THE FINANCIAL ACCELERATOR (CONTINUED)

APRIL 29, 2009

OUTLINE OF FRAMEWORK

Major ideas underlying Financial Accelerator Framework

1. **Firms' financial assets (i.e., stocks and bonds) matter for their ability to purchase physical assets (i.e., machines and equipment)**

2. **Market prices of financial assets matter for firm financing constraints**

3. **Government regulation affects the linkage between financial markets and real (i.e., goods and physical capital) markets *through* financing constraints**

Financial Accelerator Framework

- **Four Building Blocks of the Financial Accelerator Framework**

1. Firm Profit Function

$$P_1 f(k_1,n_1)+P_1k_1+(S_1+D_1)a_0-P_1w_1n_1-P_1k_2-S_1a_1+\frac{P_2 f(k_2,n_2)}{1+i}+\frac{P_2k_2}{1+i}+\frac{(S_2+D_2)a_1}{1+i}-\frac{P_2w_2n_2}{1+i}-\overbrace{\frac{P_2k_3}{1+i}}^{=0}-\overbrace{\frac{S_2a_2}{1+i}}^{=0}$$

2. Financing Constraint

$$P_1\cdot(k_2-k_1)=\quad S_1\cdot a_1$$

3. Government Regulation of Financial Relationships (imposition of *R* on financing constraint)

$$P_1\cdot(k_2-k_1)=R\cdot S_1\cdot a_1$$

4. Relationship between firm profits and dividends

NEXT TIME

Firm Profit Maximization

Maximize two-period profits

$$P_1 f(k_1,n_1)+P_1k_1+(S_1+D_1)a_0-P_1w_1n_1-P_1k_2-S_1a_1+\frac{P_2 f(k_2,n_2)}{1+i}+\frac{P_2k_2}{1+i}+\frac{(S_2+D_2)a_1}{1+i}-\frac{P_2w_2n_2}{1+i}-\overbrace{\frac{P_2k_3}{1+i}}^{=0}-\overbrace{\frac{S_2a_2}{1+i}}^{=0}$$

Subject to financing constraint

$$P_1\cdot(k_2-k_1)=R\cdot S_1\cdot a_1$$

Construct Lagrangian

$$P_1 f(k_1,n_1)+P_1k_1+(S_1+D_1)a_0-P_1w_1n_1-P_1k_2-S_1a_1+\frac{P_2 f(k_2,n_2)}{1+i}+\frac{P_2k_2}{1+i}+\frac{(S_2+D_2)a_1}{1+i}-\frac{P_2w_2n_2}{1+i}$$
$$+\lambda\left[R\cdot S_1\cdot a_1-P_1\cdot(k_2-k_1)\right]$$

Lagrange multiplier on financing constraint

CRUCIAL OBSERVATION: in basic firm theory (i.e., Chapter 6), value of this multiplier was….

$\lambda = 0$ i.e., there was no financing constraint!

KEY QUESTION: What regulatory and/or market features make the financing constraint effectively "disappear" (i.e., cause $\lambda = 0$)?

Model Analysis

Firm Profit Maximization

$$P_1 f(k_1,n_1) + P_1 k_1 + (S_1+D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2,n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2+D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i}$$
$$+ \lambda\left[R\cdot S_1\cdot a_1 - P_1\cdot(k_2-k_1)\right]$$

- **FOCs with respect to n_1, n_2**

Identical except for time subscripts

with respect to n_1: $\cancel{P_1} f_n(k_1,n_1) - \cancel{P_1} w_1 = 0$ — **Equation 1**

with respect to n_2: $\frac{\cancel{P_2} f_n(k_2,n_2)}{\cancel{1+i}} - \frac{\cancel{P_2} w_2}{\cancel{1+i}} = 0$ — **Equation 2**

  - **Financing constraint does not affect profit-maximizing choices of labor hiring…**
  - **…thus same analysis from Chapter 6 of labor demand curve, etc, applies**

- **FOCs with respect to k_2, a_1**

Model Analysis

Firm Profit Maximization

$$P_1 f(k_1,n_1) + P_1 k_1 + (S_1+D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2,n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2+D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i}$$
$$+ \lambda\left[R\cdot S_1\cdot a_1 - P_1\cdot(k_2-k_1)\right]$$

