

CONSUMPTION-SAVINGS MODEL (CONTINUED)

FEBRUARY 16, 2009

LAGRANGE ANALYSIS: LIFETIME APPROACH

- ❑ Apply Lagrange tools to consumption-savings optimization
- ❑ Objective function: $u(c_1, c_2)$
- ❑ Constraint (assuming $A_0 = 0$): $g(c_1, c_2) = Y_1 + \frac{Y_2}{1+i} - P_1 c_1 - \frac{P_2 c_2}{1+i} = 0$

- ❑ **Step 1: Construct Lagrange function**

$$L(c_1, c_2, \lambda) = u(c_1, c_2) + \lambda \left[Y_1 + \frac{Y_2}{1+i} - P_1 c_1 - \frac{P_2 c_2}{1+i} \right]$$

- ❑ **Step 2: Compute first-order conditions with respect to c_1, c_2, λ**

- ❑ **Step 3: Solve (with focus on eliminating multiplier)**

CONSUMPTION-SAVINGS OPTIMALITY CONDITION

$$\frac{u_1(c_1^*, c_2^*)}{u_2(c_1^*, c_2^*)} = \frac{1+i}{1+\pi_2} = 1+r$$

MRS (between consumption in consecutive time periods)
price ratio (across consecutive time periods)

using Fisher equation

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Sequential formulation highlights the role of net wealth (A_1) between period 1 and period 2**
 - Accords better with the explicit timing of economic events than the lifetime approach...
 - ...but yields the same result
 - Advantage: allows us to think about interaction between asset prices and macroeconomic events (intersection of finance theory and macro theory in Chapter 8)

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Sequential formulation highlights the role of net wealth (A_1) between period 1 and period 2**
 - Accords better with the explicit timing of economic events than the lifetime approach...
 - ...but yields the same result
 - Advantage: allows us to think about interaction between asset prices and macroeconomic events (intersection of finance theory and macro theory in Chapter 8)
 - **Apply Lagrange tools to consumption savings optimization**
 - **Objective function: $u(c_1, c_2)$**
 - **Constraints:**
 - **Period 1 budget constraint:** $Y_1 + (1+i)A_0 - P_1c_1 - A_1 = 0$
 - **Period 2 budget constraint:** $Y_2 + (1+i)A_1 - P_2c_2 - A_2 = 0$
- } **TWO constraints**
- **Sequential Lagrange formulation requires **two** multipliers**
 - See Math Refresher, Chapter -1

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- ❑ **Sequential formulation highlights the role of net wealth (A_1) between period 1 and period 2**
 - ❑ Accords better with the explicit timing of economic events than the lifetime approach...
 - ❑ ...but yields the same result
 - ❑ **Advantage:** allows us to think about interaction between asset prices and macroeconomic events (intersection of finance theory and macro theory in Chapter 8)
 - ❑ **Apply Lagrange tools to consumption savings optimization**
 - ❑ **Objective function:** $u(c_1, c_2)$
 - ❑ **Constraints:**
 - ❑ **Period 1 budget constraint:** $Y_1 + (1+i)A_0 - P_1c_1 - A_1 = 0$
 - ❑ **Period 2 budget constraint:** $Y_2 + (1+i)A_1 - P_2c_2 - A_2 = 0$
- } **TWO constraints**
- ❑ **Sequential Lagrange formulation requires two multipliers**
 - ❑ See Math Refresher, Chapter -1
 - ❑ **Could pursue sequential approach in either nominal or real terms**

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- ❑ **Step 1: Construct Lagrange function**

$$L(c_1, c_2, A_1, \lambda_1, \lambda_2) = u(c_1, c_2) + \lambda_1 [Y_1 + (1+i)A_0 - P_1c_1 - A_1] + \lambda_2 [Y_2 + (1+i)A_1 - P_2c_2 - A_2]$$

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- Step 1: Construct Lagrange function

$$L(c_1, c_2, A_1, \lambda_1, \lambda_2) = u(c_1, c_2) + \lambda_1 [Y_1 + (1+i)A_0 - P_1c_1 - A_1] + \lambda_2 [Y_2 + (1+i)A_1 - P_2c_2 - A_2]$$

- Step 2: Compute FOCs with respect to $c_1, c_2, A_1, \lambda_1, \lambda_2$

(FOC on A_1 will be the key to asset pricing in Chapter 8...)

