Department of Economics University of Maryland

Economics 325

Intermediate Macroeconomic Analysis

Problem Set 1 Suggested Solutions

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Instructions: Written (typed is strongly preferred, but not required) solutions must be submitted no later than 11:00am on the date listed above (either in class or in the Economics Department Main Office, Tydings Hall 3105). Your solutions, which likely require some combination of mathematical derivations, economic reasoning, graphical analysis, and pure logic, should be thoroughly presented and not leave the reader (i.e., your TAs and I) guessing about what you actually meant.

You must submit your own independently-written solutions. You are permitted (in fact, encouraged) to work in groups to think through issues and ideas, but your "writing up" of solutions should be done independently of anyone else. **Under no circumstances will multiple verbatim identical solutions be considered acceptable.**

There are three problems in total, each with multiple subparts.

Problem 1: Optimal Choice in the Consumption-Savings Model During a Credit Crunch: A Numerical Analysis. Consider a two-period economy (with no government and hence neither government spending nor taxation), in which the representative consumer has no control over his income. The lifetime utility function of the representative consumer is $u(c_1, c_2) = \ln c_1 + \ln c_2$, where $\ln$ stands for the natural logarithm. We will work here in purely real terms: suppose the consumer's real income in period 1 is y_1 = 10 and the consumer's real income in period 2 is y_2 = 22. Suppose that the real interest rate between period 1 and period 2 is ten percent (i.e., r = 0.10), and also suppose the consumer begins period 1 with **real** net wealth (inclusive of interest) of $(1+r)a_0 = 2$.

Set up the lifetime Lagrangian formulation of the consumer's problem, and use it to answer part a, b, and c. Show all steps in your logic/arguments.

Solution: The Lagrangian in real terms, using the given functional form for utility, is

$$\ln c_1 + \ln c_2 + \lambda\left[y_1 + \frac{y_2}{1+r} + (1+r)a_0 - c_1 - \frac{c_2}{1+r} \right],$$

where the term in square brackets (when set equal to zero) is simply the LBC in real terms. The first-order conditions of this problem are (recognizing that the FOC with respect to λ is simply the LBC in real terms):

$$\frac{1}{c_1} - \lambda = 0$$

$$\frac{1}{c_2} - \frac{\lambda}{1+r} = 0$$

Combining these two equations to eliminate the multiplier as usual gives the consumption-savings optimality condition for this particular utility function:

$$\frac{c_2}{c_1} = 1 + r .$$

In what follows, you must use the consumption-savings optimality condition along with the LBC (which together constitute two equations in the two unknowns c_1 and c_2) to proceed.

a. **Is** it possible to numerically compute the consumer's optimal choice of consumption in period 1? If so, compute it; if not, explain why not.

Solution: From the consumption-savings optimality condition just derived, we have a relationship between c_1 and c_2 at the optimal choice: $c_2 = (1+r)c_1$. Inserting this relation into the LBC, we have

$$c_1 + \frac{(1+r)c_1}{1+r} = y_1 + \frac{y_2}{1+r} + (1+r)a_0 .$$

Solving this for c_1, we have that the optimal choice of period-one consumption is $c_1 = \frac{1}{2}\left[y_1 + \frac{y_2}{1+r} + (1+r)a_0 \right]$. Inserting the given numerical values, we have $c_1 = 16$ as the quantity of optimal period-one consumption.

b. Is it possible to numerically compute the consumer's optimal choice of consumption in period 2? If so, compute it; if not, explain why not.

Solution: Using the relation at the optimum derived above, namely, $c_2 = (1+r)c_1$, and using the computed solution in part a and the given value for r, we have $c_2 = 17.6$.

c. Is it possible to numerically compute the consumer's real asset position at the end of period 1? If so, compute it; if not, explain why not.