- **FOCs with respect to n_1, n_2**

Identical except for time subscripts

with respect to n_1: $\cancel{P_1} f_n(k_1,n_1) - \cancel{P_1} w_1 = 0$ — **Equation 1**

with respect to n_2: $\frac{\cancel{P_2} f_n(k_2,n_2)}{\cancel{1+i}} - \frac{\cancel{P_2} w_2}{\cancel{1+i}} = 0$ — **Equation 2**

  - **Financing constraint does not affect profit-maximizing choices of labor hiring…**
  - **…thus same analysis from Chapter 6 of labor demand curve, etc, applies**

- **FOCs with respect to k_2, a_1**
  - **The interesting aspects of this framework**
  - **The heart of the financial accelerator framework**

Firm Profit Maximization

$$P_1 f(k_1,n_1)+P_1k_1+(S_1+D_1)a_0-P_1w_1n_1-P_1k_2-S_1a_1+\frac{P_2 f(k_2,n_2)}{1+i}+\frac{P_2k_2}{1+i}+\frac{(S_2+D_2)a_1}{1+i}-\frac{P_2w_2n_2}{1+i}$$
$$+\lambda\left[R\cdot S_1\cdot a_1-P_1\cdot(k_2-k_1)\right]$$

- **FOCs with respect to k_2, a_1**

  with respect to k_2:

  with respect to a_1:

Firm Profit Maximization

$$P_1 f(k_1,n_1)+P_1k_1+(S_1+D_1)a_0-P_1w_1n_1-P_1k_2-S_1a_1+\frac{P_2 f(k_2,n_2)}{1+i}+\frac{P_2k_2}{1+i}+\frac{(S_2+D_2)a_1}{1+i}-\frac{P_2w_2n_2}{1+i}$$
$$+\lambda\left[R\cdot S_1\cdot a_1-P_1\cdot(k_2-k_1)\right]$$

- **FOCs with respect to k_2, a_1**

  with respect to k_2: $-P_1+\frac{P_2 f_k(k_2,n_2)}{1+i}+\frac{P_2}{1+i}-\lambda P_1=0$ **Equation 3**

  with respect to a_1: $-S_1+\frac{S_2+D_2}{1+i}+\lambda\cdot R\cdot S_1=0$ **Equation 4**

- **Analysis of Equation 4 in isolation**
  - **Answers the central question: under what conditions does λ = 0?**
  - **Reveals how stock market returns affect financing constraints**
  - **Reveals how government regulation affects financing constraints**
- **Analysis of Equation 3 and Equation 4 jointly**
  - **Demonstrates how/why financial market prices (i.e., stock prices/returns) matter for macroeconomic activity**
  - **The financial accelerator effect**

Why is Financing a *Constraint*?

$$-S_1 + \frac{S_2 + D_2}{1+i} + \lambda \cdot R \cdot S_1 = 0$$

Equation 4

Why is Financing a *Constraint*?

$$-S_1 + \frac{S_2 + D_2}{1+i} + \lambda \cdot R \cdot S_1 = 0$$

Equation 4

Solve for λ

$$\lambda = \left[S_1 - \frac{S_2 + D_2}{1+i} \right] \cdot \frac{1}{R \cdot S_1}$$

WHY IS FINANCING A *CONSTRAINT*?

$$-S_1 + \frac{S_2 + D_2}{1+i} + \lambda \cdot R \cdot S_1 = 0$$

Equation 4

Solve for λ

$$\lambda = \left[S_1 - \frac{S_2 + D_2}{1+i} \right] \cdot \frac{1}{R \cdot S_1}$$

Pull $1/S_1$ term inside

$$\lambda = \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{1}{1+i} \right] \cdot \frac{1}{R}$$

WHY IS FINANCING A *CONSTRAINT*?

$$-S_1 + \frac{S_2 + D_2}{1+i} + \lambda \cdot R \cdot S_1 = 0$$

Equation 4

Solve for λ

$$\lambda = \left[S_1 - \frac{S_2 + D_2}{1+i} \right] \cdot \frac{1}{R \cdot S_1}$$

