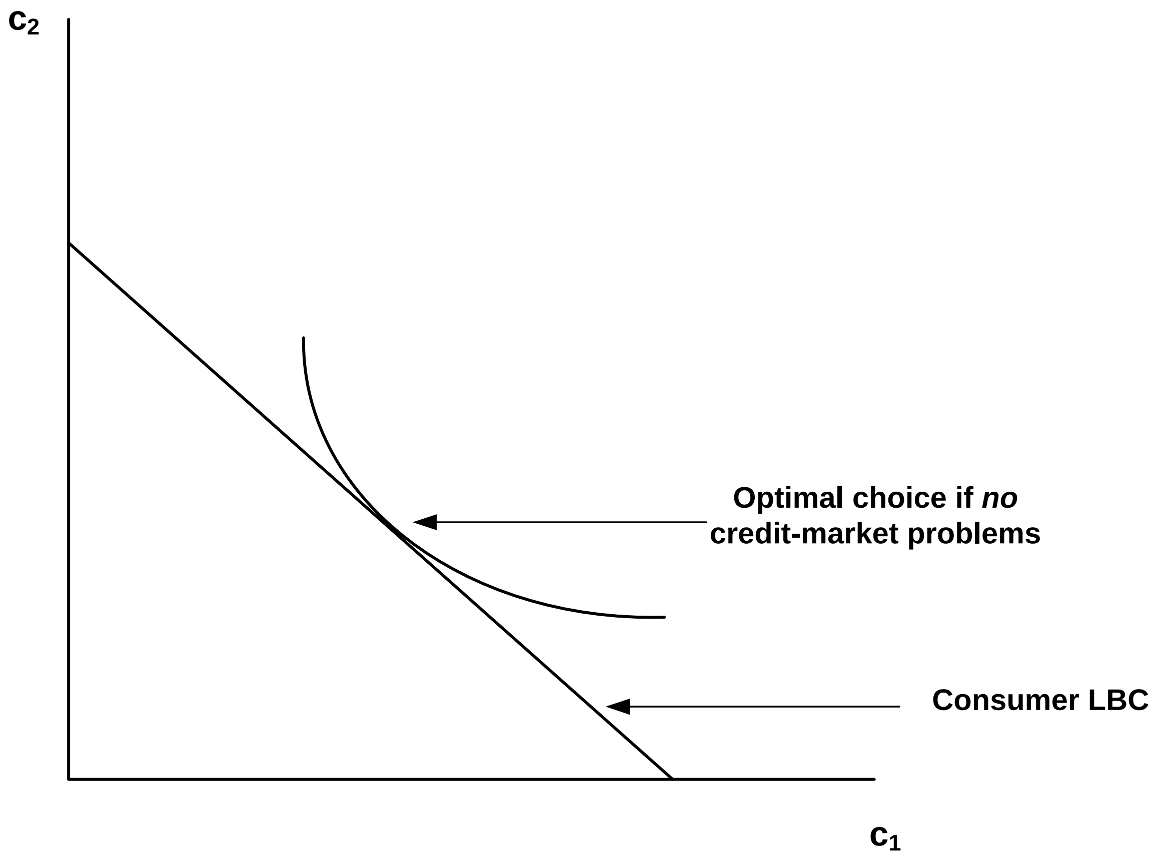
Solution: The consumption-savings optimality condition (given the natural-log utility function) is given by $c_2/c_1 = 1+r = 1$ (the second equality follows because $r = 0$ here). Thus, at the optimal choice, it is the case that $c_1 = c_2$. Using this relationship (and again using the fact that $r = 0$ here), we can express the consumer's LBC as $c_1 + c_1 = y_1 + y_2 = 20$, which obviously implies the optimal choice of period-1 consumption is $c_1 = 10$.

