Department of Economics University of Maryland

Economics 325

Intermediate Macroeconomic Analysis

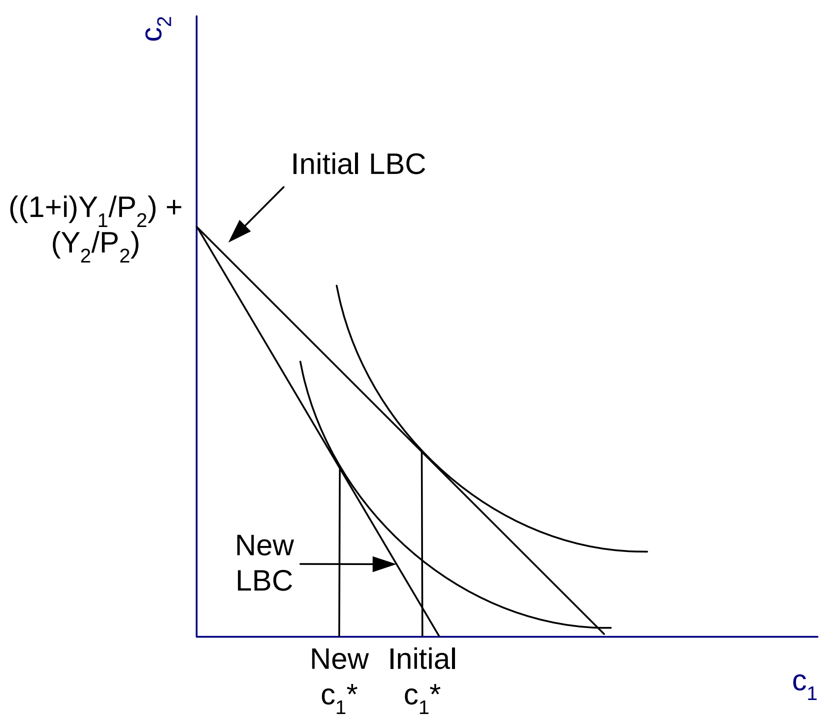
Practice Problem Set 3 Suggested Solutions

Professor Sanjay Chugh

Spring 2009

1. **The Wealth Effect on Consumption.** Consider the two-period consumption-savings model we have been developing in class.
   a. As in class, maintain the simplifying assumption that $A_0 = 0$. Show graphically how a rise in the period-1 nominal price of consumption can lead to a decrease in optimal consumption in period 1.

Solution: Recall from the standard two-period consumption-savings model that the when plotting the lifetime budget constraint (LBC) with c_2 on the vertical axis and c_1 on the horizontal axis, the price P_1 affects the slope but not the vertical intercept (refer to Figure 17 in Chapter 3). It follows that if P_1 rises, then the vertical intercept is unaffected but the budget line becomes steeper. As shown in the figure below, this can lead to lower consumption in period 1.



Notice that as drawn, the optimal choice of c_2 has also fallen. There is nothing in the structure of the model as we have discussed it that can lead us to definitively conclude that this must be the case – however, it is the most likely case.

b. Now suppose that $A_0 \neq 0$. Show graphically how a decrease in A_0 can lead to a decrease in optimal consumption in period 1.

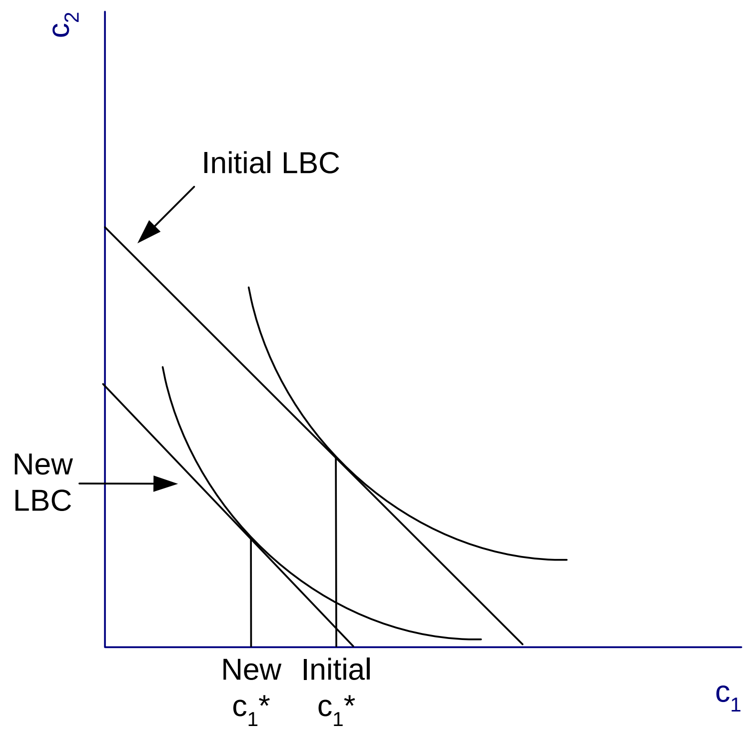
Solution: We saw that the LBC in the two-period model is