Pull $1/S_1$ term inside

$$\lambda = \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{1}{1+i} \right] \cdot \frac{1}{R}$$

Multiply ***and*** divide second term in parentheses by P_1 ***and*** P_2

$$\lambda = \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{P_1}{P_2} \cdot \frac{P_2}{P_1} \cdot \frac{1}{1+i} \right] \cdot \frac{1}{R}$$

Model Analysis

WHY IS FINANCING A *CONSTRAINT*?

$$-S_1 + \frac{S_2 + D_2}{1+i} + \lambda \cdot R \cdot S_1 = 0$$

Equation 4

Solve for λ

$$\lambda = \left[S_1 - \frac{S_2 + D_2}{1+i} \right] \cdot \frac{1}{R \cdot S_1}$$

Pull $1/S_1$ term inside

$$\lambda = \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{1}{1+i} \right] \cdot \frac{1}{R}$$

Multiply *and* divide second term in parentheses by P_1 *and* P_2

$$\lambda = \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{P_1}{P_2} \cdot \frac{P_2}{P_1} \cdot \frac{1}{1+i} \right] \cdot \frac{1}{R}$$

Use definition of inflation, $1 + \pi_2 = P_2 / P_1$, and regroup terms

$$\lambda = \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{P_1}{P_2} \cdot \frac{1+\pi_2}{1+i} \right] \cdot \frac{1}{R}$$

Model Analysis

WHY IS FINANCING A *CONSTRAINT*?

$$\lambda = \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{P_1}{P_2} \cdot \frac{1+\pi_2}{1+i} \right] \cdot \frac{1}{R}$$

(from previous page)

Use definition of "nominal interest rate on stock", $1 + i^{STOCK} = (S_2 + D_2)/ S_1$

Use definition of inflation, $1 + \pi_2 = P_2 / P_1$

$$\lambda = \left[1 - \frac{1+i^{STOCK}}{1+\pi_2} \cdot \frac{1+\pi_2}{1+i} \right] \cdot \frac{1}{R}$$

WHY IS FINANCING A *CONSTRAINT*?

$$\lambda = \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{P_1}{P_2} \cdot \frac{1+\pi_2}{1+i}\right] \cdot \frac{1}{R}$$ **(from previous page)**

Use definition of "nominal interest rate on stock", $1 + i^{STOCK} = (S_2 + D_2)/ S_1$
Use definition of inflation, $1 + \pi_2 = P_2 / P_1$

$$\lambda = \left[1 - \frac{1+i^{STOCK}}{1+\pi_2} \cdot \frac{1+\pi_2}{1+i}\right] \cdot \frac{1}{R}$$

Fisher equation for stock: $1 + r^{STOCK} = (1 + i^{STOCK})/ (1 + \pi_2)$
Fisher equation for bonds: $1 + r = (1 + i) / (1 + \pi_2)$

$$\lambda = \left[1 - \frac{1+r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$$

WHY IS FINANCING A *CONSTRAINT*?

$$\lambda = \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{P_1}{P_2} \cdot \frac{1+\pi_2}{1+i}\right] \cdot \frac{1}{R}$$ **(from previous page)**

Use definition of "nominal interest rate on stock", $1 + i^{STOCK} = (S_2 + D_2)/ S_1$
Use definition of inflation, $1 + \pi_2 = P_2 / P_1$

$$\lambda = \left[1 - \frac{1+i^{STOCK}}{1+\pi_2} \cdot \frac{1+\pi_2}{1+i}\right] \cdot \frac{1}{R}$$

Fisher equation for stock: $1 + r^{STOCK} = (1 + i^{STOCK})/ (1 + \pi_2)$
Fisher equation for bonds: $1 + r = (1 + i) / (1 + \pi_2)$

$$\lambda = \left[1 - \frac{1+r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$$

Final rewrite!

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$$ **The Lagrange multiplier on firm's financing constraint**

WHY IS FINANCING A *CONSTRAINT*?

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$$

The Lagrange multiplier on firm's financing constraint

- **Basic firm theory (Chapter 6)**
  - **No financing constraint**

WHY IS FINANCING A *CONSTRAINT*?