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- Step 1: Construct Lagrange function

$$L(c_1, c_2, A_1, \lambda_1, \lambda_2) = u(c_1, c_2) + \lambda_1 [Y_1 + (1+i)A_0 - P_1c_1 - A_1] + \lambda_2 [Y_2 + (1+i)A_1 - P_2c_2 - A_2]$$

- Step 2: Compute FOCs with respect to $c_1, c_2, A_1, \lambda_1, \lambda_2$

(FOC on A_1 will be the key to asset pricing in Chapter 8...)

- Step 3: Solve (with focus on eliminating multipliers)

CONSUMPTION-SAVINGS
OPTIMALITY CONDITION

$$\frac{u_1(c_1^*, c_2^*)}{u_2(c_1^*, c_2^*)} = \frac{1+i}{1+\pi_2} = 1+r$$

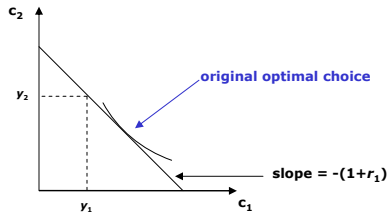
using Fisher equation

MRS (between consumption in consecutive time periods)
price ratio (across consecutive time periods)

- Identical to result of lifetime formulation

MICRO-LEVEL SAVINGS

How do micro-level consumption/savings choices change as the real interest rate changes (continue assuming $A_0 = 0$ for simplicity)?

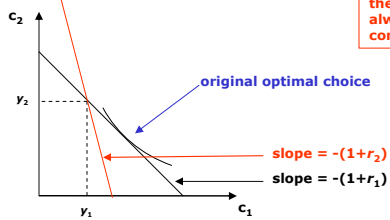


MICRO-LEVEL SAVINGS

How do micro-level consumption/savings choices change as the real interest rate changes (continue assuming $A_0 = 0$ for simplicity)?

REAL INTEREST RATE: $r_1 < r_2$

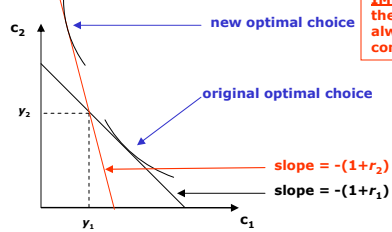
IMPORTANT: LBC *pivots* around the point (y_1, y_2) because (y_1, y_2) is always a possible choice of consumption



MICRO-LEVEL SAVINGS

How do micro-level consumption/savings choices change as the real interest rate changes (continue assuming $A_0 = 0$ for simplicity)?

REAL INTEREST RATE: $r_1 < r_2$



IMPORTANT: LBC *pivots* around the point (y_1, y_2) because (y_1, y_2) is always a possible choice of consumption

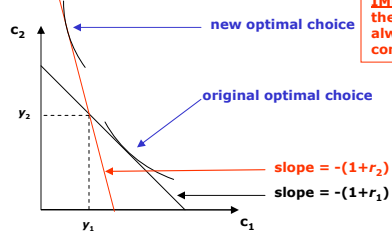
February 16, 2009

11

MICRO-LEVEL SAVINGS

How do micro-level consumption/savings choices change as the real interest rate changes (continue assuming $A_0 = 0$ for simplicity)?

REAL INTEREST RATE: $r_1 < r_2$



IMPORTANT: LBC *pivots* around the point (y_1, y_2) because (y_1, y_2) is always a possible choice of consumption

RESULT: optimal choice of c_1 falls as r rises \rightarrow optimal choice of savings $(= y_1 - c_1)$ rises as r rises

February 16, 2009

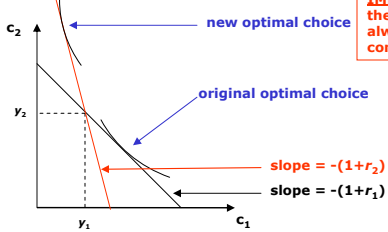
12

MICRO-LEVEL SAVINGS

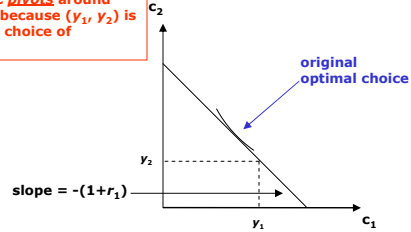
How do micro-level consumption/savings choices change as the real interest rate changes (continue assuming $A_0 = 0$ for simplicity)?

