

# THE FINANCIAL ACCELERATOR: FINANCIAL MARKETS AND THE MACROECONOMY

APRIL 27, 2009

*Introduction*

## FINANCIAL ACCELERATOR

- ❑ **“Financial accelerator” framework**
  - ❑ The most widely-used and applied framework in macroeconomic theory and policy for thinking about financial markets
  - ❑ Developed in series of studies by Bernanke, Gertler, and Gilchrist in late 1980’s and early 1990’s
- ❑ **Popular-press language**
  - ❑ “Financial accelerator”
  - ❑ “Financial feedback loops”
  - ❑ “Loan spirals”
- ❑ **Describes well many of the financial-macroeconomic linkages underpinning the dynamics of**
  - ❑ **The Great Depression**
  - ❑ **Current macroeconomic conditions**

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  - ❑ The Great Depression
  - ❑ Current macroeconomic conditions
- ❑ Will develop idea in context of firm theory (Chapter 6)
- ❑ Can also develop idea in context of consumer theory (Chapter 3, Chapter 4, Chapter 8)
  - ❑ Recall “credit constraint” analysis of consumption/savings decisions (Problem Set 1)

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## OUTLINE OF FRAMEWORK

### Major ideas underlying Financial Accelerator Framework

1. Firms’ **financial** assets (i.e., stocks and bonds) matter for their ability to purchase **physical** assets (i.e., machines and equipment)
2. Market **prices** of financial assets matter for **firm financing constraints**
3. Government regulation affects the linkage between financial markets and real (i.e., goods and physical capital) markets through financing constraints

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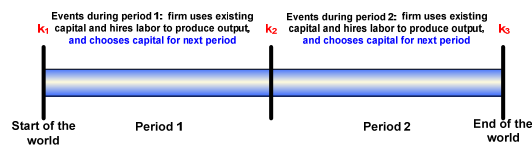
## OUTLINE OF FRAMEWORK

### Four Building Blocks of the Financial Accelerator Framework

1. **Two-Period Model of Firms**
  - Based on Chapter 6
  - Enriched to allow for both **physical** assets (machines and equipment) and **financial** assets (stocks and bonds)
2. **Financing Constraint**
  - Quantity of **physical** capital firms can purchase depends on the market value of their **financial** assets
  - Reflects market and regulatory structures designed to mitigate **informational asymmetries**
  - (Recall basic Chapter 6 theory of firms featured no constraints on firm profit maximization)
3. **Government Regulation/Oversight of Financial Relationships**
4. **Relationship between Firm Profits and Dividends**

## BASIC FIRM THEORY

### □ Timeline of events

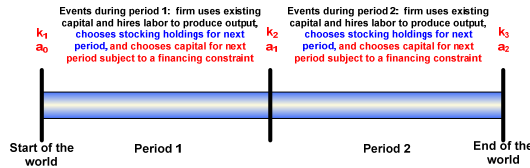


### □ Notation

- $k_1$ : capital used for production in period 1 (decided upon in "period 0")
- $n_1$ : labor used for production in period 1
- $w_1$ : real wage rate for labor in period 1 ( $w_1 = W_1/P_1$ )
- $i$ : nominal interest rate (between period 1 and period 2)
- $P_1$ : nominal price of output produced and sold by firm in period 1 AND nominal price of one unit of capital bought by the firm in period 1 for use in period 2

## ENRICHING THE BASIC FIRM THEORY

### Timeline of events



### Notation

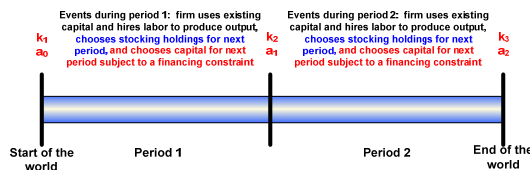
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  - $a_0$ : real wealth (stock) holdings at beginning of period 1/end of period 0
  - $S_1$ : nominal price of a unit of stock in period 1
  - $D_1$ : nominal dividend paid in period 1 by each unit of stock held at the start of period 1
- The "defining features" of stock

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## ENRICHING THE BASIC FIRM THEORY

### Timeline of events



### Notation

- $k_2$ : capital used for production in period 2 (decided upon in period 1)
  - $n_2$ : labor used for production in period 2
  - $w_2$ : real wage rate for labor in period 2 ( $w_2 = W_2/P_2$ )
  - $i$ : nominal interest rate (between period 1 and period 2)
  - $P_2$ : nominal price of output produced and sold by firm in period 2 AND nominal price of one unit of capital bought by the firm in period 2 for use in period 3
  - $a_1$ : real wealth (stock) holdings at beginning of period 2/end of period 1
  - $S_2$ : nominal price of a unit of stock in period 2
  - $D_2$ : nominal dividend paid in period 2 by each unit of stock held at the start of period 2
  - $\pi_2$ : net inflation rate between period 1 and period 2 (recall:  $\pi_2 = P_2/P_1 - 1$ )
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## RATES OF RETURN

- "Interest rates" can be defined for any type of asset
  - There is no single interest rate in the economy
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$$1+i^{STOCK} = \frac{S_2 + D_2}{S_1}$$
    - Measures the net dollar return (in period 2) on one share of stock (whose purchase price was  $S_1$  in period 1)

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REAL INTEREST RATE ON GOVERNMENT BONDS: A "SAFE" ASSET
      - Thus can think of bonds (one type of financial asset) as being in the background of the analysis
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REAL INTEREST RATE ON STOCKS: A "RISKY" ASSET
      - Measures the net dollar return (in period 2) on one share of stock (whose purchase price was  $S_1$  in period 1)
- Can distinguish two measures of **real** interest rates in this framework

## FIRM PROFIT FUNCTION

- A **dynamic** profit maximization problem
  - Because firm exists for both periods
  - All analysis conducted from the perspective of the very beginning of period 1
  - → Must consider present-discounted-value (PDV) of lifetime (i.e., two-period) profits

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- **Dynamic profit function**
  - (specified in nominal terms – could specify in real terms...)