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$$

The Lagrange multiplier on firm's financing constraint

- **Basic firm theory (Chapter 6)**
  - **No financing constraint**
  - **Can interpret basic firm theory analysis as featuring $\lambda = 0$**
    - **Interpretation: under "normal market conditions," financing constraints don't matter (much…)**

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1) a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2) a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i}$$
$$+ \underbrace{\lambda}_{= 0}\left[R \cdot S_1 \cdot a_1 - P_1 \cdot (k_2 - k_1)\right]$$

WHY IS FINANCING A *CONSTRAINT*?

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$$

The Lagrange multiplier on firm's financing constraint

- Basic firm theory (Chapter 6)
  - No financing constraint
  - Can interpret basic firm theory analysis as featuring $\lambda = 0$
    - Interpretation: under "normal market conditions," financing constraints don't matter (much…)

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i}$$

$$+ \underbrace{\lambda}_{= 0} \left[R \cdot S_1 \cdot a_1 - P_1 \cdot (k_2 - k_1)\right]$$

  - If $\lambda = 0$ (i.e., "normal market conditions")
    - Labor demand decisions unaffected by financial market conditions
    - Capital demand decisions unaffected by financial market conditions

- Key question: what causes $\lambda = 0$?

WHY IS FINANCING A *CONSTRAINT*?

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$$

The Lagrange multiplier on firm's financing constraint

- Two conditions for $\lambda = 0$
  - Market returns on risky assets equal returns on safe assets
    - Risky assets: stocks
    - Safe assets
      - Bonds (financial)

WHY IS FINANCING A *CONSTRAINT*?

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$$

The Lagrange multiplier on firm's financing constraint

- Two conditions for λ = 0
  - Market returns on risky assets equal returns on safe assets
    - Risky assets: stocks
    - Safe assets
      - Bonds (financial)
      - Machines and equipment (physical) – most directly relevant for firms' production and sales activity!
        - Basic firm theory prediction: *r = mpk*

Can think of both government bonds (financial assets) and machines & equipment (physical assets) as "safe": you (pretty much…) know what you're going to get from them.

WHY IS FINANCING A *CONSTRAINT*?

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$$

The Lagrange multiplier on firm's financing constraint

- Two conditions for λ = 0
  - Market returns on risky assets equal returns on safe assets
    - Risky assets: stocks
    - Safe assets
      - Bonds (financial)
      - Machines and equipment (physical) – most directly relevant for firms' production and sales activity!
        - Basic firm theory prediction: *r = mpk*

Can think of both government bonds (financial assets) and machines & equipment (physical assets) as "safe": you (pretty much…) know what you're going to get from them.

$$r = r^{STOCK} \longrightarrow \lambda = 0$$

Interpretation: if returns on financial assets match up with returns on physical assets, financing constraints "don't matter"

WHY IS FINANCING A *CONSTRAINT*?

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$$

The Lagrange multiplier on firm's financing constraint

- **Two conditions for $\lambda = 0$**
  - **Market returns on risky assets equal returns on safe assets**
    - **Risky assets: stocks**
    - **Safe assets**
      - **Bonds (financial)**
      - **Machines and equipment (physical) – most directly relevant for firms' production and sales activity!**
        - **Basic firm theory prediction: *r = mpk***

Can think of both government bonds (financial assets) and machines & equipment (physical assets) as "safe": you (pretty much…) know what you're going to get from them.

$$r = r^{STOCK} \longrightarrow \lambda = 0$$

Interpretation: if returns on financial assets match up with returns on physical assets, financing constraints "don't matter"

  - **Government oversight of borrowing/lending relationships very lax**
    - **The larger is *R*, the lower is λ**

Finance Fundamentals

WHY IS FINANCING A *CONSTRAINT*?