Note: although you were not asked to compute it, you could have computed the implied value of the consumer's asset position at the end of period one. Because $a_0 = 0$, $y_1 = 12$, and we just computed $c_1 = 10$, the asset position at the end of period one is $a_1 = y_1 - c_1 = 2$ (i.e., **positive** 2).

b. **(9 points)** Now suppose that because of problems in the financial sector, no consumers are allowed to be in debt at the end of period 1. With this credit restriction in place, compute the numerical value of the optimal quantity of period-1 consumption. **ALSO,** on the diagram on the next page, qualitatively and **clearly** sketch the optimal choice with this credit restriction in place (qualitatively sketched already for you is the optimal choice if there are no problems in credit markets). Your sketch should indicate **both** the new optimal choice **and** an appropriately-drawn and labeled indifference curve that contains the new optimal choice. (**Note:** if you can solve this problem without setting up a Lagrangian, you are free to do so as long as you explain your logic.)

Solution: Because in part a (ie, without any credit restrictions), the representative consumer was choosing to NOT be in debt at the end of period 1 (ie, $a_1 > 0$ under the optimal choice in part a), the imposition of the credit restriction, **nothing changes compared to part a.** That is, the optimal choice of period-1 consumption is still 10. Hence, in the diagram below, the optimal choice in the presence of credit constraints is **exactly the same as the optimal choice without credit constraints.** The general lesson to draw from this example and our analysis in class is that it is not *necessarily* the case that financial market problems *must* and *always* spill over into real economic activity (i.e., consumption in this case).

Problem 4b continued



Problem 5: Two Types of Stock (23 points). Consider a variation of our usual infinite-period "stock-pricing" model. The variation here is that there are **two distinct "types" of stock** (rather than just one) that the representative consumer can buy: "Dow" stock and "S&P" stock. Denote by a_{t-1}^{DOW} the representative consumer's holdings of Dow stock at the beginning of period t and by a_{t-1}^{SP} the representative consumer's holdings of S&P stock at the beginning of period t. Likewise, let S_t^{DOW} and S_t^{SP} denote, respectively, the nominal price of Dow and S&P stock in period t, and D_t^{DOW} and D_t^{SP} denote, respectively, the per-share nominal dividend that Dow and S&P stock pay in period t. The period-t budget constraint of the representative consumer is thus

$$P_t c_t + S_t^{SP} a_t^{SP} + S_t^{DOW} a_t^{DOW} = Y_t + (S_t^{SP} + D_t^{SP}) a_{t-1}^{SP} + (S_t^{DOW} + D_t^{DOW}) a_{t-1}^{DOW},$$

in which all of the other notation is standard: Y_t denotes nominal income (over which the consumer has no control) in period t, c_t is real units of consumption, and P_t is the nominal price of each unit of consumption. Also as usual, the lifetime utility of the consumer starting from period t onwards is $u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \dots$, where $\beta \in (0,1]$ is the usual measure of consumer impatience.

The sequential Lagrangian for this problem is

$$\begin{aligned}
& u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \dots \\
& + \lambda_t \left[Y_t + S_t^{SP} a_{t-1}^{SP} + D_t^{SP} a_{t-1}^{SP} + S_t^{DOW} a_{t-1}^{DOW} + D_t^{DOW} a_{t-1}^{DOW} - P_t c_t - S_t^{SP} a_t^{SP} - S_t^{DOW} a_t^{DOW} \right] \\
& + \beta \lambda_{t+1} \left[Y_{t+1} + S_{t+1}^{SP} a_t^{SP} + D_{t+1}^{SP} a_t^{SP} + S_{t+1}^{DOW} a_t^{DOW} + D_{t+1}^{DOW} a_t^{DOW} - P_{t+1} c_{t+1} - S_{t+1}^{SP} a_{t+1}^{SP} - S_{t+1}^{DOW} a_{t+1}^{DOW} \right] \\
& + \dots.
\end{aligned}$$

a. **(8 points)** Based on the Lagrangian presented above, compute the first-order conditions with respect to both a_t^{SP} and a_t^{DOW} .

Solution: Taking FOCs with respect to c_t, a_t^{SP}, and a_t^{DOW} and combining these FOCs as usual yields two (very similar) stock-pricing equations. Note you could have stopped simply at the FOCs on the two types of stock, as directly asked in the question.

$$S_t^{DOW} = \frac{\beta u'(c_{t+1})}{u'(c_t)} \cdot \left(S_{t+1}^{DOW} + D_{t+1}^{DOW} \right) \cdot \frac{P_t}{P_{t+1}}$$

$$S_t^{SP} = \frac{\beta u'(c_{t+1})}{u'(c_t)} \cdot \left(S_{t+1}^{SP} + D_{t+1}^{SP} \right) \cdot \frac{P_t}{P_{t+1}}$$

Problem 5a continued (if you need more space)

b. **(7 points)** Based on the expressions you obtained in part a above, determine whether it is the case that $S_t^{DOW} = S_t^{SP}$? If so, briefly explain why; if not, briefly explain why not; if it's not possible to tell, explain why not.