Solution: To compute the asset position at the end of period one, use the period-one budget constraint in real terms: $c_1 + a_1 = y_1 + (1+r)a_0$. Using the given values and the solution for c_1 computed above, we have that the consumer's asset position at the end of period 1 is $a_1 = -4$. Thus, in the absence of any restrictions on borrowing (as occurs in parts e and f below) the representative consumer is in **debt** at the end of period 1. (Note: a common error here may have been to simply compute $a_1 = y_1 - c_1$; this is incorrect here due to the presence of non-zero initial net assets.)

d. Is it possible to numerically compute the consumer's level of **borrowing or savings** (be explicit about the sign) during period 1? If so, compute it; if not, explain why not.

Solution: Savings (recall that savings is a **flow**) during period 1 is defined as $a_1 - a_0$. You just computed $a_1 = -4$ above. From the given values $r = 0.10$ and $(1+r)a_0 = 2$, you can compute that $a_0 = 1.818$. Hence, savings during period 1 is $(-4) - 1.818 = -5.818$. The consumer thus not only dissaved his entire his initial net assets, he also further borrowed resources (which will be repaid during period 2).

For parts e and f of this problem, suppose that lenders to this consumer impose **credit constraints** (a "credit crunch") on the consumer. Specifically, they impose the tightest possible credit constraint – the consumer is not allowed to have any debt at all at the end of period one, which means that the consumer's real wealth at the end of period one must be nonnegative ($a_1 \geq 0$).

e. With this credit constraint in place, compute numerically the consumer's optimal choice of period-one and period-two consumption? Briefly explain, either logically or graphically or both. (**Hint:** is it even possible to set up a Lagrangian "as usual" here?)

Solution: The analysis here is purely logical. Above, we found that the consumer **optimally** chose to be in debt at the end of period 1 (i.e., we found $a_1 < 0$). Now, with the credit constraint, this is no longer possible. The goal of the consumer is still to maximize his lifetime utility. Thus, maximizing utility here entails choosing a level of consumption in period 1 such that the credit restriction is satisfied as "loosely" as possible – which involves choosing a level of c_1 such that a_1 ends up being **exactly equal to zero** (because this is the point at which the credit restriction is just barely satisfied). From the period-one budget constraint, $c_1 + a_1 = y_1 + (1+r)a_0$, and imposing $a_1 = 0$, we find that the choice of period-1 consumption **in the presence of the credit crunch** is $c_1 = 12$ (i.e., it is simply equal to the consumer's period-1 income **and** initial assets).

f. Compared to your analysis in parts a, b, c, and d, does the credit constraint enhance or diminish welfare (i.e., does it increase or decrease total lifetime utility)? Justify your answer with some appropriate combination of logic, graphs, and/or mathematics.

Solution: With $a_1 = 0$, we can readily compute from the period-2 budget constraint, $c_2 + a_2 = y_2 + (1+r)a_1$, that (with the maintained assumption $a_2 = 0$) that consumption in period 2 is $c_2 = y_2 = 22$. Thus, the credit crunch in this case ends up making period-2 consumption exactly equal to period-2 income.

We can compute the total utility of the optimal combinations of (c_1, c_2) in both parts a and b as well as parts e and f. Comparing these two levels of total utility, it is clear that total utility is **smaller** in the presence of the credit crunch than when consumers can freely borrow. Thus, in this case, because of the reduced willingness of lenders to let consumers be in debt to them due to the "credit crunch," consumers cannot borrow as much as they would like and total lifetime welfare (utility) is diminished.

Suppose now that the consumer experiences a temporary increase in real income in period one to $y_1 = 18$, with real income in period two still $y_2 = 22$.

g. Calculate numerically the effect of this positive surprise in income on c_1 and c_2, supposing that there is **no** credit constraint on the consumer.

Solution: The algebra here proceeds exactly as in parts a and b; using the new numerical value for y_1, we have that the optimal choices are $c_1 = 20$ and $c_2 = 22$. Note that, although you did not need to calculate it, you could have calculated from the period-1 budget constraint, $c_1 + a_1 = y_1 + (1+r)a_0$, that $a_1 = 0$ is the optimal amount of indebtedness at the end of period 1.

h. Finally, suppose that the credit constraint you analyzed in parts e and f is back in place. In this case, will the credit crunch affect the consumer's choices (compared to part g, that is)? (**Note:** in this part, there is no need to compute anything – all that is required is a logical and/or graphical argument.)