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$$

The Lagrange multiplier on firm's financing constraint

- **Two conditions for $\lambda = 0$**
  - **Market returns on risky assets equal returns on safe assets**
    - **Risky assets: stocks**
    - **Safe assets**
      - **Bonds (financial)**
      - **Machines and equipment (physical) – most directly relevant for firms' production and sales activity!**
        - **Basic firm theory prediction: *r = mpk***

Can think of both government bonds (financial assets) and machines & equipment (physical assets) as "safe": you (pretty much…) know what you're going to get from them.

$$r = r^{STOCK} \longrightarrow \lambda = 0$$

Interpretation: if returns on financial assets match up with returns on physical assets, financing constraints "don't matter"

  - **Government oversight of borrowing/lending relationships very lax**
    - **The larger is *R*, the lower is λ**
    - **Financing constraint:** $P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$ Market value of financial assets
      - **Holding constant market value of financial assets, higher *R* allows higher k_2**

12

WHY IS FINANCING A *CONSTRAINT*?

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$$

The Lagrange multiplier on firm's financing constraint

- **Two conditions for λ = 0**
  - **Market returns on risky assets equal returns on safe assets**
    - **Risky assets: stocks**
    - **Safe assets**
      - **Bonds (financial)**
      - **Machines and equipment (physical) – most directly relevant for firms' production and sales activity!**
        - **Basic firm theory prediction: *r = mpk***

Can think of both government bonds (financial assets) and machines & equipment (physical assets) as "safe": you (pretty much…) know what you're going to get from them.

$$r = r^{STOCK} \longrightarrow \lambda = 0$$

Interpretation: if returns on financial assets match up with returns on physical assets, financing constraints "don't matter"

  - **Government oversight of borrowing/lending relationships very lax**
    - **The larger is *R*, the lower is λ**
    - **Financing constraint:** $P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$
      - **Holding constant market value of financial assets, higher *R* allows higher k_2**

Market value of financial assets

$$R = \infty \longrightarrow \lambda = 0$$

WHY IS FINANCING A *CONSTRAINT*?

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$$

The Lagrange multiplier on firm's financing constraint

- **Two conditions for λ = 0**
  - **Market returns on risky assets equal returns on safe assets**
    - **Risky assets: stocks**
    - **Safe assets**
      - **Bonds (financial)**
      - **Machines and equipment (physical) – most directly relevant for firms' production and sales activity!**
        - **Basic firm theory prediction: *r = mpk***

Can think of both government bonds (financial assets) and machines & equipment (physical assets) as "safe": you (pretty much…) know what you're going to get from them.

$$r = r^{STOCK} \longrightarrow \lambda = 0$$

Interpretation: if returns on financial assets match up with returns on physical assets, financing constraints "don't matter"

  - **Government oversight of borrowing/lending relationships very lax**
    - **The larger is *R*, the lower is λ**
    - **Financing constraint:** $P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$
      - **Holding constant market value of financial assets, higher *R* allows higher k_2**

Market value of financial assets

In practice, not literally infinity…

$$R = \infty \longrightarrow \lambda = 0$$

Interpretation: if government regulations allow high borrowing with little assets, financing constraints "don't matter"

Financing Constraint and Capital Demand

- **Assume R = 1 under "normal conditions" (but keep R in rest of analysis)**
  - **R > 1 is "lax regulation" (because it lowers λ, all else constant)**
  - **R < 1 is "tight regulation" (because it increases λ, all else constant)**
  - **→ Whether or not financing constraint matters (i.e., whether or not λ = 0) all depends on whether or not $r^{STOCK} = r$ or not**

Capital Demand in the Micro

Financing Constraint and Capital Demand

- **Assume R = 1 under "normal conditions" (but keep R in rest of analysis)**
  - **R > 1 is "lax regulation" (because it lowers λ, all else constant)**
  - **R < 1 is "tight regulation" (because it increases λ, all else constant)**
  - **→ Whether or not financing constraint matters (i.e., whether or not λ = 0) all depends on whether or not $r^{STOCK} = r$ or not**

KEY IDEA: if returns on safe assets = returns on risky assets → financing constraints "don't matter" for firm production decisions

FINANCING CONSTRAINT AND CAPITAL DEMAND

- **Assume R = 1 under "normal conditions" (but keep R in rest of analysis)**
  - **R > 1 is "lax regulation" (because it lowers λ, all else constant)**
  - **R < 1 is "tight regulation" (because it increases λ, all else constant)**
  - **→ Whether or not financing constraint matters (i.e., whether or not λ = 0) all depends on whether or not $r^{STOCK} = r$ or not**

