FIRMS IN THE TWO-PERIOD FRAMEWORK (CONTINUED)

MARCH 25, 2009

Model Structure

FIRM PROFIT MAXIMIZATION

- A ***dynamic*** profit maximization problem
  - Because firm exists for both periods
  - All analysis conducted from the perspective of the very beginning of period 1
  - → Must consider present-discounted-value (PDV) of lifetime (i.e., two-period) profits
- Dynamic profit function
  - (specified in nominal terms – could specify in real terms…)

Period-1 profits: first four terms. (PDV of) period-2 profits: last four terms.

$$P_1 f(k_1,n_1) + P_1 k_1 - P_1 w_1 n_1 - P_1 k_2 + \frac{P_2 f(k_2,n_2)}{1+i} + \frac{P_2 k_2}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i}$$

k_3 = 0

- $P_1 f(k_1,n_1)$: Total revenue in period 1 (price x output)
- $P_1 k_1$: Value of pre-existing capital (an asset for firms)
- $P_1 w_1 n_1$: Total labor cost in period 1
- $P_1 k_2$: Total cost of buying capital for period 2 (time to build → must purchase period-2 capital in period 1)
- $\frac{P_2 f(k_2,n_2)}{1+i}$: Total revenue in period 2 (price x output)
- $\frac{P_2 k_2}{1+i}$: Value of pre-existing capital (an asset for firms)
- $\frac{P_2 w_2 n_2}{1+i}$: Total labor cost in period 2
- $\frac{P_2 k_3}{1+i}$: Total cost of buying capital for period 3 (time to build → must purchase period-3 capital in period 2)

- Two-period model: k_3 = 0 (no machines needed in "period 3")

Firm Profit Maximization

$$P_1 f(k_1,n_1)+P_1k_1-P_1w_1n_1-P_1k_2+\frac{P_2 f(k_2,n_2)}{1+i}+\frac{P_2k_2}{1+i}-\frac{P_2w_2n_2}{1+i}-\frac{P_2k_3}{1+i}$$

= 0

- **FOCs with respect to n_1, n_2, k_2**

Identical except for time subscripts

with respect to n_1: $P_1 f_n(k_1,n_1)-P_1w_1=0$ Equation 1

with respect to n_2: $\frac{P_2 f_n(k_2,n_2)}{1+i}-\frac{P_2w_2}{1+i}=0$ Equation 2

with respect to k_2: $-P_1+\frac{P_2 f_k(k_2,n_2)}{1+i}+\frac{P_2}{1+i}=0$ Equation 3

- **Marginal product of labor**
  - $f_n(k_t,n_t)$
  - Sometimes denote by mpn_t
- **Marginal product of capital**
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These FOCs are foundation for:
1. Labor Demand
2. Capital / Investment Demand

COBB-DOUGLAS PRODUCTION FUNCTION

- **A commonly-used functional form in modern quantitative macroeconomic models**

$$f(k_t, n_t) = k_t^{\alpha} n_t^{1-\alpha}$$

(saw Cobb-Douglas utility function on Practice Problem Set 1)

- **Describes the empirical relationship between aggregate GDP, aggregate capital, and aggregate labor quite well**

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- $\alpha \in (0,1)$ **measures capital's share of output**
  - **Hence** $(1-\alpha) \in (0,1)$ **measures labor's share of output**
  - **Interpretation**
    - **The relative importance of (either) capital (or labor) in the production process**

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  - Estimates for Chinese economy: $\alpha \approx 0.15$ (not (yet) a very capital-rich economy)
- Cobb-Douglas form useful for illustrating factor demands
  - $mpn_t = f_n(k_t, n_t) = (1-\alpha)k_t^{\alpha} n_t^{-\alpha}$
  - $mpk_t = f_k(k_t, n_t) = \alpha k_t^{\alpha-1} n_t^{1-\alpha}$

MICRO-LEVEL LABOR DEMAND

- **Firm-level demand for labor defined by the relation**

Follows from Equation 1 and Equation 2 $w_t = (1-\alpha)k_t^{\alpha}n_t^{-\alpha}(= mpn_t)$ for both t = 1 and t = 2

Labor Demand in the Micro

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Because exponent ($-\alpha$) is a negative number, can move to denominator

$$w_t = (1-\alpha)\left(\frac{k_t}{n_t}\right)^{\alpha}$$

A NEGATIVE RELATIONSHIP BETWEEN w_t and n_t

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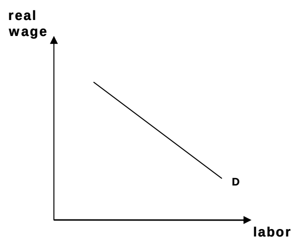