$$P_1 c_c + \frac{P_2 c_2}{(1+i)} = Y_1 + \frac{Y_2}{(1+i)} + (1+i)A_0 .$$

If we keep $A_0 \neq 0$, then solving for c_2 gives us the LBC

$$c_2 = -\left(\frac{P_1(1+i)}{P_2}\right)c_1 + \left(\frac{1+i}{P_2}\right)Y_1 + \frac{Y_2}{P_2} + \left(\frac{(1+i)^2}{P_2}\right)A_0 .$$

Clearly, A_0 (regardless of whether it is positive or negative) affects only the intercepts of the LBC but not the slope. Thus a decrease in A_0 leads to a parallel shift inwards of the LBC, thus leading to a fall in the optimal choice of c_1 as shown in the figure below.



c. The two effects you analyzed in parts a and b work through seemingly different channels. Actually, they are usefully thought of as operating through the same broadly-defined channel. Explain this broadly-defined channel.

Solution: The perfectly-rational representative-agent considers **all** his lifetime resources (both labor income as well as initial wealth) when making his optimal consumption-savings decision. Thus, both the change in price in part a and the change in initial wealth in part b (as well as possible changes in the nominal interest rate, the price of consumption in period 2, or labor income in either period!) have their effect by impacting the **real** value (as opposed to nominal value) of lifetime resources. The real value of lifetime resources is sometimes called "lifetime wealth" – thus, it is lifetime wealth that the individual considers when making his optimal consumption-savings choice.

2. **A Three-Period Economy.** Rather than the two-period consumption-savings model economy we have been developing in class, consider a three-period model that is analogous to the two-period model.
   a. Derive a relation similar to expression 11 on page 46 in the Lecture Text for the three-period economy (that is, derive the lifetime budget constraint (LBC) for the three-period economy). Define any new notation you introduce, and briefly explain the logic you use in deriving your final expression.

Solution: The period-3 budget constraint would be

$$P_3c_3 + A_3 = Y_3 + (1+i)A_2 \,,$$

where, analogous to our existing notation, c_3 is consumption in period 3, P_3 is the nominal price of consumption in period 3, Y_3 is nominal labor income in period 3, and A_3 is the wealth the individual chooses to carry from period 3 to period 4. Because there is no period 4 to save for, however, we know the individual will choose $A_3 = 0$. This condition **replaces** the condition $A_2 = 0$ we had in the two-period economy (that is, we can no longer simply say that $A_2 = 0$). Then perform the following steps of algebra (watch your algebra!): solve the period-3 budget constraint for A_2, then insert this expression for A_2 into the period-2 budget constraint. Solve the resulting expression for A_1 (tedious algebra here), and insert this into the period-1 budget constraint – the final expression you now have is the LBC for the three-period economy,

$$P_1c_1 + \frac{P_2c_2}{(1+i)} + \frac{P_3c_3}{(1+i)^2} = Y_1 + \frac{Y_2}{(1+i)} + \frac{Y_3}{(1+i)^2} + (1+i)A_0 \,.$$

Notice carefully the squared $(1+i)$ terms which now appear in the LBC.

b. Provide a brief interpretation of the LBC you derive in part a.

Solution: The interpretation of this LBC is exactly as before: it states that the present discounted value of all lifetime resources (which takes account both initial wealth as well as all lifetime labor income) equals the present discounted value of all lifetime consumption. Over the course of his lifetime, the individual spends all his lifetime resources on lifetime consumption.

c. In reality, there are an "infinite" number of periods. Write down the LBC for an infinite-period economy. (No need to be very mathematical – just use what you've learned in class and what you derived above).