KEY IDEA: if returns on safe assets = returns on risky assets → financing constraints "don't matter" for firm production decisions

$$-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} - \lambda P_1 = 0$$

Equation 3 (FOC on k_2)

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$$

Equation 4 (FOC on a_1)

- **Basic firm theory (Chapter 6)**
  - **Capital demand function derived from Equation 3**
  - **Idea still the same…but now complicated by the financing constraint**

FINANCING CONSTRAINT AND CAPITAL DEMAND

- **Assume R = 1 under "normal conditions" (but keep R in rest of analysis)**
  - **R > 1 is "lax regulation" (because it lowers λ, all else constant)**
  - **R < 1 is "tight regulation" (because it increases λ, all else constant)**
  - **→ Whether or not financing constraint matters (i.e., whether or not λ = 0) all depends on whether or not $r^{STOCK} = r$ or not**

KEY IDEA: if returns on safe assets = returns on risky assets → financing constraints "don't matter" for firm production decisions

$$-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} - \lambda P_1 = 0$$

Equation 3 (FOC on k_2)

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$$

Equation 4 (FOC on a_1)

- **Basic firm theory (Chapter 6)**
  - **Capital demand function derived from Equation 3**
  - **Idea still the same…but now complicated by the financing constraint**

Substitute λ from Equation 4 into Equation 3

$$-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R} P_1 = 0$$

Rearrange

FINANCING CONSTRAINT AND CAPITAL DEMAND

$$-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R}P_1 = 0$$ **(from previous page)**

$\downarrow$ **Divide by P_1**

$$\frac{P_2 f_k(k_2, n_2)}{P_1(1+i)} + \frac{P_2}{P_1(1+i)} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R} = 1$$

FINANCING CONSTRAINT AND CAPITAL DEMAND

$$-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R}P_1 = 0$$ **(from previous page)**

$\downarrow$ **Divide by P_1**

$$\frac{P_2 f_k(k_2, n_2)}{P_1(1+i)} + \frac{P_2}{P_1(1+i)} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R} = 1$$

$\downarrow$ **Use definition of inflation, $1 + \pi_2 = P_2 / P_1$**

$$\left(\frac{1+\pi_2}{1+i}\right) f_k(k_2, n_2) + \frac{1+\pi_2}{1+i} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R} = 1$$

Financing Constraint and Capital Demand

$$-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R} P_1 = 0$$ (from previous page)

Divide by P_1

$$\frac{P_2 f_k(k_2, n_2)}{P_1(1+i)} + \frac{P_2}{P_1(1+i)} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R} = 1$$

Use definition of inflation, $1 + \pi_2 = P_2 / P_1$

$$\left(\frac{1+\pi_2}{1+i}\right) f_k(k_2, n_2) + \frac{1+\pi_2}{1+i} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R} = 1$$

Apply Fisher relation for "safe" assets

$$\frac{f_k(k_2, n_2)}{1+r} + \frac{1}{1+r} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R} = 1$$

Financing Constraint and Capital Demand

$$-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R} P_1 = 0$$ (from previous page)

Divide by P_1

$$\frac{P_2 f_k(k_2, n_2)}{P_1(1+i)} + \frac{P_2}{P_1(1+i)} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R} = 1$$

Use definition of inflation, $1 + \pi_2 = P_2 / P_1$

$$\left(\frac{1+\pi_2}{1+i}\right) f_k(k_2, n_2) + \frac{1+\pi_2}{1+i} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R} = 1$$

Apply Fisher relation for "safe" assets

$$\frac{f_k(k_2, n_2)}{1+r} + \frac{1}{1+r} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R} = 1$$

Multiply by (1+r)

$$f_k(k_2, n_2) + 1 - \frac{r - r^{STOCK}}{R} = 1 + r$$

FINANCING CONSTRAINT AND CAPITAL DEMAND

$$-P_1 + \frac{P_2 f_k(k_2,n_2)}{1+i} + \frac{P_2}{1+i} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R}P_1 = 0$$ **(from previous page)**

Divide by P_1

$$\frac{P_2 f_k(k_2,n_2)}{P_1(1+i)} + \frac{P_2}{P_1(1+i)} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R} = 1$$