REAL INTEREST RATE: $r_1 < r_2$

IMPORTANT: LBC pivots around the point (y_1, y_2) because (y_1, y_2) is always a possible choice of consumption



OR



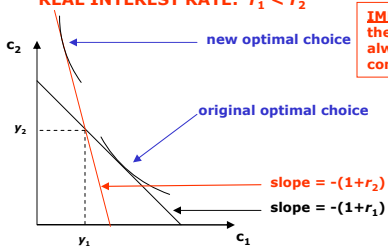
RESULT: optimal choice of c_1 falls as r rises \rightarrow optimal choice of savings $(= y_1 - c_1)$ rises as r rises

MICRO-LEVEL SAVINGS

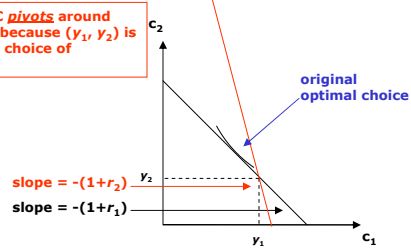
How do micro-level consumption/savings choices change as the real interest rate changes (continue assuming $A_0 = 0$ for simplicity)?

REAL INTEREST RATE: $r_1 < r_2$

IMPORTANT: LBC pivots around the point (y_1, y_2) because (y_1, y_2) is always a possible choice of consumption



OR



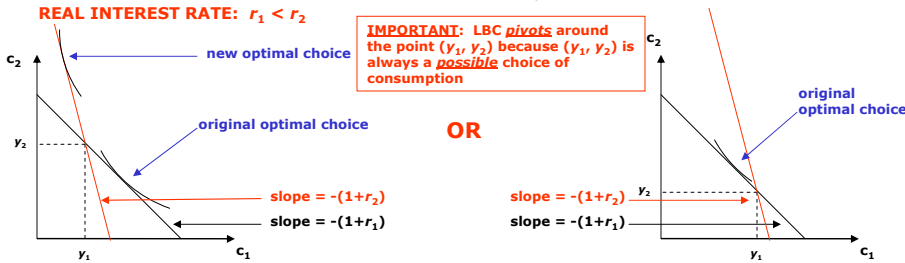
RESULT: optimal choice of c_1 falls as r rises \rightarrow optimal choice of savings $(= y_1 - c_1)$ rises as r rises

MICRO-LEVEL SAVINGS

How do micro-level consumption/savings choices change as the real interest rate changes (continue assuming $A_0 = 0$ for simplicity)?

REAL INTEREST RATE: $r_1 < r_2$

IMPORTANT: LBC pivots around the point (y_1, y_2) because (y_1, y_2) is always a possible choice of consumption



RESULT: optimal choice of c_1 falls as r rises \rightarrow optimal choice of savings $(= y_1 - c_1)$ rises as r rises

OR

RESULT: optimal choice of c_1 falls as r rises \rightarrow optimal choice of savings $(= y_1 - c_1)$ rises as r rises

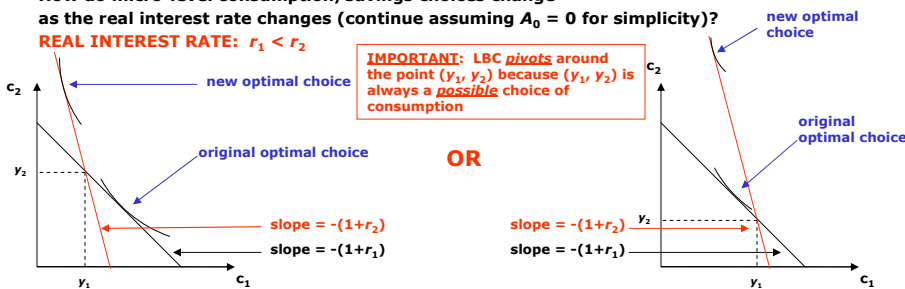
Empirical evidence shows that when r rises, period-1 (i.e., "short-run") consumption of all types of consumers falls

MICRO-LEVEL SAVINGS

How do micro-level consumption/savings choices change as the real interest rate changes (continue assuming $A_0 = 0$ for simplicity)?

