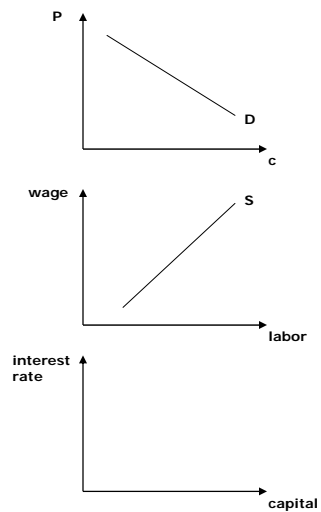


# CONSUMPTION-SAVINGS MODEL

FEBRUARY 9, 2009

## THE THREE MACRO (AGGREGATE) MARKETS

- ❑ **Goods Markets**
  - ❑ Demand derived from C-L model
  
- ❑ **Labor Markets**
  - ❑ Supply derived from C-L model
  
- ❑ **Capital/Savings/Funds/Asset Markets (aka Financial Markets)**



## BASICS

- ❑ Consumption-Savings Model – provides foundation for
  - ❑ Goods-market demand function (again...but w/different interpretation)
  - ❑ Capital-market supply function
  - ❑ An application of the basic consumer theory model...
  - ❑ ...we will put a macro interpretation on it

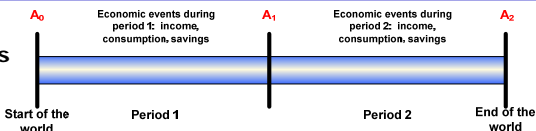
## BASICS

- ❑ Consumption-Savings Model – provides foundation for
  - ❑ Goods-market demand function (again...but w/different interpretation)
  - ❑ Capital-market supply function
  - ❑ An application of the basic consumer theory model...
  - ❑ ...we will put a macro interpretation on it
  - ❑ **Two time periods**
    - ❑ **Important: all of our analysis will be conducted from the perspective of the very beginning of period 1...**
    - ❑ **...so a “future” (period 2) for which to save**

## BASICS

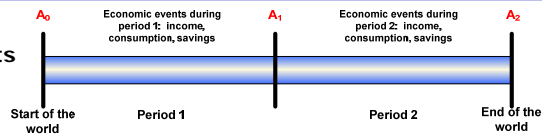
- Consumption-Savings Model – provides foundation for
  - Goods-market demand function (again...but w/different interpretation)
  - Capital-market supply function
  - An application of the basic consumer theory model...
  - ...we will put a macro interpretation on it
  - Two time periods
    - Important: all of our analysis will be conducted from the perspective of the very beginning of period 1...
    - ...so a “future” (period 2) for which to save
- Dynamic models are the staple of modern macroeconomic theory
- An explicit accounting of time
- Two periods are sufficient to illustrate the basic principles
  - Soon will extend to infinite number of periods (Chapter 8)

## BASICS

- Timeline of events
 
- Notation
  - $c_1$ : consumption in period 1
  - $c_2$ : consumption in period 2
  - $P_1$ : nominal price of consumption in period 1
  - $P_2$ : nominal price of consumption in period 2
  - $Y_1$ : nominal income in period 1 (“falls from the sky”)
  - $Y_2$ : nominal income in period 2 (“falls from the sky”)

## BASICS

### Timeline of events



### Notation

- $c_1$ : consumption in period 1
- $c_2$ : consumption in period 2
- $P_1$ : nominal price of consumption in period 1
- $P_2$ : nominal price of consumption in period 2
- $Y_1$ : nominal income in period 1 ("falls from the sky")
- $Y_2$ : nominal income in period 2 ("falls from the sky")
- $A_0$ : nominal wealth at the beginning of period 1/end of period 0
- $A_1$ : nominal wealth at the beginning of period 2/end of period 1
- $A_2$ : nominal wealth at the beginning of period 3/end of period 2
- $i$ : nominal interest rate between periods
- $r$ : real interest rate between periods
- $\pi_2$ : net inflation rate between period 1 and period 2  $\pi_2 = \frac{P_2 - P_1}{P_1} \left( = \frac{P_2}{P_1} - 1 \right)$
- $y_1$ : real income in period 1 ( $= Y_1/P_1$ )
- $y_2$ : real income in period 2 ( $= Y_2/P_2$ )

