Department of Economics University of Maryland

Economics 325
Intermediate Macroeconomic Analysis
Practice Problem Set 6 Suggested Solutions

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1. **Deriving a Money Demand Function.** Denote by $\phi(c_t, i_t)$ the **real** money demand function. Here you will generate particular functional forms for $\phi(\cdot)$ using the MIU model we have studied.

   In an MIU model, recall that the consumption-money optimality condition can be expressed as

$$\frac{u_{m_t}}{u_{c_t}} = \frac{i_t}{1+i_t},$$

   where u_{m_t} denotes marginal utility with respect to **real** money balances and u_{c_t} denotes marginal utility with respect to consumption. In each of the following, you are given a utility function and its associated marginal utility functions. For each case, construct the consumption-money optimality condition and use it to generate the function $\phi(\cdot)$. In each case, your money demand function should end up being an increasing function of c_t and a decreasing function of i_t. (**Note:** Be careful to make the distinction between real money holdings and nominal money holdings. The marginal utility function u_{m_t} is marginal utility with respect to **real** money holdings.)

Solution: For each utility function, we have now written the marginal utility functions u_{c_t} and u_{m_t}. Also note that you are, in each question, being asked to solve for $\frac{M_t}{P_t}$ as a function of c_t and i_t, which is the consumer's real money demand.

a. $u\left(c_t, \frac{M_t}{P_t}\right) = \ln c_t + \ln\left(\frac{M_t}{P_t}\right)$, with $u_{c_t} = \frac{1}{c_t}$ and $u_{m_t} = \frac{1}{M_t / P_t}$.

Solution: Constructing the consumption-money optimality condition with the given functions, we have

$$\frac{u_{m_t}}{u_{c_t}} = \frac{1/(M_t / P_t)}{1/c_t} = \frac{P_t c_t}{M_t} = \frac{i_i}{1+i_t}.$$

Solving for M_t / P_t, we have

$$\frac{M_t}{P_t} = \frac{c_t(1+i_t)}{i_t}.$$

Thus, the function $\phi(\cdot)$ function is $\phi(c_t, i_t) = \frac{c_t(1+i_t)}{i_t}$, which is increasing in consumption and decreasing in the nominal interest rate, as expected.

b. $u\left(c_t, \frac{M_t}{P_t}\right) = 2\sqrt{c_t} + 2\sqrt{\frac{M_t}{P_t}}$, with $u_{c_t} = \frac{1}{\sqrt{c_t}}$ and $u_{m_t} = \frac{1}{\sqrt{M_t / P_t}}$.

Solution: Proceeding as above, the consumption-money optimality condition is

$$\frac{u_m}{u_{c_t}} = \frac{1/\sqrt{M_t / P_t}}{1/\sqrt{c_t}} = \frac{\sqrt{P_t}\sqrt{c_t}}{\sqrt{M_t}} = \frac{i_i}{1+i_t}.$$

Solving for M_t / P_t, we have

$$\frac{M_t}{P_t} = \frac{c_t(1+i_t)^2}{i_t^2}$$

(be careful with the algebra here – notice the squared terms in the solution). Thus, the function $\phi(\cdot)$ function is $\phi(c_t, i_t) = \frac{c_t(1+i_t)^2}{i_t^2}$, which is increasing in consumption and decreasing in the nominal interest rate, again as expected.

c. $u\left(c_t, \frac{M_t}{P_t}\right) = c_t^{\sigma} \cdot \left(\frac{M_t}{P_t}\right)^{1-\sigma}$, with $u_{c_t} = \sigma c_t^{\sigma-1}\left(\frac{M_t}{P_t}\right)^{1-\sigma}$ and $u_{m_t} = (1-\sigma)c_t^{\sigma}\left(\frac{M_t}{P_t}\right)^{-\sigma}$.

Solution: The consumption-money optimality condition is

$$\frac{u_m}{u_{c_t}} = \frac{1-\sigma}{\sigma} \cdot \frac{c_t^{\sigma}(M_t / P_t)^{-\sigma}}{c_t^{\sigma-1}(M_t / P_t)^{1-\sigma}} = \frac{i_t}{1+i_t}.$$

After combining exponents, we can write this as

$$\frac{1-\sigma}{\sigma} \cdot c_t \cdot \frac{P_t}{M_t} = \frac{i_i}{1+i_t}.$$

Solving for M_t / P_t, we have

$$\frac{M_t}{P_t} = \frac{1-\sigma}{\sigma} \cdot \frac{c_t(1+i_t)}{i_t}.$$

Thus, the function $\phi(\cdot)$ is $\phi(c_t, i_t) = \frac{1-\sigma}{\sigma} \cdot \frac{c_t(1+i_t)}{i_t}$.

