MONEY IN THE INFINITE-PERIOD ECONOMY

APRIL 13, 2009

BASICS

- Extend our infinite-period framework
  - Introduce money and bonds into the Chapter 8 framework
  - So now three types of assets (stocks, bonds, money) for representative consumer to use for savings purposes

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- Will allow us to think about connection between bond prices and stock prices
- Will allow us to think about issue of monetary neutrality (the main issue in the RBC vs. New Keynesian debate)
  - i.e., does money (and thus monetary policy) have important consequences for ***real*** (i.e., consumption and real GDP) variables?

Introduction

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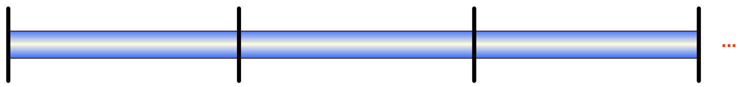
- Index time periods by arbitrary indexes t, $t+1$, $t+2$, etc.
  - Important: all of our analysis will be conducted from the perspective of the very beginning of period t…
- Sequential Lagrangian analysis (with money in the utility function)

BASICS

- Timeline of events



- Notation
  - c_t: consumption in period t
  - P_t: nominal price of consumption in period t
  - Y_t: nominal income in period t ("falls from the sky")
  - a_{t-1}: real stock holdings at beginning of period t / end of period t-1

a_{t-1} Economic events during period t: income, consumption, savings a_t Economic period t consump

Period t Peri

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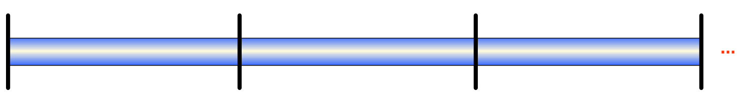

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Now *three* types of assets consumers can use for savings purposes

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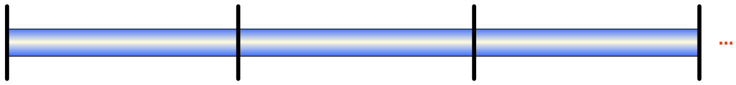

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  - S_t: nominal price of a unit of stock in period t
  - D_t: nominal dividend paid in period t by each unit of stock held at the start of t
  - P^b_t: nominal price of a bond in period t
  - i_t: nominal interest rate on a bond purchased in t and which pays off in t+1

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a_{t-1} B_{t-1} M_{t-1}

Economic events during period t: income, consumption, savings

a_t B_t M_t

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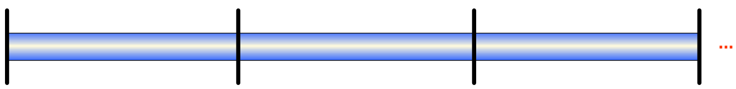

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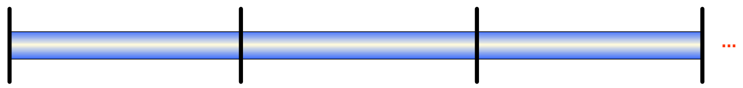

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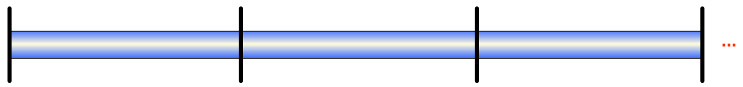

- Notation
  - And so on for period $t+2$, $t+3$, etc…

BUDGET CONSTRAINT(S)

- **Extend budget constraints from Chapter 8 stock-pricing framework to now include the three distinct types of assets: stocks, money, and bonds**
- **Need infinite budget constraints to describe economic opportunities and possibilities**
  - **One for each period**

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$$\underbrace{P_t c_t + P_t^b B_t + M_t + S_t a_t}_{} = \underbrace{Y_t + M_{t-1} + B_{t-1} + S_t a_{t-1} + D_t a_{t-1}}_{}$$

Total outlays in period *t*: period-*t* consumption + stocks to *carry into period t*+1 + money to *carry into period t*+1 + bond purchases

Total income in period *t*: period-*t* *Y* + income from stock-holdings *carried into period t* (has value S_t and pays dividend D_t) + money-holdings *carried into period t* + bond-holdings *carried into period t* (each unit repays *FV* = 1)

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Total outlays in period *t*+1: period-*t*+1 consumption + stocks to *carry into period t*+2 + money to *carry into period t*+2 + bond purchases

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Model Structure

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  - And identical-looking budget constraints in period *t*+2, *t*+3, *t*+4, etc.