Period-1 profits

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1) a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1$$

Total revenue  
in period 1  
(price x  
output)

Value of  
pre-existing  
physical  
capital (an  
asset for  
firms)

Total labor  
cost in  
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Total cost of  
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Total revenue in period 1 (price x output)  
 Value of pre-existing physical capital (an asset for firms)  
 Total labor cost in period 1  
 Total cost of buying physical capital for period 2 (time to build → must purchase period-2 capital in period 1)  
 Value (inclusive of dividends) of pre-existing financial assets (i.e., stock-holdings in other firms)  
 Total cost of buying financial assets (i.e., stock-holdings in other firms) for period 2

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Period-1 profits (PDV of) period-2 profits

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 Total revenue in period 2 (price x output)  
 Value of pre-existing physical capital (an asset for firms)  
 Total labor cost in period 2  
 Total cost of buying physical capital for period 3 (time to build → must purchase period-3 capital in period 2)  
 Value (inclusive of dividends) of pre-existing financial assets (i.e., stock-holdings in other firms)  
 Total cost of buying financial assets (i.e., stock-holdings in other firms) for period 3

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(PDV of) period-2 profits = 0 = 0

As usual: no physical or financial assets needed for "period 3"

## INFORMATIONAL ASYMMETRIES

- "Informational asymmetries" pervasive in borrowing/lending relationships
- Borrower (whether consumer or firm) **much** more likely to know his own ability/willingness to repay a loan
  - Lenders often have very little information about the "quality" or "trustworthiness" of a borrower
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  - ❑ **On firm side**
    - ❑ Capital investment (building factories, etc) very expensive!
    - ❑ → firms have need to borrow
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- ❑ **Note:** the basic informational asymmetry **itself** cannot be mitigated...

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## FINANCING CONSTRAINT

- ❑ Capture this idea through a **financing constraint** on firm's ability to purchase capital between period 1 and period 2
- ❑ **Financing constraint**
  - ❑ **Total expenditures on period-1 physical investment must be equal to market value of firm's financial (stock) holdings**
  - ❑ (Technically, smaller than or equal to, so an inequality constraint...but will only analyze constraint with equality)

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- **Important:  $a_1$  appears in the financing constraint, not  $a_0$** 
  - Idea this assumption captures: firm might try to strategically manipulate the value of *financial* assets it holds in order to affect the quantity of *physical* investment it can engage in
  - (From the perspective of beginning of period 1,  $a_1$  has not yet been chosen, whereas  $a_0$  is pre-determined)

## GOVERNMENT OVERSIGHT OF FINANCIAL MARKETS

- **Government oversight of informational asymmetries in borrower/lender relationships**
  - **Filing of proper documentation**
  - **Full disclosure (“truth-in-lending”) laws**
  - **Direct lending in some markets**
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- **Capture government Regulation of financial dealings in our framework in very simple way**
  - **Firm can only borrow up to a fraction  $R$  of the market value of its financial assets for physical investment purposes**
  - **e.g., if government mandates that expenditures on physical investment can only comprise 80% of market value of financial assets,  $R = 0.80$**

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**Impose this financing constraint on firm profit maximization problem**

## FINANCIAL ACCELERATOR FRAMEWORK

□ Four Building Blocks of the Financial Accelerator Framework

1. Firm Profit Function

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1) a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2) a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i}$$

= 0      = 0

2. Financing Constraint

$$P_1 \cdot (k_2 - k_1) = S_1 \cdot a_1$$

3. Government Regulation of Financial Relationships (imposition of **R** on financing constraint)

$$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$$

4. Relationship between firm profits and dividends

NEXT TIME

## FIRM PROFIT MAXIMIZATION

Maximize two-period profits

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1) a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2) a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i}$$

= 0      = 0

Subject to financing constraint

$$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$$

## FIRM PROFIT MAXIMIZATION

Maximize two-period profits

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1) a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2) a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i}$$

Subject to financing constraint

$$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$$

Construct Lagrangian

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1) a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2) a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} + \lambda [R \cdot S_1 \cdot a_1 - P_1 \cdot (k_2 - k_1)]$$

Lagrange multiplier on financing constraint

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$\lambda = 0$  i.e., there was no financing constraint!

**NEXT TIME:** will think about what regulatory and/or market features make the financing constraint effectively "disappear" (i.e., cause  $\lambda = 0$ )

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□ FOCs with respect to  $n_1, n_2$

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□ **FOCs with respect to  $n_1, n_2$**

Identical except for time subscripts

→ with respect to  $n_1$ :  $\cancel{P_1} f_n(k_1, n_1) - \cancel{P_1} w_1 = 0$  Equation 1

→ with respect to  $n_2$ :  $\frac{\cancel{P_2} f_n(k_2, n_2)}{1+i} - \frac{\cancel{P_2} w_2}{1+i} = 0$  Equation 2

- Financing constraint does not affect profit-maximizing choices of labor hiring...
- ...thus same analysis from Chapter 6 of labor demand curve, etc, applies

□ **FOCs with respect to  $k_2, a_1$**

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- The interesting aspects of this framework
- Next: when do financing constraints matter?
- Next: the financial accelerator framework in action