February 9, 2009

7

## STOCKS VS. FLOWS

- Stock variables**
  - Variables whose natural measurement occurs at a **particular moment in time**
- Flow variables**
  - Variables whose natural measurement occurs over the course of a **given interval of time**

February 9, 2009

8

## STOCKS VS. FLOWS

- ❑ **Stock variables**
  - ❑ Variables whose natural measurement occurs at a **particular moment in time**
- Economic examples {
  - ❑ Checking account balance
  - ❑ Credit card indebtedness
  - ❑ Mortgage loan payoff
 } Interpret  $A$  in our model as net wealth  
(= total assets – total debts)
- ❑ **Flow variables**
  - ❑ Variables whose natural measurement occurs over the course of a **given interval of time**

## STOCKS VS. FLOWS

- ❑ **Stock variables**
  - ❑ Variables whose natural measurement occurs at a **particular moment in time**
- Economic examples {
  - ❑ Checking account balance
  - ❑ Credit card indebtedness
  - ❑ Mortgage loan payoff
 } Interpret  $A$  in our model as net wealth  
(= total assets – total debts)
- ❑ **Flow variables**
  - ❑ Variables whose natural measurement occurs over the course of a **given interval of time**
- Economic examples {
  - ❑ Income
  - ❑ Consumption
  - ❑ Savings
 }

## STOCKS VS. FLOWS

❑ **Stock variables**

- ❑ Variables whose natural measurement occurs at a **particular moment in time**

Economic examples {

- ❑ Checking account balance
- ❑ Credit card indebtedness
- ❑ Mortgage loan payoff

} Interpret  $A$  in our model as net wealth (= total assets – total debts)

❑ **Flow variables**

- ❑ Variables whose natural measurement occurs over the course of a **given interval of time**

Economic examples {

- ❑ Income
- ❑ Consumption
- ❑ Savings

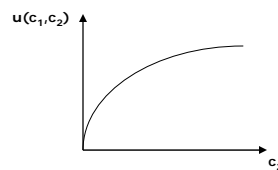
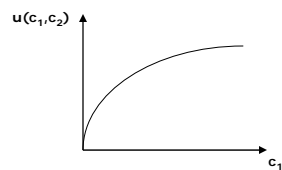
❑ **The two broad categories of income**

- All income is a FLOW regardless of source
- ❑ Labor income
  - ❑ Asset income (generated by interest rate(s) on (components of) wealth)

## UTILITY

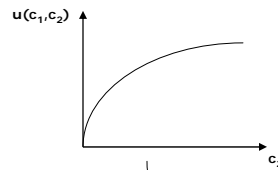
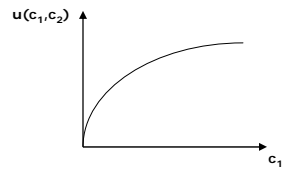
❑ Preferences  $u(c_1, c_2)$  with all the “usual properties”

- ❑ **Lifetime utility function**
- ❑ Strictly increasing in  $c_1$
- ❑ Strictly increasing in  $c_2$
- ❑ Diminishing marginal utility in  $c_1$
- ❑ Diminishing marginal utility in  $c_2$

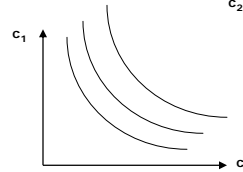


## UTILITY

- Preferences  $u(c_1, c_2)$  with all the "usual properties"
  - Lifetime utility function
  - Strictly increasing in  $c_1$
  - Strictly increasing in  $c_2$
  - Diminishing marginal utility in  $c_1$
  - Diminishing marginal utility in  $c_2$