Solution: The LBC for the infinite-period economy is

$$P_1c_1 + \frac{P_2c_2}{(1+i)} + \frac{P_3c_3}{(1+i)^2} + \frac{P_4c_4}{(1+i)^3} + \frac{P_5c_5}{(1+i)^4} + ... =$$
$$(1+i)A_0 + Y_1 + \frac{Y_2}{(1+i)} + \frac{Y_3}{(1+i)^2} + \frac{Y_4}{(1+i)^3} + \frac{Y_5}{(1+i)^4} + ...$$

where the ellipsis on both sides of the equality indicate the summations continue indefinitely. Alternatively, using sigma-notation from mathematics, the LBC can be written more compactly as

$$\sum_{i=1}^{\infty} \frac{P_ic_i}{(1+i)^{i-1}} = (1+i)A_0 + \sum_{j=1}^{\infty} \frac{Y_j}{(1+i)^{j-1}}.$$

d. The Permanent Income Hypothesis states that individuals consider their future lifetime earnings when making their current consumption decision. Discuss briefly how the multi-period models we are considering here (regardless of two-period, three-period, n-period, or infinite-period) are consistent with the Permanent Income Hypothesis.

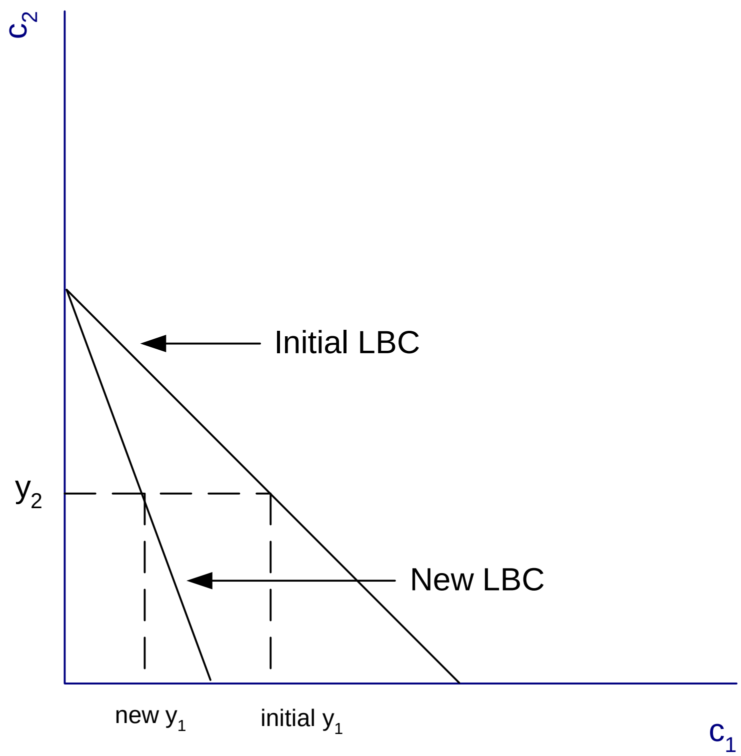
Solution: The Permanent Income Hypothesis is embodied in the very set-up of the consumption-savings model. The LBC shows that consumption in all periods depends on income over the course of the individual's entire lifetime – thus, in particular, consumption in any single period (period 1, say) depends on income in that period and all future periods.

3. **Mechanics of the Consumption-Savings Model.** Recall that in our two-period consumption-savings model, real labor income in any period is given by nominal

labor income divided by the price level (that is, recall $y_1 = Y_1 / P_1$ and $y_2 = Y_2 / P_2$). Suppose that nominal labor income in both periods is held constant. Clearly indicating the position of real labor income before and after each change on your diagrams, illustrate how the LBC is affected by the following events. As in class, make the simplifying assumption that the individual has zero initial wealth (i.e., $A_0 = 0$).