REAL INTEREST RATE: $r_1 < r_2$

IMPORTANT: LBC pivots around the point (y_1, y_2) because (y_1, y_2) is always a possible choice of consumption



RESULT: optimal choice of c_1 falls as r rises \rightarrow optimal choice of savings $(= y_1 - c_1)$ rises as r rises

RESULT: optimal choice of c_1 falls as r rises \rightarrow optimal choice of savings $(= y_1 - c_1)$ rises as r rises

Empirical evidence shows that when r rises, period-1 (i.e., "short-run") consumption of all types of consumers falls

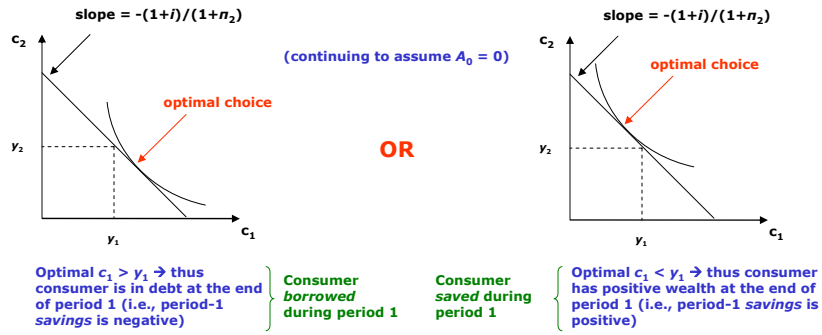
ASSESSING THE EFFECTS OF THE CREDIT CRUNCH

- **"Credit crunch" – financial sector has restricted the quantity of loans it is willing to extend to consumers in the "short run"**
 - **Financial market data and bank surveys show quantity of loans made to consumers (car loans, home loans, personal loans, etc.) has shrunk over the past year**

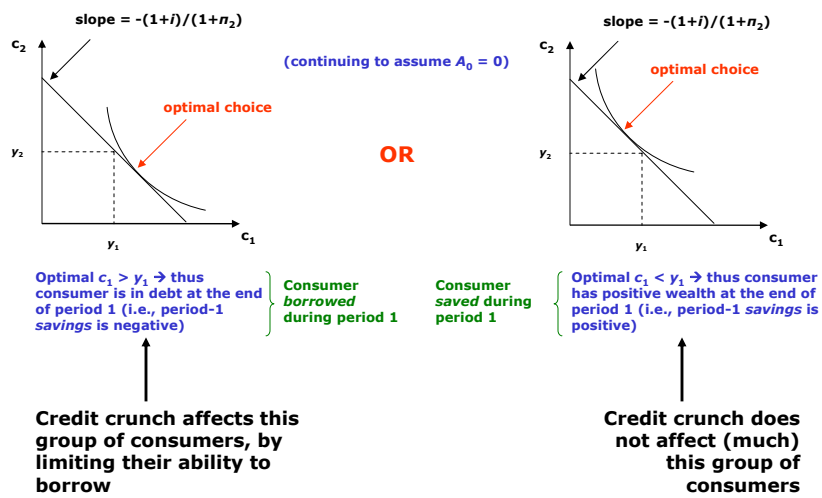
ASSESSING THE EFFECTS OF THE CREDIT CRUNCH

- **"Credit crunch" – financial sector has restricted the quantity of loans it is willing to extend to consumers in the "short run"**
 - **Financial market data and bank surveys show quantity of loans made to consumers (car loans, home loans, personal loans, etc.) has shrunk over the past year**
- **Can analyze macroeconomic consequences of shrinkage of credit availability using two-period model**
 - **Interpret "short run" to be period 1 (i.e., 2008-2009)**