Solution: Given that the optimal amount of indebtedness at the end of period 1 is now zero, the onset of the credit crunch has **no effect whatsoever**. This is simply because with this level of y_1, consumers have no need to borrow from the financial sector; this is an instance of problems in credit markets **not** spilling over to consumption (and hence real macroeconomic) activity.

Problem 2: A Contraction in Credit Availability. The graph below shows our usual two-period indifference-curve/budget constraint diagram, with period-1 consumption plotted on the horizontal axis, period-2 consumption plotted on the vertical axis, and the downward-sloping line representing, as always, the consumer's LBC. Throughout all of the analysis here, assume that ***r* = 0 always.** Furthermore, there is no government, hence never any taxes.

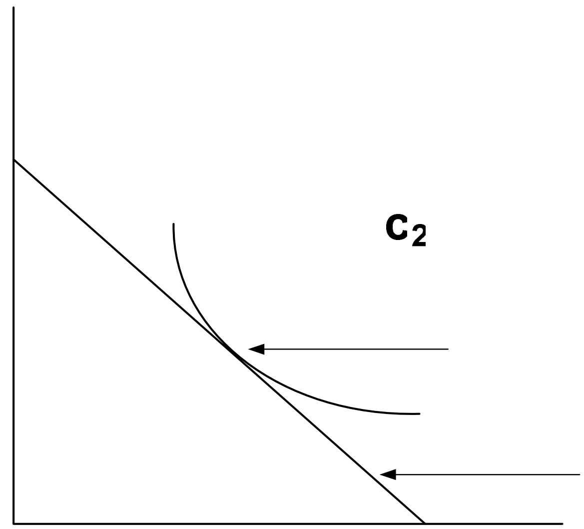
Suppose that the representative consumer has lifetime utility function $u(c_1, c_2) = \ln c_1 + \ln c_2$, and that the **real** income of the consumer in period 1 and period 2 is $y_1 = 12$ and $y_2 = 8$. Finally, suppose that the initial quantity of net assets the consumer has is $a_0 = 0$. **EVERY** consumer in the economy is described by this utility function and these values of y_1, y_2, and a_0.

a. If there are no problems in credit markets whatsoever (so that consumers can borrow or save as much or as little as they want), compute the numerical value of the optimal quantity of period-1 consumption and period-2 consumption. In solving this problem, you should set up an appropriate Lagrangian to do so and show/explain all steps in your logic.

Solution: The consumption-savings optimality condition (given the natural-log utility function) is given by $c_2/c_1 = 1+r = 1$ (the second equality follows because $r = 0$ here). Thus, at the optimal choice, it is the case that $c_1 = c_2$. Using this relationship (and again using the fact that $r = 0$ here), we can express the consumer's LBC as $c_1 + c_1 = y_1 + y_2 = 20$, which obviously implies the optimal choice of period-1 consumption is $c_1 = 10$.

Note: although you were not asked to compute it, you could have computed the implied value of the consumer's asset position at the end of period one. Because $a_0 = 0$, $y_1 = 12$, and we just computed $c_1 = 10$, the asset position at the end of period one is $a_1 = y_1 - c_1 = 2$ (i.e., **positive** 2).

b. Now suppose that because of problems in the financial sector, no consumers are allowed to be in debt at the end of period 1 because banks will not allow them to be. With this credit restriction in place, compute the numerical value of the optimal quantity of period-1 consumption. **ALSO,** on the diagram below (which you should reproduce in your solutions), qualitatively and **clearly** sketch the optimal choice with this credit restriction in place (qualitatively sketched already for you is the optimal choice if there are no problems in credit markets). Your sketch should indicate **both** the new optimal choice **and** an appropriately-drawn and labeled indifference curve that contains the new optimal choice. (**Note:** the analysis here is largely logical/qualitative, with very little mathematics required.)