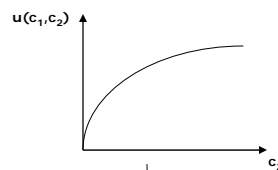
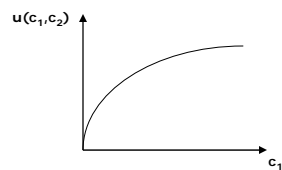


- Plotted as indifference curves

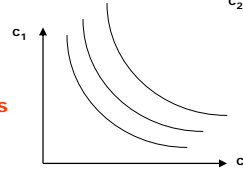


## UTILITY

- Preferences  $u(c_1, c_2)$  with all the "usual properties"
  - Lifetime utility function
  - Strictly increasing in  $c_1$
  - Strictly increasing in  $c_2$
  - Diminishing marginal utility in  $c_1$
  - Diminishing marginal utility in  $c_2$



- Plotted as indifference curves



- Utility side of consumption-savings model no different than Chapter 1 model

## BUDGET CONSTRAINT(S)

- ❑ Suppose again  $Y$  "falls from the sky"
  - ❑  $Y_1$  in period 1,  $Y_2$  in period 2
- ❑ Need **two** budget constraints to describe economic opportunities and possibilities
  - ❑ One for each period

## BUDGET CONSTRAINT(S)

- ❑ Suppose again  $Y$  "falls from the sky"
  - ❑  $Y_1$  in period 1,  $Y_2$  in period 2
- ❑ Need **two** budget constraints to describe economic opportunities and possibilities
  - ❑ One for each period
  - ❑ Period-1 budget constraint

$$P_1 c_1 + A_1 = Y_1 + (1+i)A_0$$

Total expenditure in period 1:  
 period-1 consumption +  
 wealth to *carry into period 2*

Total income in period 1:  
 period-1  $Y$  + income from  
 wealth *carried into period 1*  
 (inclusive of interest)

## BUDGET CONSTRAINT(S)

- Suppose again  $Y$  "falls from the sky"
  - $Y_1$  in period 1,  $Y_2$  in period 2
- Need **two** budget constraints to describe economic opportunities and possibilities
  - One for each period
  - Period-1 budget constraint

$$P_1c_1 + A_1 = Y_1 + (1+i)A_0$$

Total expenditure in period 1:  
period-1 consumption +  
wealth to carry into period 2

Total income in period 1:  
period-1  $Y$  + income from  
wealth carried into period 1  
(inclusive of interest)

- Period-2 budget constraint

$$P_2c_2 + A_2 = Y_2 + (1+i)A_1$$

Total expenditure in period 2:  
period-2 consumption +  
wealth to carry into period 3

Total income in period 2:  
period-2  $Y$  + income from  
wealth carried into period 2  
(inclusive of interest)

## BUDGET CONSTRAINT(S)

- Suppose again  $Y$  "falls from the sky"
  - $Y_1$  in period 1,  $Y_2$  in period 2
- Need **two** budget constraints to describe economic opportunities and possibilities
  - One for each period
  - Period-1 budget constraint

$$P_1c_1 + A_1 = Y_1 + (1+i)A_0$$

← can rewrite as →

$$P_1c_1 + A_1 - A_0 = Y_1 + iA_0$$

Savings during period 1 (a flow)     Asset income during period 1 (a flow)  
↓

Total expenditure in period 1:  
period-1 consumption +  
wealth to carry into period 2

Total income in period 1:  
period-1  $Y$  + income from  
wealth carried into period 1  
(inclusive of interest)

- Period-2 budget constraint

$$P_2c_2 + A_2 = Y_2 + (1+i)A_1$$

← can rewrite as →

$$P_2c_2 + A_2 - A_1 = Y_2 + iA_1$$

Savings during period 2 (a flow)     Asset income during period 2 (a flow)  
↓

Total expenditure in period 2:  
period-2 consumption +  
wealth to carry into period 3

Total income in period 2:  
period-2  $Y$  + income from  
wealth carried into period 2  
(inclusive of interest)

## BUDGET CONSTRAINT(S)

- Suppose again  $Y$  "falls from the sky"
  - $Y_1$  in period 1,  $Y_2$  in period 2
- Need **two** budget constraints to describe economic opportunities and possibilities