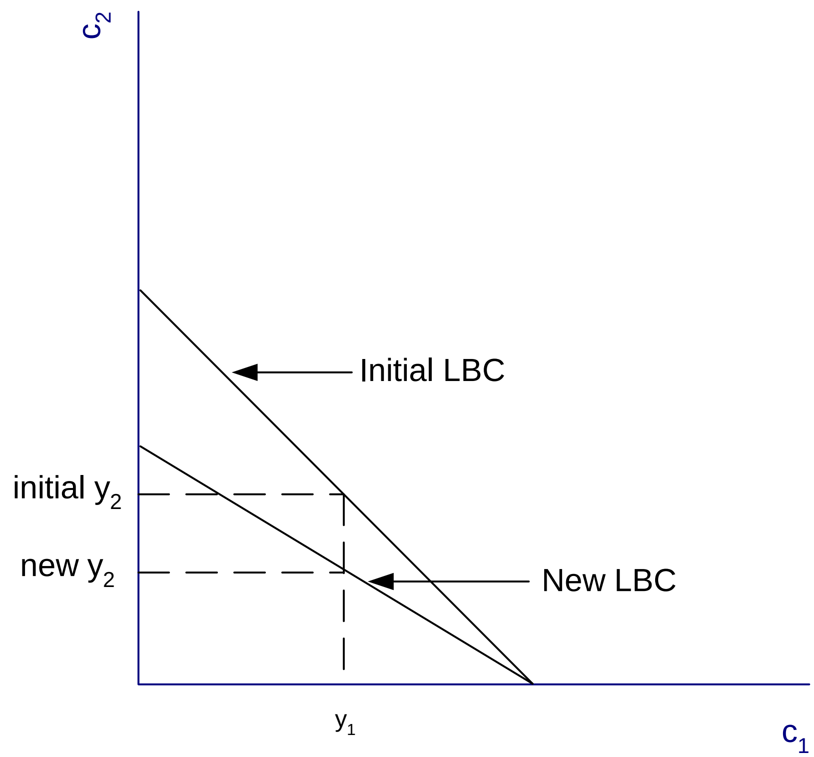
a. The price level in period 1, P_1, rises, while P_2 is held constant.

Solution: A rise in the price level in period 1 lowers real labor income in period 1 because $y_1 = Y_1 / P_1$ and Y_1 remains constant by assumption. Real labor income in period 2 is unaffected. The LBC becomes steeper because, recall, the slope of the LBC is $-(P_1(1+i)/P_2)$. The vertical intercept is unaffected because P_1 does not enter the expression for the vertical intercept of the LBC in nominal terms (see page 44 of the Lecture Notes). These effects are shown in the figure below:



b. The price level in period 2, P_2, rises, while P_1 is held constant.

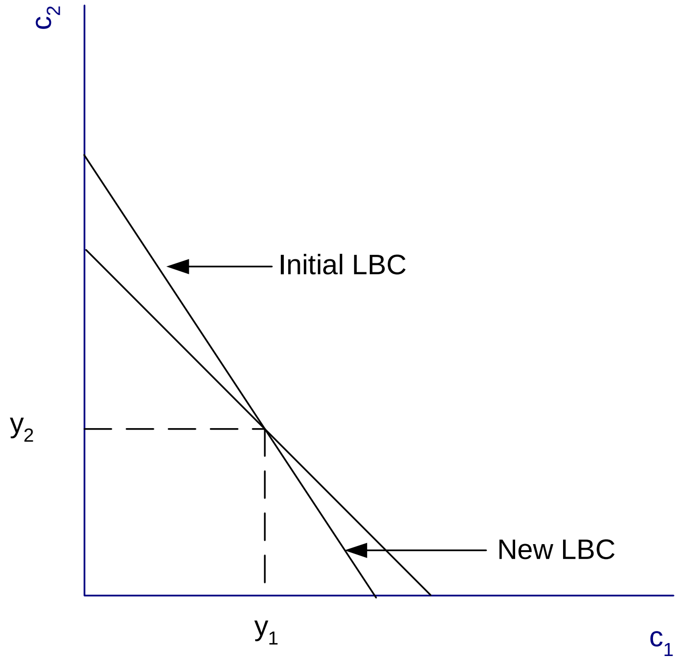
Solution: A rise in the price level in period 2 lowers real labor income in period 2 because $y_2 = Y_2 / P_2$ and Y_2 remains constant by assumption. Real labor income in period 1 is unaffected. The LBC becomes flatter because, recall again, the slope of the LBC is $-(P_1(1+i)/P_2)$. Here, the vertical intercept is now lower because P_2 does enter the expression for the vertical intercept of the LBC in nominal terms (again see page 44 of the Lecture Notes), and it is the horizontal intercept which remains fixed. These effects are shown in the figure below:



Note: In both questions 1a and 1b, you might have been thrown off if you were trying to use Figure 18 on page 52 of the Lecture Notes, in which you see that both y_1 and y_2 affect the vertical intercept. If you tried to base your analysis on this diagram and your answers were incorrect, it is probably because you failed to take account of the fact that the real interest rate rises in an exactly offsetting way in part a and falls in an exactly offsetting way in part b. This points out that the LBC in nominal terms and the LBC in real terms highlight different issues. In any given problem, it is usually more straightforward to use one rather than the other. As you might expect, when you are considering changes in nominal variables (prices, nominal interest rate, inflation), it is usually more straightforward to use the LBC in nominal terms.

c. The nominal interest rate i rises, while both P_1 and P_2 are held constant.