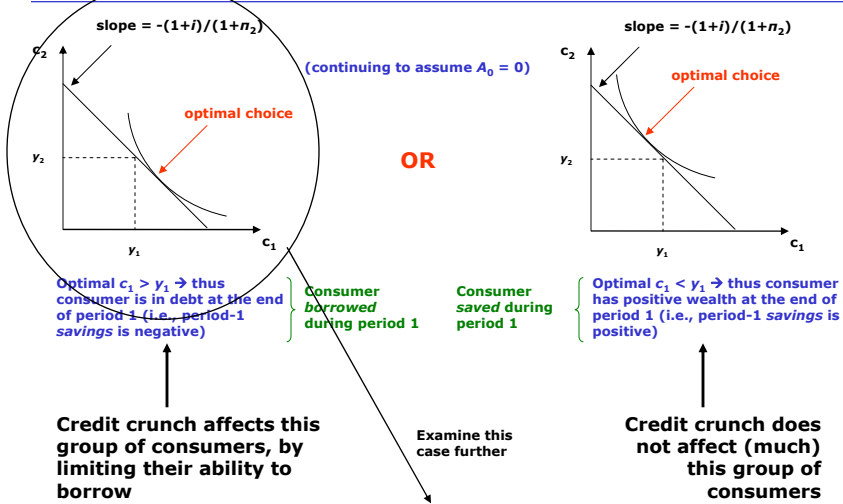
ASSESSING THE EFFECTS OF THE CREDIT CRUNCH



ASSESSING THE EFFECTS OF THE CREDIT CRUNCH



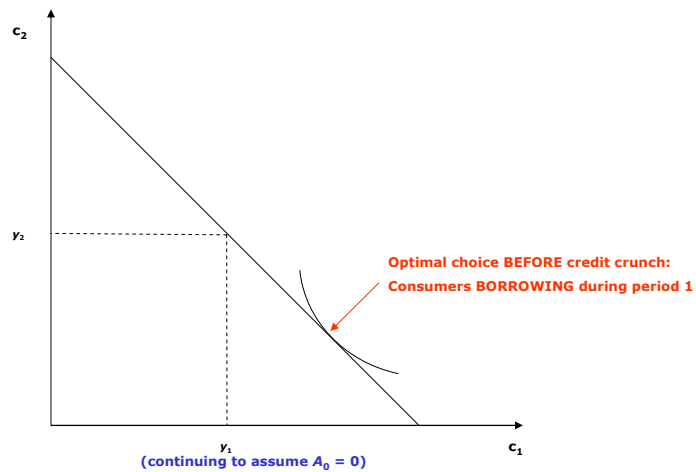
ASSESSING THE EFFECTS OF THE CREDIT CRUNCH



February 16, 2009

27

ASSESSING THE EFFECTS OF THE CREDIT CRUNCH



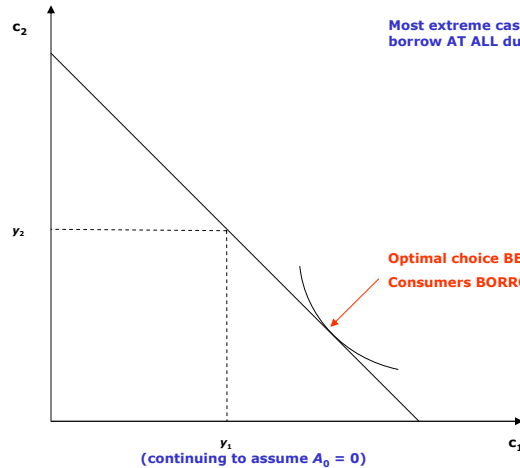
February 16, 2009

28

ASSESSING THE EFFECTS OF THE CREDIT CRUNCH

"Credit crunch" in period 1 limits the ability of consumers to borrow during period 1.

Most extreme case: consumers cannot borrow AT ALL during period 1.