- One for each period

- Period-1 budget constraint

$$P_1c_1 + A_1 = Y_1 + (1+i)A_0$$

Total expenditure in period 1:  
period-1 consumption +  
wealth to carry into period 2

Total income in period 1:  
period-1  $Y$  + income from  
wealth carried into period 1  
(inclusive of interest)

← can rewrite as →

$$P_1c_1 + A_1 - A_0 = Y_1 + iA_0$$

Savings during period 1 (a flow)      Asset income during period 1 (a flow)  
↓

- Period-2 budget constraint

$$P_2c_2 + A_2 = Y_2 + (1+i)A_1$$

Total expenditure in period 2:  
period-2 consumption +  
wealth to carry into period 3

Total income in period 2:  
period-2  $Y$  + income from  
wealth carried into period 2  
(inclusive of interest)

← can rewrite as →

$$P_2c_2 + A_2 - A_1 = Y_2 + iA_1$$

Savings during period 2 (a flow)      Asset income during period 2 (a flow)  
↓

**DEFINITION:** A consumer's **savings** during a given period is the **change in his wealth** during that period

## BUDGET CONSTRAINT(S)

- Adopt a **lifetime** view of the budget constraint(s)
  - All analysis conducted from perspective of beginning of period 1
  - Period-1 budget constraint  $P_1c_1 + A_1 = Y_1 + (1+i)A_0$
  - Period-2 budget constraint  $P_2c_2 + A_2 = Y_2 + (1+i)A_1$

## BUDGET CONSTRAINT(S)

- Adopt a **lifetime** view of the budget constraint(s)
  - All analysis conducted from perspective of beginning of period 1
  - Period-1 budget constraint  $P_1c_1 + A_1 = Y_1 + (1+i)A_0$
  - Period-2 budget constraint  $P_2c_2 + A_2 = Y_2 + (1+i)A_1$

Asset position at end of period 1/beginning of period 2 the key link  
- will think further about this soon...

## BUDGET CONSTRAINT(S)

- Adopt a **lifetime** view of the budget constraint(s)
  - All analysis conducted from perspective of beginning of period 1
  - Period-1 budget constraint  $P_1c_1 + A_1 = Y_1 + (1+i)A_0$
  - Period-2 budget constraint  $P_2c_2 + A_2 = Y_2 + (1+i)A_1$

Asset position at end of period 1/beginning of period 2 the key link  
- will think further about this soon...

Assume = 0 (fully-rational, informed, selfish individuals)

## BUDGET CONSTRAINT(S)

- Adopt a **lifetime** view of the budget constraint(s)
  - All analysis conducted from perspective of beginning of period 1
  - Period-1 budget constraint  $P_1c_1 + A_1 = Y_1 + (1+i)A_0$ 
    - Asset position at end of period 1/beginning of period 2 the key link
  - Period-2 budget constraint  $P_2c_2 + A_2 = Y_2 + (1+i)A_1$ 
    - will think further about this soon...
- Assume = 0 (fully-rational, informed, selfish individuals)
- Combine into **lifetime budget constraint (LBC)**
  - Solve period-2 budget constraint for  $A_1$ ...
  - ...and substitute into period-1 budget constraint

## BUDGET CONSTRAINT(S)

- Adopt a **lifetime** view of the budget constraint(s)
  - All analysis conducted from perspective of beginning of period 1
  - Period-1 budget constraint  $P_1c_1 + A_1 = Y_1 + (1+i)A_0$ 
    - Asset position at end of period 1/beginning of period 2 the key link
  - Period-2 budget constraint  $P_2c_2 + A_2 = Y_2 + (1+i)A_1$ 
    - will think further about this soon...
- Assume = 0 (fully-rational, informed, selfish individuals)
- Combine into **lifetime budget constraint (LBC)**
  - Solve period-2 budget constraint for  $A_1$ ...
  - ...and substitute into period-1 budget constraint

$$P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i} + (1+i)A_0$$

Present discounted value (PDV) of all lifetime expenditure      Present discounted value (PDV) of all lifetime income