Solution: No, it is not possible to tell whether or not $S_t^{DOW} = S_t^{SP}$ simply because you are thus far given no information on the dividends that each of these two different assets pay.

Problem 5 continued

c. **(8 points – Harder)** Assume here for simplicity that $\beta = 1$. Suppose the economy eventually reaches a steady-state. In this steady state, Dow stock continue to pay zero dividends but S&P stock pay a nominal dividend that is **always one-tenth the nominal price of a share of S&P stock.** That is, in the steady state, $D^{SP} = 0.1S^{SP}$. Further suppose that in the steady-state, the inflation rate of consumer goods prices between one period and the next is always 10 percent (i.e., $\pi = 0.10$). Compute numerically the **steady-state rate at which the nominal price of *each* type of stock grows every period** (i.e., what you're being asked to compute is the "inflation" or "appreciation" rates of each of the two types of stock). Justify your answer with any appropriate combination of mathematical, graphical, or qualitative arguments. **Also provide brief economic rationale/intuition for your findings.**

Solution: Recall that we can express things in terms of the MRS between period-t and period-t+1 consumption. Doing this using **each type** of stock, we have

$$\frac{u'(c_t)}{u'(c_{t+1})} = \frac{\beta(S_{t+1}^{DOW} + D_{t+1}^{DOW})}{S_t^{DOW}} \cdot \frac{P_t}{P_{t+1}}$$

and

$$\frac{u'(c_t)}{u'(c_{t+1})} = \frac{\beta(S_{t+1}^{SP} + D_{t+1}^{SP})}{S_t^{SP}} \cdot \frac{P_t}{P_{t+1}}.$$

The reason that we have two alternative ways of expressing the consumption-savings optimality condition here is simply because we are considering two alternative assets. With the assumption that $\beta = 1$, $D_{t+1}^{DOW} = 0$, $D_{t+1}^{SP} = 0.1S_{t+1}^{SP}$, and the given information $\frac{P_t}{P_{t+1}} = \frac{1}{1+\pi_{t+1}} = \frac{1}{1.1}$, we can

write the above two expressions as

$$\frac{u'(c_t)}{u'(c_{t+1})} = \frac{S_{t+1}^{DOW}}{S_t^{DOW}} \cdot \frac{1}{1.1}$$

and

$$\frac{u'(c_t)}{u'(c_{t+1})} = \frac{1.1S_{t+1}^{SP}}{S_t^{SP}} \cdot \frac{1}{1.1}.$$

Next, recall that a steady state means that consumption becomes constant from one period to the next. If consumption is constant from one period to the next, clearly marginal utility of consumption becomes constant from one period to the next as well, meaning the left-hand-side of the last two expressions equals one in steady state:

$$1 = \frac{S_{t+1}^{DOW}}{S_t^{DOW}} \cdot \frac{1}{1.1}$$

and

$$1 = \frac{1.1S_{t+1}^{SP}}{S_t^{SP}} \cdot \frac{1}{1.1}.$$

From these two steady-state expressions, it is clear how Dow prices and S&P prices are changing over time: from the latter expression, clearly S&P prices are **not** changing over time, while from the former expression, Dow prices are rising at a rate of 10 percent, the same as the rate of consumer price inflation.

The intuition behind these results is as follows. No matter which we way we measure the "real interest rate" (whether using Dow returns or S&P returns), they must both must be equal to the consumer's MRS. The Dow stock pays no dividend, hence its entire return must come through changes in the price of the stock itself – i.e., there are capital gains on the Dow stock. In contrast, because S&P stocks do pay a dividend, the required capital gains on S&P stock are lower. With the particular numerical values given, the required capital gain on S&P stock turn out to be zero.