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Step 1:** **Construct Lagrange function (starting from *t*)**

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$$u(c_t, M_t / P_t) + \beta u(c_{t+1}, M_{t+1} / P_{t+1}) + \beta^2 u(c_{t+2}, M_{t+2} / P_{t+2}) + \ldots$$

First the lifetime utility function….(no different than Chapter 8)

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$$+\lambda_t \left[Y_t + (S_t + D_t)a_{t-1} + M_{t-1} + B_{t-1} - P_t c_t - S_t a_t - M_t - P_t^b B_t \right]$$

...then the period *t* constraint...

Infinite-Period Model: Sequential Formulation

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...then the period *t* constraint...

$$+\beta\lambda_{t+1} \left[Y_{t+1} + (S_{t+1} + D_{t+1})a_t + M_t + B_t - P_{t+1}c_{t+1} - S_{t+1}a_{t+1} - M_{t+1} - P_{t+1}^b B_{t+1} \right]$$

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$$+\beta^2\lambda_{t+2}\left[Y_{t+2} + (S_{t+2} + D_{t+2})a_{t+1} + M_{t+1} + B_{t+1} - P_{t+2}c_{t+2} - S_{t+2}a_{t+2} - M_{t+2} - P_{t+2}^b B_{t+2}\right]$$

…then the period *t*+2 constraint…

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$$+\ldots$$
Infinite number of terms

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- **Step 2: Compute FOCs with respect to c_t, a_t, B_t, M_t, …**

  with respect to c_t:

  with respect to a_t:

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with respect to c_t: $u_1(c_t, M_t/P_t) - \lambda_t P_t = 0$ Equation 1

with respect to a_t: $-\lambda_t S_t + \beta\lambda_{t+1}(S_{t+1} + D_{t+1}) = 0$ Equation 2

with respect to B_t: $-\lambda_t P_t^b + \beta\lambda_{t+1} = 0$ Equation 3

with respect to M_t: $\dfrac{u_2(c_t, M_t/P_t)}{P_t} - \lambda_t + \beta\lambda_{t+1} = 0$ Equation 4 (need chain rule of calculus to derive this)

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- **Equation 2 →** $S_t = \left(\dfrac{\beta\lambda_{t+1}}{\lambda_t}\right)(S_{t+1} + D_{t+1})$ STOCK-PRICING EQUATION

Period-*t* stock price = Pricing kernel x Future return

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- **Much of finance theory concerned with pricing kernel**
  - **Theoretical properties**
  - **Empirical models of kernels**
- **Pricing kernel where macro theory and finance theory intersect**

Finance Fundamentals

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- **Equation 3 →** $P_t^b = \frac{\beta\lambda_{t+1}}{\lambda_t}$ BOND-PRICING EQUATION

ASSET PRICING REVISITED

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$$-\lambda_t S_t + \beta\lambda_{t+1}(S_{t+1} + D_{t+1}) = 0$$ Equation 2

$$-\lambda_t P_t^b + \beta\lambda_{t+1} = 0$$ Equation 3

$$\frac{u_2(c_t, M_t / P_t)}{P_t} - \lambda_t + \beta\lambda_{t+1} = 0$$ Equation 4

- **Equation 2 →** $S_t = \left(\frac{\beta\lambda_{t+1}}{\lambda_t}\right)(S_{t+1} + D_{t+1})$ STOCK-PRICING EQUATION

  Period-*t* stock price = Pricing kernel x Future return

- **Equation 3 →** $P_t^b = \frac{\beta\lambda_{t+1}}{\lambda_t}$ BOND-PRICING EQUATION

- **Price of a bond *is* the pricing kernel**
  - **Stock prices and bond prices are connected**
  - **Most (all?) asset prices fundamentally connected to bond prices**
  - **(Advanced finance course: pricing kernel reflects the price/return of the least risky asset in the economy – U.S. Treasury bonds)**