Solution: Real labor income in both periods is unaffected by the change in the nominal interest rate. The rise in i makes the LBC steeper by pivoting around the unchanged point (y_1, y_2), as shown in the diagram below:



4. **Taxes on Interest Earnings.** In our two-period consumption-savings model (with no leisure), suppose **positive** interest income in period 2 is taxed at the rate t_s, where $0 < t_s < 1$. That is, if interest income in period 2 is positive, then the government takes a fraction t_s of the interest income, while if interest income in period 2 is non-positive, then there is no tax. As in class, make the simplifying assumption that the individual has zero initial wealth (i.e., $A_0 = 0$). Also suppose that the interest tax has no effect on the nominal price level in either period.
   a. In this modified version of the model, algebraically express the period-1 budget constraint and the period-2 budget constraint of the individual.

   Solution: In the standard model we have studied, interest income in period 2 is given by the term iA_1. Here, when interest income is positive, the government taxes part of it away, leaving the individual with only part of his original interest income. In mathematical terms, we have here that

$$\text{interest income} = \begin{cases} i(1-t_s)A_1 & \text{if } c_1 < y_1 \\ iA_1 & \text{if } c_1 \geq y_1 \end{cases}$$

   The tax is only levied if the individual has positive wealth at the end of period 1, which only occurs if his consumption is less than his real labor income in period 1. Thus, notice that interest income, net of the tax, is a piecewise function. With this, the period-2 budget constraint is also piecewise:

$$P_2c_2 + A_2 = \begin{cases} Y_2 + (1+i(1-t_s))A_1 & \text{if } c_1 < y_1 \\ Y_2 + (1+i)A_1 & \text{if } c_1 \geq y_1 \end{cases}$$

The period-1 budget constraint is unaffected by the tax, so it is simply $P_1c_1 + A_1 = Y_1$ as usual.

b. Using your period-1 and period-2 budget constraints from part a, derive the individual's lifetime budget constraint (LBC). **(Hint:** Is the slope of this LBC continuous?)

Solution: As usual, we have that $A_2 = 0$ because period 2 is the last period of the economy. Solving the period-2 budget constraint for A_1 (our usual next step at this point), we get

$$A_1 = \begin{cases} \dfrac{P_2c_2}{1+i(1-t_s)} - \dfrac{Y_2}{1+i(1-t_s)} & \text{if } c_1 < y_1 \\ \dfrac{P_2c_2}{1+i} - \dfrac{Y_2}{1+i} & \text{if } c_1 \geq y_1 \end{cases}$$

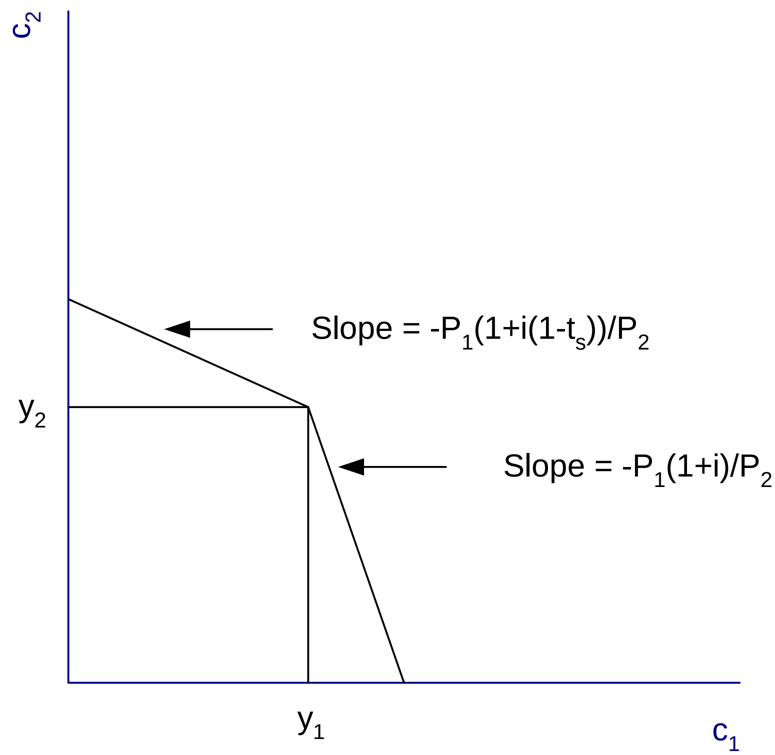
again piecewise. Substitute this expression for A_1 into the period-1 budget constraint to get the LBC