2. **The Keynesian-RBC-New Keynesian Evolution.** Here you will briefly analyze aspects of the evolution in macroeconomic theory over the past 25 years.

a. Describe **briefly** what the Lucas critique is and how/why it led to the demise of (old) Keynesian models.

Solution: The old Keynesian models were large estimated systems of equations, and the estimated coefficients could not (because they were just based on historical observations) take into account how behavior might change if policy changed. In the 1970's, this led to the downfall of such models as policy-makers tried more and more to exploit these relationships, but the "coefficients" began to vary a lot (for some reason…) with policy, eventually causing the profession (through the Lucas critique) to understand that such models really were not all that useful for policy advice after all.

b. Briefly define and describe the neutrality vs. nonneutrality debate surrounding monetary policy today. Which type of shock does this debate concern?

Solution: The RBC view holds that money shocks do not affect real variables (i.e., consumption or GDP) in the economy (neutrality), while the New Keynesian view holds that they do (nonneutrality) because prices take time to adjust (are "sticky").

3. **A Quantitative Look at Monetary Policy.** On June 25, 2003, the Federal Reserve's main policy-setting committee, the Federal Open Market Committee (FOMC), announced it was immediately lowering its target for the Federal Funds interest rate from 1.25% to 1.00%. To implement this policy, it thus needed to conduct an open-market operation on June 25, 2003. Assume the following:

- The Fed could purchase or sell as many bonds as it wanted to at the equilibrium price of bonds which prevailed on June 24, 2003 (i.e., the day before it announced its policy);
- On June 24, there were 1,000 bonds on the open market, **each of which promised to pay a face value of $1.00 (one dollar) on June 24 (or June 25 if you like…), 2004;**
- The money supply curve is vertical;
- Before the Fed action, the domestic money supply was $499.50, and the Fed has complete control over the domestic money supply;
- The money demand function and the bond demand function are given, respectively, by

$$M^D = 993 - 39,480i$$

$$B^D = 4,185 - 3,227P_b,$$

where, as in class, i denotes the nominal interest rate and P_b denotes the price of each bond (which, recall, pays $1 in one year's time). A reminder that a nominal interest rate of, for example, 1.25% , means that $i = 0.0125$.

Recall from introductory macroeconomics the notion of the **money multiplier.** For our purposes here, the presence of a money multiplier means that the total change in the money supply of an economy is equal to the money multiplier times the initial injection of money into the economy. Suppose that the money multiplier is 10 – that is, if the Fed injects $1 into the economy, the money supply actually increases by $10. Accurate to at least three decimal places, compute each of the following:

- The price of a bond before the Fed action;
- Total domestic money supply following the Fed action;
- The amount of dollars given (or taken) by the Federal Reserve to (or from) the banking system (i.e., the "initial injection");
- Total bond supply on the open market following the Fed action;
- The price of a bond following the Fed action

Solution: You had to proceed very systematically through this problem, although there were in a few places alternative paths through the same forest. The point of this problem is to think through how in practice the Fed must implement the policy actions it announces.

In the given money demand function, first note that inserting $i = 0.0125$ (the target rate before the policy change) gives $M = 499.5$, which is exactly the given money supply before the Fed policy (this is simply a check to show that the money market was indeed in equilibrium before the Fed policy change – you did not need to conduct this check).
To compute the price of each bond, we can use the bond demand function along with the given initial 1000 unit of bonds – inserting $B = 1000$ (because the bond market must always be in equilibrium) in the bond demand function gives us $P_b = \$0.987$, which is the price of each bond before the Fed action. Alternatively, we could have computed the price of the bond before the Fed action by recalling that $P_b = \frac{FV}{1+i}$. Here, $FV = 1$ (because $1 will be paid by the bond one year hence) and $i = 0.0125$ before the Fed action, thus $P_b = 0.987$.

Using the new target rate $i = 0.01$ in the money demand function, we find that $M^D = 598.2$ following the Fed action. Since the money market must be in equilibrium, this immediately implies that the money supply following the Fed action is 598.2, thus the quantity of money in the economy increased by (598.2 - 499.5) = 98.7 due to the Fed action. **But this is after the money multiplier process has unfolded.**

You're told that the Fed's initial injection is one-tenth the ultimate increase of the money supply (the money multiplier process), so the Fed actually gave to the banking system $9.87.

Because the Fed spent \$9.87 on bond purchases, and the price it paid per bond was 98.7 cents, it clearly purchases 10 bonds – thus, bond supply on the open market fell by 10 bonds, so bond supply following the Fed action is 990 bonds.

Using the bond demand curve and the new value $B = 990$, we can compute that $P_b = \$0.9901$ following the Fed action. Alternatively, since $P_b = \frac{1}{1+i}$, and the new nominal interest rate is $i = 0.01$, we can compute $P_b = \$0.9901$ using this relationship as well.