Steady-State (Long-Run) of Infinite-Period Economy: Why are Interest Rates Positive?

March 4, 2009

A Long-Run Theory of Macro

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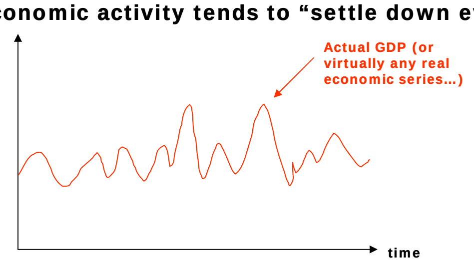

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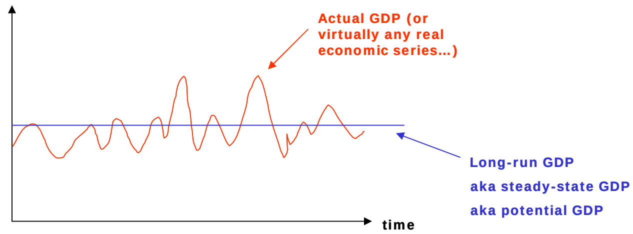

  - The "ups and downs" are business cycles
  - The "average" is the long-run
    - Technical terminology: steady-state
  - Business-cycle theory after midterm exam

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KEY RELATIONSHIP

Inverse of subjective discount factor (one plus) real interest rate

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  - (Chapter 3 & 4: ***r*** was the price of period-1 consumption in terms of period-2 consumption)

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Macro Fundamentals

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- Which came first, β or *r*?
  - Modern macro view: $\beta < 1$ causes $r > 0$, not the other way around
  - A deep view of ***why*** positive interest rates exist in the world