$$P_1c_1 + \frac{P_2c_2}{1+i(1-t_s)} = Y_1 + \frac{Y_2}{1+i(1-t_s)} \quad \text{if } c_1 < y_1$$
$$P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i} \quad \text{if } c_1 \geq y_1$$

Notice that if $c_1 \geq y_1$, the LBC is the same as in our standard model (because $t_s = 0$). If we solve the LBC for c_2 as a function of c_1, we find

$$c_2 = \begin{cases} -\left(\dfrac{P_1(1+i(1-t_s))}{P_2}\right)c_1 + \left(\dfrac{1+i(1-t_s)}{P_2}\right)Y_1 + \dfrac{Y_2}{P_2} & \text{if } c_1 < y_1 \\ -\left(\dfrac{P_1(1+i)}{P_2}\right)c_1 + \left(\dfrac{1+i}{P_2}\right)Y_1 + \dfrac{Y_2}{P_2} & \text{if } c_1 \geq y1 \end{cases}$$

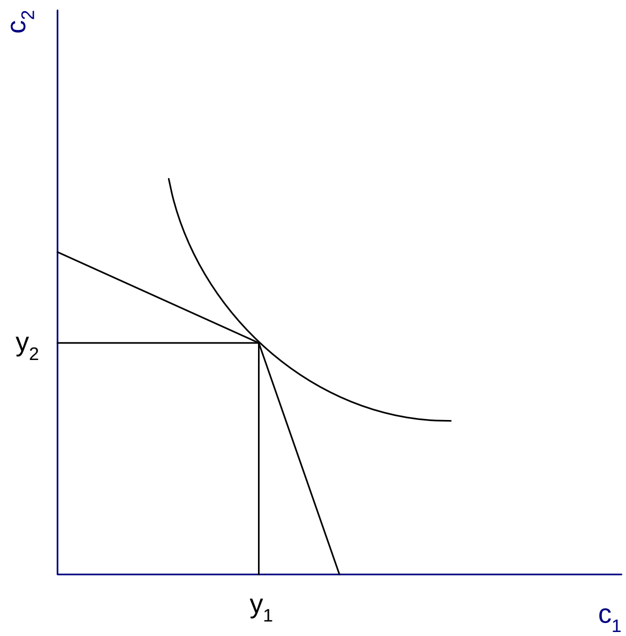
Thus, the LBC is a **piecewise linear function.** It changes slope discontinuously at the point (y_1, y_2): to the right of this point, the LBC is steeper than it is to the left of this point. Graphically, we have that the LBC has a kink in it:



c. Recall our assumption (based on empirical evidence) that the aggregate private savings function is an increasing function of the real interest rate. Suppose that at the representative agent's current optimal choice, he is choosing to consume exactly his real labor income in period 1.

   i. At his current optimal choice, is his marginal rate of substitution between present consumption and future consumption equal to (one plus) the real interest rate? Explain why or why not.

