

# FIRMS IN THE TWO-PERIOD FRAMEWORK

MARCH 23, 2009

*Where Things Are Going*

## THE PLAN

- ❑ **First half of course: basic theory**
  - ❑ Consumption-leisure framework
  - ❑ Consumption-savings framework
  - ❑ Infinite-period framework
- ❑ **Second half of course**
  - ❑ **Round out details of the basic theory**
    - ❑ Firms (Chapter 6)
    - ❑ Shocks (Chapter 9)
    - ❑ Putting together consumption-leisure with consumption-savings (Chapter 5)

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    - ❑ How/why did macro theory and practice evolve the way it has?
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Chapter 11-13  
and  
supplementary  
readings

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  - ❑ **Monetary economics**
    - ❑ Monetary policy: the neutrality/non-neutrality debate (Chapter 14)
    - ❑ Interactions between monetary policy and fiscal policy (Chapter 15)
  - ❑ **Link between financial markets and the macroeconomy ("accelerator")**

Chapter 11-13  
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# FIRMS IN THE TWO-PERIOD FRAMEWORK

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*Introduction*

## BASICS

- Embed firms in two-period (multi-period) economy
- In each period  $t$ , representative firm produces according to a production technology  $A_t f(k_t, n_t)$ 
  - $n_t$ : labor used for production
  - $k_t$ : capital ("machines and equipment") used for production
  - $A_t$ : total factor productivity
    - A catch-all measure for level of sophistication of technology
    - Real Business Cycle (RBC) view: the driving force behind the periodic ups and downs of macroeconomic activity (Chapter 13)

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- ❑ **Broad macro view of the factors of production**
  - ❑ **Labor** – all types
  - ❑ **Capital**
    - ❑ Machines and equipment
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Can also think of education and other intangibles (i.e., experience, brand name) as "capital"

The function  $f(k, n)$  describes how capital and labor combine with each other to yield output (goods)

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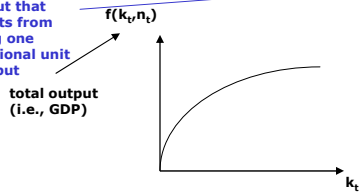
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## PRODUCTION FUNCTION

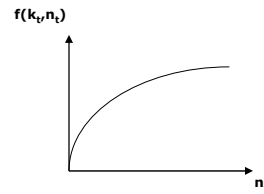
- Production function  $f(k_t, n_t)$  with all the “usual properties” of production functions
  - Strictly increasing in  $k_t$  and  $n_t$
  - Diminishing marginal product in  $k_t$  and  $n_t$

Recall from intro micro

The extra output that results from using one additional unit of input



total output (i.e., GDP)



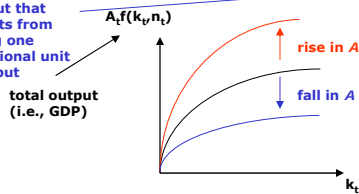
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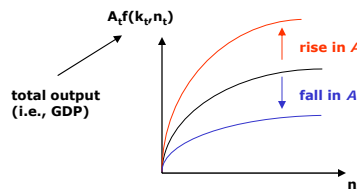
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  - Source of business cycle fluctuations in RBC theory

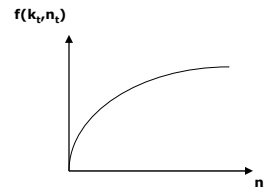
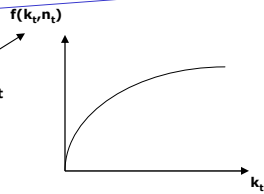
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## CAPITAL AND INVESTMENT

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  - ❑ A technical term
  - ❑ Does *not* refer to consumers' purchase of stocks, bonds, etc

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  - ❑ Analogous to consumers' *savings*
- ❑ Capital: a stock variable
  - ❑ Analogous to consumers' *wealth/asset position*
  - ❑ Except  $k$  cannot be negative (negative machines?...)

