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Economics 325
Intermediate Macroeconomic Analysis
Practice Problem Set 8
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1. **Consumption, Savings, and Financing Constraints.** Because the consumption expenditures that occur later in an individual's lifetime are typically on "bigger-ticket items" (e.g., cars, refrigerators, etc.) and thus more expensive, an individual often has to begin planning for his/her future consumption expenditures well in advance of their actual purchase and use. In contrast, consumption expenditures during the earlier years of an individual's life are predominantly on "smaller" items (e.g., movies, food, entertainment, etc.) and thus might not require as much "advance planning."

We can analyze this idea in a two-period representative consumer framework. As always, suppose the representative consumer has utility function of period-1 consumption and period-2 consumption given by $u(c_1, c_2)$. Naturally, period 1 is the "early stage" of an individual's economic life, and period 2 is the "later stage" of an individual's economic life.

Suppose the financial assets that individuals have at their disposal are "stocks," just as we studied in Chapter 8. The representative consumer begins period 1 with zero stock holdings (i.e., $a_0 = 0$). The period-1 and period-2 budget constraints of the representative consumer are thus

$$P_1 c_1 + S_1 a_1 = Y_1$$
$$P_2 c_2 + S_2 a_2 = Y_2 + (S_2 + D_2) a_1$$

in which the rest of the notation is as always: P denotes the per-unit nominal price of a consumption good (in a given time period), Y denotes the nominal income the consumer earns (in a given time period), S denotes the nominal price of each share of stock (in a given time period), and D denotes the per-unit nominal dividend each share of stock pays (in a given time period).

Because period-2 goods are weighted more towards "big-ticket items," consumers typically have to borrow to purchase them. This means that asymmetric information issues may be a factor in lenders being willing to extend credit to consumers for their period-2 purchases. Suppose the financing constraint (aka credit constraint) that has evolved in markets to deal with these information issues is

$$P_2 c_2 = S_1 a_1 .$$

The interpretation of this constraint is that the market value of assets accumulated during period 1 of the individual's life forms the basis for period-2 consumption. Finally, just as in class, define the "nominal interest rate on stock" as

$$1+i^{STOCK}=\frac{S_2+D_2}{S_1},$$

define the "real interest rate on stock" as $1+r^{STOCK}=\frac{1+i^{STOCK}}{1+\pi_2}$, and, because this is a two-period framework, we know $a_2 = 0$.

a. Formulate the **sequential Lagrangian** for the representative consumer's utility maximization problem, starting, as usual, from the perspective of the beginning of period 1. (Several hints and notes are useful here: i) Do **not** substitute the financing constraint directly into any of the other constraints; it will be most informative to conduct the analysis with the financing constraint as a separate constraint; ii) Use the multiplier μ (the Greek letter "mu") for the financing constraint; iii) Be extremely careful about your setup of the Lagrangian here, because it is the basis for almost all of the analysis that follows!)

b. Based on the Lagrangian in part a, compute the first-order conditions with respect to c_1, c_2, and a_1.

c. If there were no financing constraint on consumers' purchases of period-2 consumption, the value of the Lagrange multiplier μ could be thought of as being equal to what numerical value? Be as precise as possible, and briefly explain. (**Note:** You can answer this part even if you were unable to get all the way through part a and part b.)

d. Using the definitions $1+i^{STOCK}=\frac{S_2+D_2}{S_1}$ and $1+r^{STOCK}=\frac{1+i^{STOCK}}{1+\pi_2}$, rearrange the first-order conditions you obtained in part b above to derive the **consumption-savings optimality condition.** Your final expression should be of the form $\frac{u_1(c_1,c_2)}{u_2(c_1,c_2)}=\ldots$, where the ellipsis on the right hand side indicate terms that you must determine. **Clearly explain/show the steps in your logic/derivation.** (**Note: It is fine if the final expression contains the multiplier μ in it.** This derivation requires a few algebraic steps, but the logic of the derivation is exactly the same as our initial study of the two-period framework.)

e. **(Harder)** Based on the consumption-savings optimality condition you derived in part d above, does the financing constraint $P_2c_2=S_1a_1$ "matter" for consumer's consumption and savings decisions over time? In other words, does the "standard" consumption-savings optimality condition studied in Chapter 3 and 4 get altered by the presence of this financing constraint? **If so, explain the economic intuition behind why; if not, explain the economic intuition behind why not.** (**Hint:** A diagrammatic explanation may be useful. In any case, there are likely several different ways to usefully describe the economic effects here.)