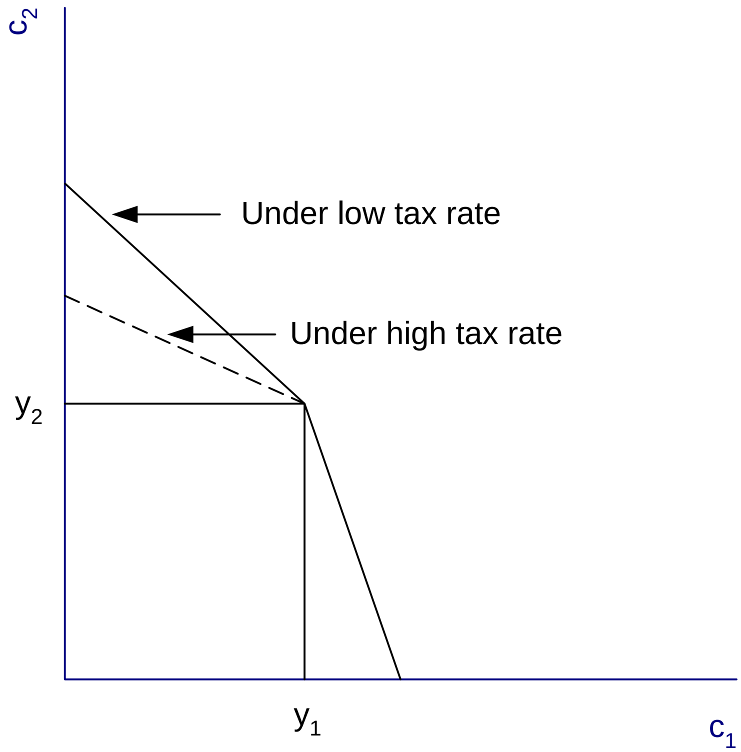
Solution: If the individual's optimal choice is $c_1^* = y_1$, then we must have the following situation:



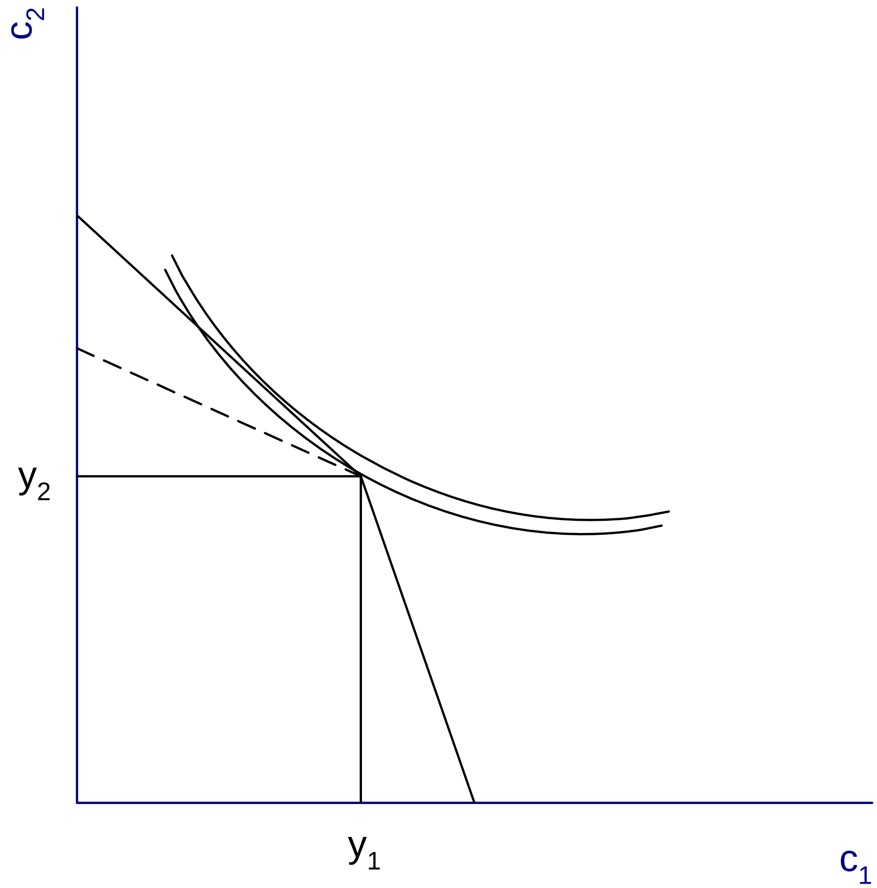
The indifference curve shown touches the LBC at the point (y_1, y_2) -- however, notice that it **not** tangent to the LBC at this point because the slope of the LBC at this point is undefined. That is, the slope has a (missing point) jump discontinuity at this point (try graphing the slope of the LBC for yourself to see it). Recall that the slope of an indifference curve is the marginal rate of substitution. From our basic microeconomics of consumer theory, we would be looking for the condition "marginal rate of substitution equals the price ratio" – however, here the price ratio (the real interest rate) is undefined at the optimal choice. Thus, the MRS does not equal the price ratio here – a perverse result that occurs here because the LBC has a kink in it **and** that kink point happens to be the point which gives the consumer the highest lifetime utility.

ii. President Bush, as part of his first-term economic agenda, lowered the tax rate on interest income from savings (one part of this package was eliminating the tax on dividends – but there are other elements of this idea in his tax package as well). Part of the rationale is that it will encourage individuals to save more. In this example, would a decrease in the tax rate t_s encourage the representative agent to save more in period 1? Explain why or why not?