## BUDGET CONSTRAINT(S)

- ❑ Adopt a **lifetime** view of the budget constraint(s)
  - ❑ All analysis conducted from perspective of beginning of period 1
  - ❑ Period-1 budget constraint  $P_1c_1 + A_1 = Y_1 + (1+i)A_0$ 
    - Asset position at end of period 1/beginning of period 2 the key link
  - ❑ Period-2 budget constraint  $P_2c_2 + A_2 = Y_2 + (1+i)A_1$ 
    - will think further about this soon...
- Assume = 0 (fully-rational, informed, selfish individuals)
- ❑ Combine into **lifetime budget constraint (LBC)**
  - ❑ Solve period-2 budget constraint for  $A_1$ ...
  - ❑ ...and substitute into period-1 budget constraint

$$P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i} + (1+i)A_0$$

Present discounted value (PDV) of all lifetime expenditure
Present discounted value (PDV) of all lifetime income

For graphical simplicity, will often assume  $A_0 = 0$  (i.e., consumer begins life with zero net wealth).  
 Note this is a *different* assumption than  $A_2 = 0$ .

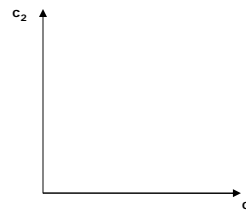
## LIFETIME BUDGET CONSTRAINT

### Graphically

$$P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$$

↓ Solve for  $c_2$

↓



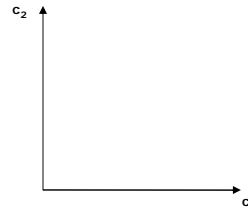
## LIFETIME BUDGET CONSTRAINT

### Graphically

$$P_1 c_1 + \frac{P_2 c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$$

↓ Solve for  $c_2$

$$c_2 = -\left(\frac{P_1(1+i)}{P_2}\right)c_1 + \left(\frac{1+i}{P_2}\right)Y_1 + \frac{Y_2}{P_2}$$



February 9, 2009

27

## LIFETIME BUDGET CONSTRAINT

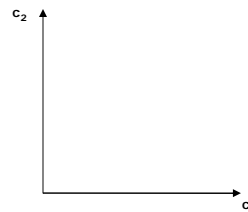
### Graphically

$$P_1 c_1 + \frac{P_2 c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$$

↓ Solve for  $c_2$

$$c_2 = -\left(\frac{P_1(1+i)}{P_2}\right)c_1 + \left(\frac{1+i}{P_2}\right)Y_1 + \frac{Y_2}{P_2}$$

Rearrange further using definition of inflation:  $1 + \pi_2 = \frac{P_2}{P_1} \Rightarrow \frac{1}{1 + \pi_2} = \frac{P_1}{P_2}$



February 9, 2009

28

## LIFETIME BUDGET CONSTRAINT

### Graphically

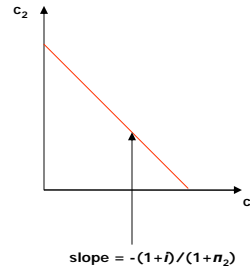
$$P_1 c_1 + \frac{P_2 c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$$

Solve for  $c_2$

$$c_2 = -\left(\frac{P_1(1+i)}{P_2}\right)c_1 + \left(\frac{1+i}{P_2}\right)Y_1 + \frac{Y_2}{P_2}$$

Rearrange further using definition of inflation:  $1 + \pi_2 = \frac{P_2}{P_1} \Rightarrow \frac{1}{1 + \pi_2} = \frac{P_1}{P_2}$

$$c_2 = -\left(\frac{1+i}{1 + \pi_2}\right)c_1 + \left(\frac{1+i}{P_2}\right)Y_1 + \frac{Y_2}{P_2}$$