Solution: Because in part a (ie, without any credit restrictions), the representative consumer was choosing to NOT be in debt at the end of period 1 (ie, $a_1 > 0$ under the optimal choice in part a), the imposition of the credit restriction, **nothing changes compared to part a.** That is, the optimal choice of period-1 consumption is still 10. Hence, in the diagram below, the optimal choice in the presence of credit constraints is **exactly the same as the optimal choice without credit constraints.** The general lesson to draw from this example and our analysis in class is that it is not *necessarily* the case that financial market problems *must* and *always* spill over into real economic activity (i.e., consumption in this case).

Problem 3: Analyzing the Fiscal Stimulus Plan (24 points). In this problem, you will analyze a couple of broad aspects of the Obama administration's fiscal stimulus plan using the two-period model of the government presented in Chapter 7.

Let's interpret "period 1" to be the two-year period January 1, 2009-December 31, 2010, and "period 2" to be January 1, 2011 and beyond (note: there is no requirement in the two-period model that the two "periods" be of equal time length). Equivalently, you can interpret "period 1" to be the "short run," and period 2 to be the "long run."

Under pre-existing fiscal plans (i.e., if the fiscal plans under the Bush administration were simply rolled over into the present and the future), the fiscal deficit during 2009-2010 (i.e., "period 1") would be approximately \$1.4 trillion. Also, the federal government debt as of the beginning of 2009 is approximately \$10 trillion. In terms of the notation of our Chapter 7 framework, this means b_0 = - \$10 trillion and $t_1 - g_1$ = -\$1.4 trillion (note the minus sign).

Interest rates are extremely low right now, so let's make the further assumption that $r = 0$.

General remarks about solutions: parts a, b, c, and d of this problem are nothing more than an application of appropriate definitions and budget constraint relationships.

a. **(3 points)** Under current fiscal plans (i.e., if the Obama administration did not change any aspects of fiscal policy whatsoever), compute the numerical value of the fiscal surplus or deficit that would need to be run in 2011 and beyond (i.e., in "period 2"). Present your logic and provide brief economic explanation.

Solution: Under the maintained assumption of $r = 0$, the lifetime government budget constraint is

$$g_1 + g_2 = t_1 + t_2 + b_0 \,.$$

We can rearrange this expression in the following way:

$$b_0 = (g_1 - t_1) + (g_2 - t_2)\,,$$

which highlights the relationship between the quantity of government debt outstanding at the start of 2009 (i.e., "period 1") and the fiscal government deficits in both "period 1" (the combined 2009-2010 years) and "period 2" (everything beyond 2010). You are given that b_0 = -10 trillion, and that under pre-existing fiscal plans, the fiscal deficit during 2009-2010 would be 1.4 trillion – this means that $g_1 - t_1 = 1.4$ trillion. Be careful about signs here! If the government is running a *deficit* in period 1, that means government spending exceeds tax revenue in period 1, which in turn means the quantity $g_1 - t_1$ is *positive*. Using the above expression (and again being carefully with signs!), we can them immediately conclude that

$$g_2 - t_2 = b_0 - (g_1 - t_1) = -10 \text{ trillion} - (1.4 \text{ trillion}) = -11.4 \text{ trillion}$$

In period 2, the government must, under pre-existing fiscal plans, run a **surplus** of \$11.4 trillion to pay off its pre-existing debt and the deficit-induced borrowing that occurs during period 1.

b. **(3 points)** Under current fiscal plans (i.e., if the Obama administration did not change any aspects of fiscal policy whatsoever), compute the numerical value of the government's asset/debt position at the end of 2010 (i.e., at the end of "period 1"). **Be clear about the sign.** Present your logic and provide brief economic explanation.

Solution: The period-1 budget constraint of the government (again under the maintained assumption that $r = 0$) is $g_1 + b_1 = t_1 + b_0$. What you are asked to solve for is b_1. Rearranging the period-1 budget constraint, we have

$$b_1 = (t_1 - g_1) + b_0$$
$$= -1.4 \text{ trillion + (-10 trillion) = -11.4 trillion}$$

The government **debt** (because $b1$ has a negative sign) is \$11.4 trillion at the end of period 1. (Note that this reconciles exactly with the **surplus** the government must run in period 2, computed in part a above – this makes sense because in a **lifetime** sense, the government has to pay for all its debts and spending.)