Use definition of inflation, $1 + \pi_2 = P_2 / P_1$

$$\left(\frac{1+\pi_2}{1+i}\right) f_k(k_2,n_2) + \frac{1+\pi_2}{1+i} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R} = 1$$

Apply Fisher relation for "safe" assets

$$\frac{f_k(k_2,n_2)}{1+r} + \frac{1}{1+r} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R} = 1$$

Multiply by (1+r)

$$f_k(k_2,n_2) + \cancel{1} - \frac{r - r^{STOCK}}{R} = \cancel{1} + r$$

FINANCING CONSTRAINT AND CAPITAL DEMAND

$$-P_1 + \frac{P_2 f_k(k_2,n_2)}{1+i} + \frac{P_2}{1+i} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R}P_1 = 0$$ **(from previous page)**

Divide by P_1

$$\frac{P_2 f_k(k_2,n_2)}{P_1(1+i)} + \frac{P_2}{P_1(1+i)} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R} = 1$$

Use definition of inflation, $1 + \pi_2 = P_2 / P_1$

$$\left(\frac{1+\pi_2}{1+i}\right) f_k(k_2,n_2) + \frac{1+\pi_2}{1+i} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R} = 1$$

Apply Fisher relation for "safe" assets

$$\frac{f_k(k_2,n_2)}{1+r} + \frac{1}{1+r} - \left[\frac{r - r^{STOCK}}{1+r}\right]\frac{1}{R} = 1$$

Multiply by (1+r)

Marginal product of capital, *mpk*

$$\underline{f_k(k_2,n_2)} + \cancel{1} - \frac{r - r^{STOCK}}{R} = \cancel{1} + r$$

Assuming R = 1 under "normal conditions," but keeping R in the analysis

COBB-DOUGLAS PRODUCTION FUNCTION

- **A commonly-used functional form in modern quantitative macroeconomic models**

$$f(k,n) = k^{\alpha} n^{1-\alpha}$$

- **Describes the empirical relationship between aggregate GDP, aggregate capital, and aggregate labor quite well**

- $\alpha \in (0,1)$ **measures capital's share of output**
  - **Hence** $(1-\alpha) \in (0,1)$ **measures labor's share of output**
  - **Interpretation**
    - **The relative importance of (either) capital (or labor) in the production process**
  - **Estimates for U.S. economy:** $\alpha \approx 0.3$
  - **Estimates for Chinese economy:** $\alpha \approx 0.15$ **(not (yet) a very capital-rich economy)**

- **Cobb-Douglas form useful for illustrating factor demands**
  - $mpn = f_n(k,n) = (1-\alpha)k^{\alpha} n^{-\alpha}$
  - $mpk = f_k(k,n) = \alpha k^{\alpha-1} n^{1-\alpha}$

FINANCING CONSTRAINT AND CAPITAL DEMAND

- **Firm-level demand for capital defined by the relation**

$$r = \alpha k^{\alpha-1} n^{1-\alpha} - \left[\frac{r - r^{STOCK}}{R}\right] \left(= mpk - \left[\frac{r - r^{STOCK}}{R}\right] \right)$$

FINANCING CONSTRAINT AND CAPITAL DEMAND

- Firm-level demand for capital **defined** by the relation

$$r = \alpha k^{\alpha-1} n^{1-\alpha} - \left[\frac{r - r^{STOCK}}{R}\right] \left(= mpk - \left[\frac{r - r^{STOCK}}{R}\right] \right)$$

$$\downarrow$$

$$r = \alpha k^{\alpha-1} n^{1-\alpha} - \frac{r}{R} + \frac{r^{STOCK}}{R}$$

$$\downarrow$$

$$\left[1 + \frac{1}{R}\right] r = \alpha k^{\alpha-1} n^{1-\alpha} + \frac{r^{STOCK}}{R}$$

$$\downarrow$$

$$\left[\frac{R+1}{R}\right] r = \alpha k^{\alpha-1} n^{1-\alpha} + \frac{r^{STOCK}}{R}$$

$$\downarrow$$

$$r = \left(\frac{R}{R+1}\right) \alpha k^{\alpha-1} n^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