Labor Demand in the Micro and the Macro

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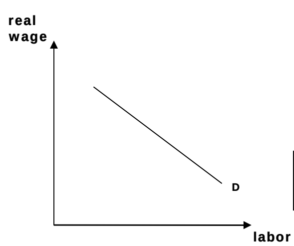
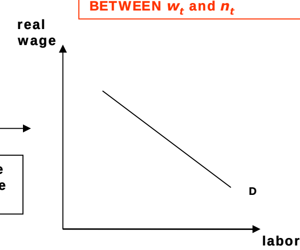


Firm-level labor demand function

Sum over all firms →

(No tension between the micro and macro as there is for labor supply)



Aggregate-level labor demand function

- Completes picture of the aggregate labor market

MICRO-LEVEL CAPITAL DEMAND

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Capital Demand in the Micro

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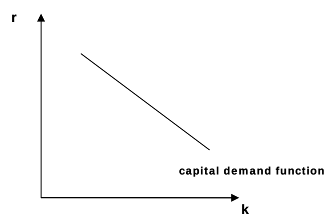

CAPITAL DEMAND

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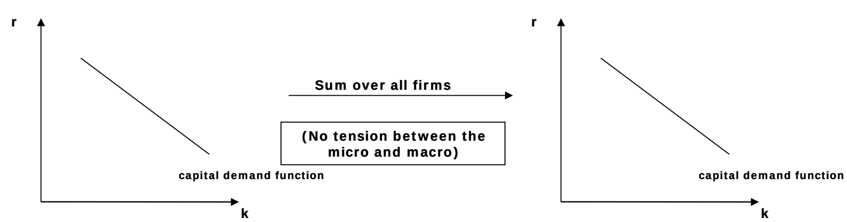


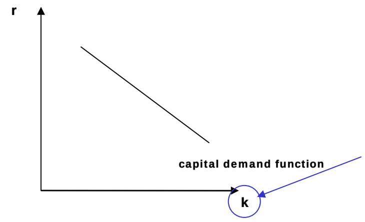
Firm-level capital demand function **Aggregate-level capital demand function**

- **(Almost…) completes picture of the aggregate capital market**

Investment Demand

FROM CAPITAL DEMAND TO INVESTMENT DEMAND

- Capital is a stock variable



Want investment (a flow) to show up here, not capital (a stock)

Investment is ***change*** in capital stock between consecutive periods

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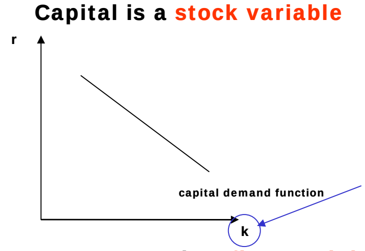

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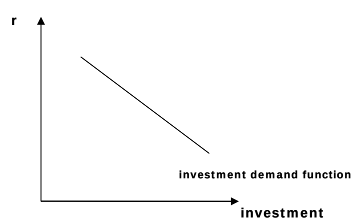
$inv_1 = k_2 - k_1$

At start of period 1, k_1 cannot be changed. Thus any rise in demand for k_2 is reflected one-for-one in a rise in inv_1.

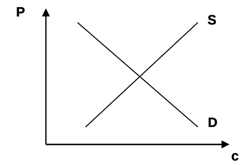
→ Capital demand and investment demand functions have same shape

- Investment is a flow variable

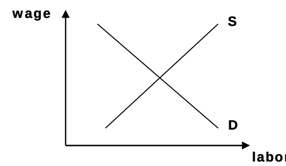


THE THREE MACRO (AGGREGATE) MARKETS

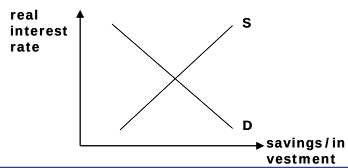
- **Goods Markets**
  - Demand derived from C-L framework
    (Supply follows from factor demand functions)



- **Labor Markets**
  - Supply derived from C-L framework
  - Demand derived from firm theory in C-S framework



- **Capital / Savings / Funds / Asset Markets (aka Financial Markets)**
  - Supply derived from C-S framework
    Demand derived from firm theory in C-S framework



REAL INTEREST RATE

- *r* a key variable for macroeconomic analysis
- Chapter 4: *r* measures the price of period-1 consumption in terms of period-2 consumption
- Chapter 8: *r* reflects degree of impatience
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$$-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} = 0$$

Equation 3 (FOC on k_2)

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$$f_k(k_2, n_2) = r$$

When firms make optimal investment decisions → *r* = *mpk*