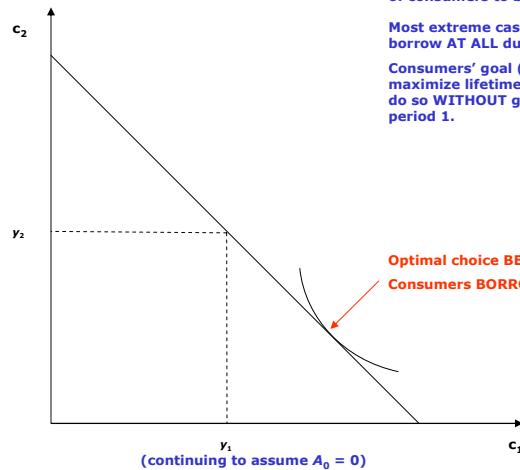
(continuing to assume $A_0 = 0$)

ASSESSING THE EFFECTS OF THE CREDIT CRUNCH

"Credit crunch" in period 1 limits the ability of consumers to borrow during period 1.

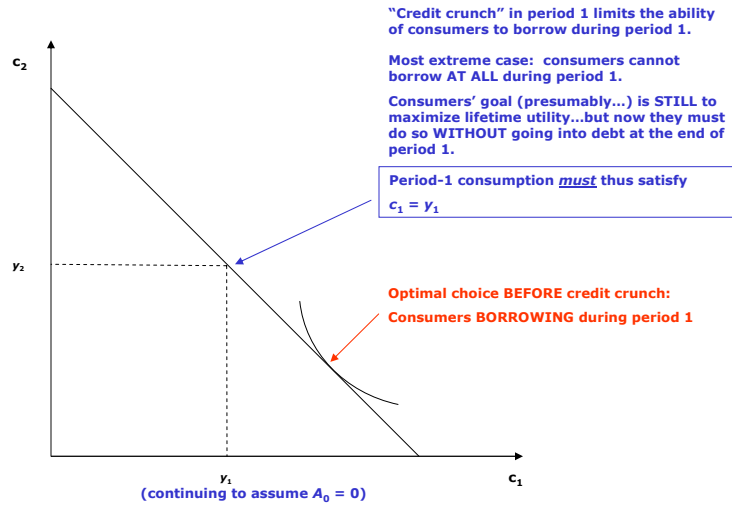
Most extreme case: consumers cannot borrow AT ALL during period 1.

Consumers' goal (presumably...) is STILL to maximize lifetime utility...but now they must do so WITHOUT going into debt at the end of period 1.

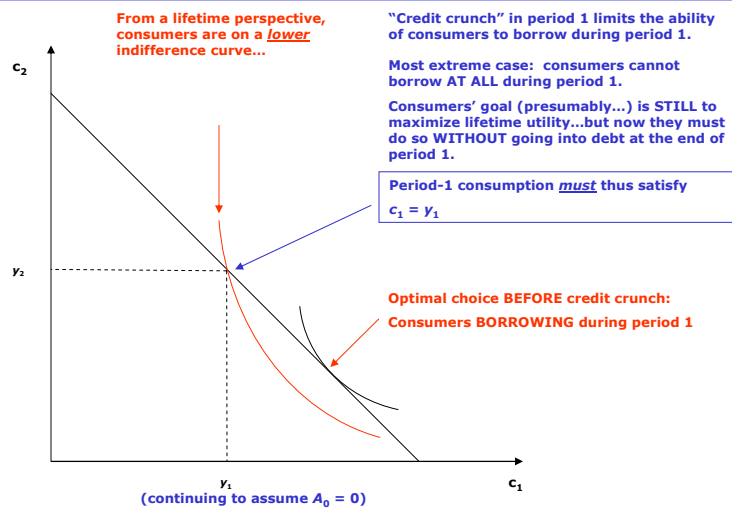


(continuing to assume $A_0 = 0$)