Finance Fundamentals

ASSET PRICING REVISITED

$$u_1(c_t, M_t / P_t) - \lambda_t P_t = 0$$ Equation 1

$$-\lambda_t S_t + \beta\lambda_{t+1}(S_{t+1} + D_{t+1}) = 0$$ Equation 2

$$-\lambda_t P_t^b + \beta\lambda_{t+1} = 0$$ Equation 3

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- **Equation 2 →** $S_t = \left(\frac{\beta\lambda_{t+1}}{\lambda_t}\right)(S_{t+1} + D_{t+1})$ STOCK-PRICING EQUATION

  Period-*t* stock price = Pricing kernel x Future return

- **Equation 3 →** $P_t^b = \frac{\beta\lambda_{t+1}}{\lambda_t}$ BOND-PRICING EQUATION

- **Recall** $P_t^b = \frac{1}{1+i_t}$

- **→ can express pricing kernel as** $\frac{\beta\lambda_{t+1}}{\lambda_t} = \frac{1}{1+i_t}$

FISHER EQUATION

$$u_1(c_t, M_t / P_t) - \lambda_t P_t = 0$$ Equation 1

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- **Combining stock-pricing equation with bond-pricing equation →**

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}}$$ FISHER EQUATION

Macro Fundamentals

FISHER EQUATION

$$u_1(c_t, M_t / P_t) - \lambda_t P_t = 0$$ Equation 1

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- **Combining stock-pricing equation with bond-pricing equation →**

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}}$$ FISHER EQUATION

  - **Fisher equation a relationship between returns on nominal bonds and returns on stock (finance theory: "no-arbitrage" condition)**
  - **(See derivation in Chapter 14)**

- **Was a building block of two-period model**

- **Recall approximate form: $r \approx i - \pi$**

CONSUMPTION-MONEY OPTIMALITY CONDITION

Begin with equation 4:

$$\frac{u_2(c_t, M_t/P_t)}{P_t} - \lambda_t = -\beta\lambda_{t+1}$$

Use $\beta\lambda_{t+1} = \lambda_t P^b_t$ from equation 2

$$\frac{u_2(c_t, M_t/P_t)}{P_t} - \lambda_t = -\lambda_t P_t^b$$

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Divide through by λ_t

$$\frac{u_2(c_t, M_t/P_t)}{\lambda_t P_t} - 1 = -P_t^b$$

Consumption-Money Optimality Condition

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Divide through by λ_t

$$\frac{u_2(c_t, M_t / P_t)}{\lambda_t P_t} - 1 = -P^b_t$$

Use $\lambda_t P_t = u_{1t}$ from equation 1

$$\frac{u_2(c_t, M_t / P_t)}{u_1(c_t, M_t / P_t)} = 1 - P^b_t$$

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$$\frac{u_2(c_t, M_t / P_t)}{u_1(c_t, M_t / P_t)} = 1 - P^b_t$$

Use $P^b_t = 1 / (1 + i_t)$

$$\frac{u_2(c_t, M_t / P_t)}{u_1(c_t, M_t / P_t)} = \frac{i_t}{1 + i_t}$$

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$$\frac{u_2(c_t, M_t/P_t)}{P_t} - \lambda_t = -\lambda_t P^b_t$$

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↓ Use $P^b_t = 1/(1+i_t)$

CONSUMPTION-MONEY OPTIMALITY CONDITION

$$\underbrace{\frac{u_2(c_t, M_t/P_t)}{u_1(c_t, M_t/P_t)}}_{\text{MRS}} = \underbrace{\frac{i_t}{1+i_t}}_{\text{price ratio}}$$

MRS (between consumption and ***real*** money holdings)

price ratio (between consumption and money)

Money Demand

- **Consumption-money optimality condition the foundation of money demand function**
- **Example: suppose** $u\left(c_t, \frac{M_t}{P_t}\right) = \ln c_t + \ln\left(\frac{M_t}{P_t}\right)$

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- Consumption-money optimality condition (for this utility function…) is

$$\frac{P_t c_t}{M_t} = \frac{i_t}{1+i_t}$$

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Isolate the M_t / P_t term

$$\frac{M_t}{P_t} = c_t \cdot \left(\frac{1+i_t}{i_t}\right)$$

REAL MONEY DEMAND FUNCTION: depends positively on c_t and negatively on i_t (i_t is the opportunity cost of money)

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REAL MONEY DEMAND FUNCTION: depends positively on c_t and negatively on i_t (i_t is the opportunity cost of money)

- Will use this money demand function to analyze
  - The monetary neutrality debate
  - The long-run (aka steady-state) connection between monetary policy and inflation