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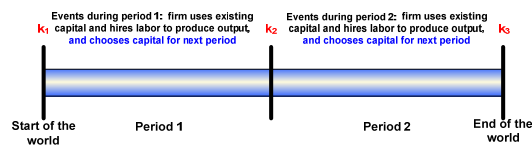
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- ❑ One of the components of GDP ( $= C + I + G + NX$ )
  - ❑ Investment comprises  $\approx 15\%$  of GDP in U.S.
  - ❑ Investment comprises  $\approx 40\%$  of GDP in China (high  $I$  drives rapid growth)

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## BASICS

- ❑ Timeline of events



- ❑ Notation

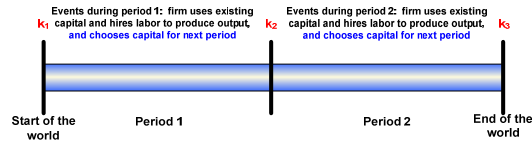
- ❑  $k_1$ : capital used for production in period 1 (decided upon in "period 0")
- ❑  $n_1$ : labor used for production in period 1
- ❑  $w_1$ : real wage rate for labor in period 1 ( $w_1 = W_1/P_1$ )
- ❑  $i$ : nominal interest rate
- ❑  $P_1$ : nominal price of output produced and sold by firm in period 1

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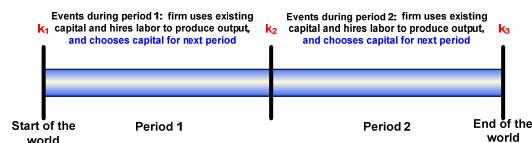
Underlying assumption/view of world: *capital goods* are not necessarily "distinct" from *consumption goods* (i.e., computers purchased by both firms and individual consumers)

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## BASICS

### Timeline of events



### Notation

- $k_2$ : capital used for production in period 2 (decided upon in period 1)
- $n_2$ : labor used for production in period 2
- $w_2$ : real wage rate for labor in period 2 ( $w_2 = W_2/P_2$ )
- $i$ : nominal interest rate
- $P_2$ : nominal price of output produced and sold by firm in period 2  
**AND nominal price of one unit capital bought by the firm in period 2 for use in period 3** (recall time to build...)

Underlying assumption/view of world: *capital goods* are not necessarily "distinct" from *consumption goods* (i.e., computers purchased by both firms and individual consumers)

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## FIRM PROFIT MAXIMIZATION

- A *dynamic* profit maximization problem
  - Because firm exists for both periods
  - All analysis conducted from the perspective of the very beginning of period 1
  - → Must consider present-discounted-value (PDV) of lifetime (i.e., two-period) profits

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  - (specified in nominal terms – could specify in real terms...)

Period-1 profits

$$P_1 f(k_1, n_1) + P_1 k_1 - P_1 w_1 n_1 - P_1 k_2$$

⏟

Total revenue in period 1 (price x output)

⏟

Value of pre-existing capital (an asset for firms)

⏟

Total labor cost in period 1

⏟

Total cost of buying capital for period 2 (time to build → must purchase period-2 capital in period 1)

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- **Two-period model:  $k_3 = 0$  (no machines needed in "period 3")**

## FIRM PROFIT MAXIMIZATION

$$P_1 f(k_1, n_1) + P_1 k_1 - P_1 w_1 n_1 - P_1 k_2 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} \stackrel{=0}{}$$

- FOCs with respect to  $n_1, n_2, k_2$

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Identical except for time subscripts	→ with respect to $n_1$ :	<del><math>P_1 f_n(k_1, n_1) - P_1 w_1 = 0</math></del>	Equation 1
	→ with respect to $n_2$ :	<del><math>\frac{P_2 f_n(k_2, n_2)}{1+i} - \frac{P_2 w_2}{1+i} = 0</math></del>	Equation 2
	with respect to $k_2$ :	<del><math>-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} = 0</math></del>	Equation 3

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□ **Marginal product of labor**

- $f_n(k_t, n_t)$
- Sometimes denote by  $mpn_t$

□ **Marginal product of capital**

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These FOCs are foundation for:  
 1. Labor Demand  
 2. Capital/Investment Demand

## COBB-DOUGLAS PRODUCTION FUNCTION

- A commonly-used functional form in modern quantitative macroeconomic models

$$f(k_t, n_t) = k_t^\alpha n_t^{1-\alpha}$$

(saw Cobb-Douglas utility function on Practice Problem Set 1)

- Describes the empirical relationship between aggregate GDP, aggregate capital, and aggregate labor quite well

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  - **Estimates for Chinese economy:**  $\alpha \approx 0.15$  (not (yet) a very capital-rich economy)

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- **Cobb-Douglas form useful for illustrating factor demands**
  - $mpn_t = f_n(k_t, n_t) = (1-\alpha)k_t^\alpha n_t^{-\alpha}$
  - $mpk_t = f_k(k_t, n_t) = \alpha k_t^{\alpha-1} n_t^{1-\alpha}$

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