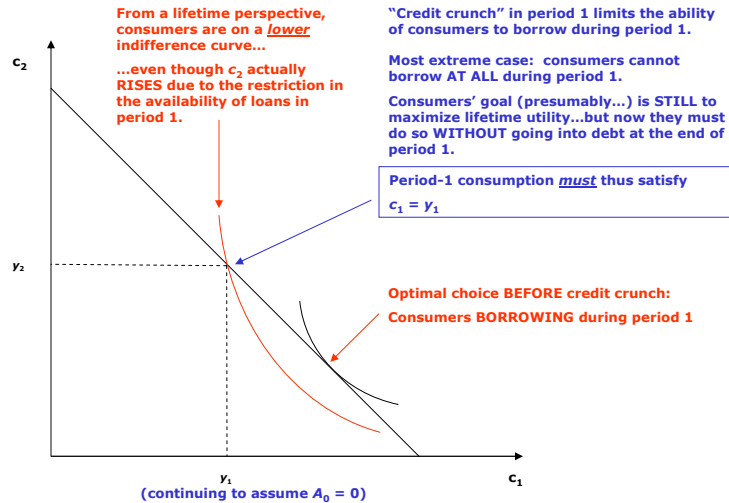
ASSESSING THE EFFECTS OF THE CREDIT CRUNCH



ASSESSING THE EFFECTS OF THE CREDIT CRUNCH



ASSESSING THE EFFECTS OF THE CREDIT CRUNCH



February 16, 2009

33

ASSESSING THE EFFECTS OF THE CREDIT CRUNCH

- ❑ "Credit crunch" – financial sector has restricted the quantity of loans it is willing to extend to consumers in the "short run"
 - ❑ Financial market data and bank surveys show quantity of loans made to consumers (car loans, home loans, personal loans, etc.) has shrunk over the past year
- ❑ Can analyze macroeconomic consequences of shrinkage of credit availability using two-period model
 - ❑ Interpret "short run" to be period 1 (i.e., 2008-2009)
- ❑ Consequences
 - ❑ A large fraction of consumers (though not all) unable to borrow to pay for their desired period-1 consumption → their period-1 (i.e., "short run") consumption falls
 - ❑ Consumption $\approx 2/3$ of GDP → period-1 (i.e., "short run") GDP falls

February 16, 2009

34

ASSESSING THE EFFECTS OF THE CREDIT CRUNCH

- ❑ "Credit crunch" – financial sector has restricted the quantity of loans it is willing to extend to consumers in the "short run"
 - ❑ Financial market data and bank surveys show quantity of loans made to consumers (car loans, home loans, personal loans, etc.) has shrunk over the past year
- ❑ Can analyze macroeconomic consequences of shrinkage of credit availability using two-period model
 - ❑ Interpret "short run" to be period 1 (i.e., 2008-2009)
- ❑ Consequences
 - ❑ A large fraction of consumers (though not all) unable to borrow to pay for their desired period-1 consumption → their period-1 (i.e., "short run") consumption falls
 - ❑ Consumption $\approx 2/3$ of GDP → period-1 (i.e., "short run") GDP falls
 - ❑ Consumption in period 2 (i.e., "the long run") actually rises
 - ❑ Interpretation: the credit-crunch is a necessary cleansing process that will eventually (i.e., in period 2) get the economy back to a healthy state

February 16, 2009

35

ASSESSING THE EFFECTS OF THE CREDIT CRUNCH

- ❑ "Credit crunch" – financial sector has restricted the quantity of loans it is willing to extend to consumers in the "short run"
 - ❑ Financial market data and bank surveys show quantity of loans made to consumers (car loans, home loans, personal loans, etc.) has shrunk over the past year
 - ❑ Can analyze macroeconomic consequences of shrinkage of credit availability using two-period model
 - ❑ Interpret "short run" to be period 1 (i.e., 2008-2009)
 - ❑ Consequences
 - ❑ A large fraction of consumers (though not all) unable to borrow to pay for their desired period-1 consumption → their period-1 (i.e., "short run") consumption falls
 - ❑ Consumption $\approx 2/3$ of GDP → period-1 (i.e., "short run") GDP falls
 - ❑ Consumption in period 2 (i.e., "the long run") actually rises
 - ❑ Interpretation: the credit-crunch is a necessary cleansing process that will eventually (i.e., in period 2) get the economy back to a healthy state
- OVERALL (i.e., lifetime utility of consumers FALLS (lower indiff. curve))

February 16, 2009

36