Solution: As with many questions in economics, the answer here is that "it depends." Specifically, it depends on the precise nature of the representative agent's indifference map. If the tax rate on interest earnings is lowered, this affects the slope of the upper portion of the LBC but not the slope of the lower portion – specifically, it makes the upper portion of the LBC steeper (just plug in a lower value of t_s in the algebraic expressions in parts a and b above). Graphically:

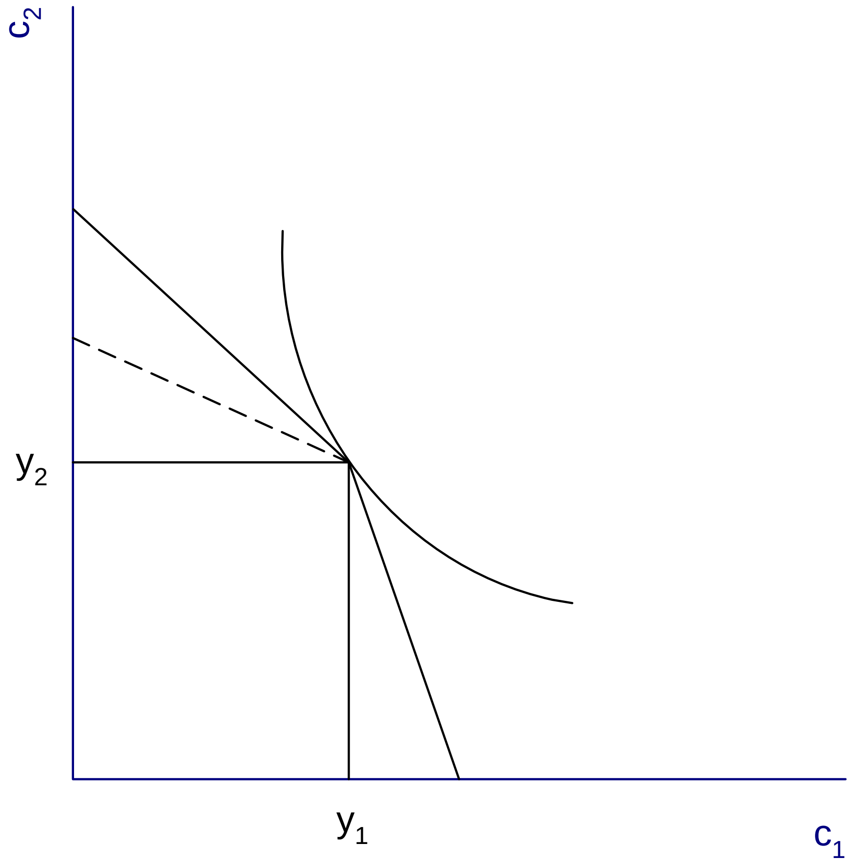


Now suppose the individual's indifference map is as follows:



With very flat indifference curves, it is possible that the indifference curve on which the point (y_1, y_2) lies intersects the new upper portion of the LBC, in which case there must be another indifference curve which is tangent to the upper portion of the LBC. Such a tangency would therefore show the new optimal choice – and because it is on the upper portion of the LBC, consumption in period 1 is less than real labor income in period 1, so indeed the reduction in the interest tax has led to an increase in savings in period 1.

However, this need not be the case. Consider instead the following situation:



If the indifference curves are very steep, as shown here, then it could be that despite the decrease in the interest tax, there still is no indifference curve that intersects the (new) upper portion of the LBC. If this is the case, then the original consumption choice (y_1, y_2) is still the optimal choice – here, savings are completely unaffected despite the decrease in the interest tax.

The economics to take away from this: despite the Bush economic team's pronouncements that lowering taxes on savings will necessarily lead to higher savings, it may or may not, depending on the "average American's" preferences over consumption in the present and consumption in the future. The second scenario shows a representative agent who simply does not "want" to save, regardless of the decrease in the interest tax. A simple analogy: if you absolutely hate pizza and it makes you sick, hearing that the pizza parlor down the street is now offering slices for one cent is not going to make you run out and buy pizza.