Solve for r (return on "safe" physical assets)

Capital Demand in the Micro

FINANCING CONSTRAINT AND CAPITAL DEMAND

- Firm-level demand for capital **defined** by the relation

$$r = \left(\frac{R}{R+1}\right) \alpha k^{\alpha-1} n^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

Financing Constraint and Capital Demand

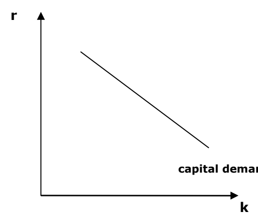
- **Firm-level demand for capital defined by the relation**

$$r = \left(\frac{R}{R+1}\right)\alpha k^{\alpha-1}n^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

Because exponent (α – 1) is a negative number, can move to denominator

$$r = \left(\frac{R}{R+1}\right)\alpha\left(\frac{n_t}{k_t}\right)^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

CHAPTER 6: NEGATIVE RELATIONSHIP BETWEEN *r* AND *k*



Financing Constraint and Capital Demand

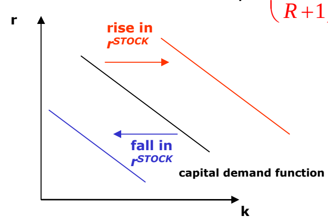
- **Firm-level demand for capital defined by the relation**

$$r = \left(\frac{R}{R+1}\right)\alpha k^{\alpha-1}n^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

Because exponent (α – 1) is a negative number, can move to denominator

$$r = \left(\frac{R}{R+1}\right)\alpha\left(\frac{n_t}{k_t}\right)^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

Rise (fall) in return on stock leads to shift out (in) of capital demand function



Financing Constraint and Capital Demand

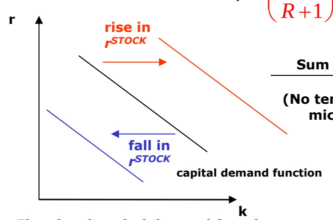
- **Firm-level demand for capital defined by the relation**

$$r = \left(\frac{R}{R+1}\right)\alpha k^{\alpha-1} n^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

Because exponent (α – 1) is a negative number, can move to denominator

$$r = \left(\frac{R}{R+1}\right)\alpha \left(\frac{n_t}{k_t}\right)^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

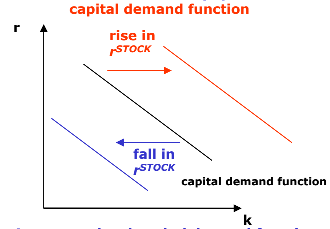
Rise (fall) in return on stock leads to shift out (in) of capital demand function



Firm-level capital demand function

Sum over all firms

(No tension between the micro and macro)



Aggregate-level capital demand function

- **<u>IMPORTANT</u>: changes in financial market returns shift capital demand (and hence investment demand – recall $inv_t = k_{t+1} - k_t$)**

Financing Constraint and Capital Demand

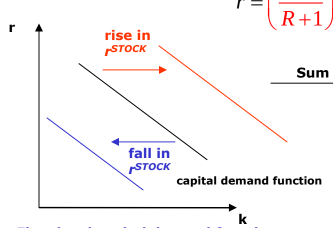
- **Firm-level demand for capital defined by the relation**

$$r = \left(\frac{R}{R+1}\right)\alpha k^{\alpha-1} n^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

Because exponent (α – 1) is a negative number, can move to denominator

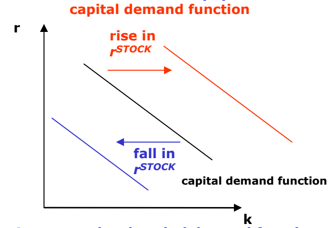
$$r = \left(\frac{R}{R+1}\right)\alpha \left(\frac{n_t}{k_t}\right)^{1-\alpha} + \frac{r^{STOCK}}{R+1}$$

Rise (fall) in return on stock leads to shift out (in) of capital demand function



Firm-level capital demand function

Sum over all firms



Aggregate-level capital demand function

- **Next: the financial accelerator**
- **Next: the role of financial oversight**