## LIFETIME BUDGET CONSTRAINT

### Graphically

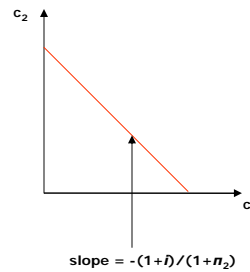
$$P_1 c_1 + \frac{P_2 c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$$

Solve for  $c_2$

$$c_2 = -\left(\frac{P_1(1+i)}{P_2}\right)c_1 + \left(\frac{1+i}{P_2}\right)Y_1 + \frac{Y_2}{P_2}$$

Rearrange further using definition of inflation:  $1 + \pi_2 = \frac{P_2}{P_1} \Rightarrow \frac{1}{1 + \pi_2} = \frac{P_1}{P_2}$

$$c_2 = -\left(\frac{1+i}{1 + \pi_2}\right)c_1 + \left(\frac{1+i}{P_2}\right)Y_1 + \frac{Y_2}{P_2}$$



The larger is  $(1+i)/(1+\pi_2)$ , the steeper is the budget line

## LIFETIME BUDGET CONSTRAINT

### Graphically

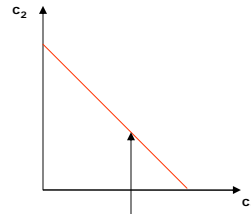
$$P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$$

Solve for  $c_2$

$$c_2 = -\left(\frac{P_1(1+i)}{P_2}\right)c_1 + \left(\frac{1+i}{P_2}\right)Y_1 + \frac{Y_2}{P_2}$$

Rearrange further using definition of inflation:  $1 + \pi_2 = \frac{P_2}{P_1} \Rightarrow \frac{1}{1 + \pi_2} = \frac{P_1}{P_2}$

$$c_2 = -\left(\frac{1+i}{1+\pi_2}\right)c_1 + \left(\frac{1+i}{P_2}\right)Y_1 + \frac{Y_2}{P_2}$$



slope =  $-(1+i)/(1+\pi_2)$

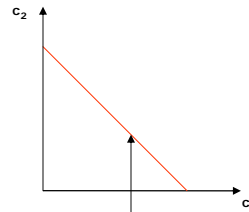
The larger is  $(1+i)/(1+\pi_2)$ , the steeper is the budget line

**IMPORTANT:** Changes in nominal interest rates (Fed) and/or inflation affect the slope of the LBC

## CONSUMER OPTIMIZATION

□ **Consumer's decision problem:** maximize lifetime utility subject to lifetime budget constraint – bring together both **cost** side and **benefit** side

- Choose  $c_1$  and  $c_2$  subject to  $P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$
- Plot budget line



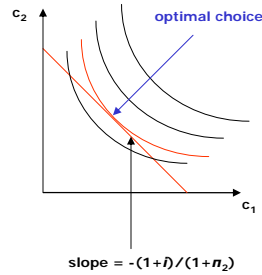
slope =  $-(1+i)/(1+\pi_2)$

## CONSUMER OPTIMIZATION

- **Consumer's decision problem:** maximize lifetime utility subject to lifetime budget constraint – bring together both **cost** side and **benefit** side

- Choose  $c_1$  and  $c_2$  subject to  $P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$
- Plot budget line

- Superimpose indifference map

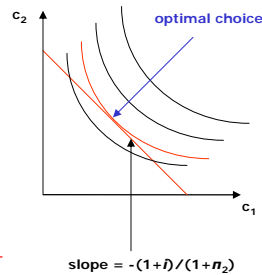


## CONSUMER OPTIMIZATION

- **Consumer's decision problem:** maximize lifetime utility subject to lifetime budget constraint – bring together both **cost** side and **benefit** side

- Choose  $c_1$  and  $c_2$  subject to  $P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$
- Plot budget line

- Superimpose indifference map



- **At the optimal choice**

CONSUMPTION-SAVINGS  
OPTIMALITY CONDITION  
- A key building block of  
modern macro models

$$\frac{u_1(c_1^*, c_2^*)}{u_2(c_1^*, c_2^*)} = \frac{1+i}{1+r_2}$$

MRS (between consumption in consecutive time periods)      price ratio (across consecutive time periods)