Under Obama's plan, the combined 2009-2010 fiscal deficit would be approximately \$2 trillion.[1]

c. **(3 points)** Under the Obama plan, compute the numerical value of the fiscal surplus or deficit that would need to be run in 2011 and beyond (i.e., in "period 2"). Present your logic and provide brief economic explanation.

Solution: Using exactly the same logic as in part a, we have, under Obama's program, that

$$g_2 - t_2 = b_0 - (g_1 - t_1) = -10 \text{ trillion} - (2 \text{ trillion}) = -12 \text{ trillion},$$

meaning a \$2 trillion surplus needs to be generated in period 2 (larger than what we found in part a). The period-2 surplus is larger because the period-1 deficit is larger under the Obama plan and the government must eventually (i.e., in the long run) repay all its debt.

d. **(3 points)** Under the Obama plan, compute the numerical value of the government's debt/asset position at the end of 2010 (i.e., at the end of "period 1"). **Be clear about the sign.** Present your logic and provide brief economic explanation.

[1] Thus, the Obama plan would *raise the fiscal deficit* by about \$600 billion in 2009 and 2010 combined. This is smaller than the approximately "\$800 billion stimulus plan" that has captured headlines; this is simply because part of the stimulus plan takes effect beyond 2010 (which itself has been a point of contention).

Solution: Using exactly the same logic as in part b, we have, under Obama's program, that

$$b_1 = (t_1 - g_1) + b_0$$
$$= -2 \text{ trillion} + (-10 \text{ trillion}) = -12 \text{ trillion}$$

obviously a larger quantity of **debt** at the end of period 1 than if the Obama plan were not enacted. (Note again that this reconciles exactly with the **surplus** the government must run in period 2, computed in part c above – this again makes sense because in a **lifetime** sense, the government has to pay for all its debts and spending.)

A point of contention regarding the stimulus plan has been whether it should be composed mostly of increases in government spending or mostly of tax cuts. In what follows, you will analyze the two polar extreme possibilities.

e. **(4 points)** Suppose that the fiscal stimulus is composed entirely of increased government spending in the years 2009-2010 (i.e., in "period 1"), with no changes in fiscal policy whatsoever beyond that. Assuming that net exports and investment are both zero AND assuming that Ricardian Equivalence holds, will the increase in government spending raise total GDP during the years 2009-2010? Carefully explain your logic, including what role Ricardian Equivalence plays in your logic.

Solution: The increased in government expenditure (should…) directly lead to an increase in GDP because it is simply a "direct effect." Ricardian Equivalence plays no role in the logic here whatsoever because Ricardian Equivalence is about the possible (non)effects on private-sector economic activity as a result of changes in *tax policy,* not as a result of changes in government spending policy. That is, even though Ricardian Equivalence may hold, it is irrelevant for the question at hand here because the change in fiscal policy is *not* regarding taxes.

f. **(4 points)** Suppose that the fiscal stimulus is composed entirely decreases in lump-sum taxes in the years 2009-2010 (i.e., in "period 1"), with no changes in fiscal policy whatsoever beyond that. Assuming that net exports and investment are both zero AND assuming that Ricardian Equivalence holds, will the tax reduction raise total GDP during the years 2009-2010? Carefully explain your logic, including what role Ricardian Equivalence plays in your logic.

Solution: Here, assuming that all taxes are lump-sum (and that there are no credit constraints on consumers), Ricardian Equivalence DOES kick in, and any changes in tax policy do nothing to stimulate consumption in either the short run (period 1) or the long run (period 2). Hence, they do nothing to raise GDP, either. (Note: you were not explicitly told here that taxes were lump sum, so if you simply assumed they were (by **explicitly** noting this), that was fine; alternatively, if you asserted that taxes were **not** lump sum, you needed to construct an appropriate argument for why Ricardian Equivalence was thus irrelevant and a tax cut in period 1 *would* in fact be expected to lead to a change in the mix of short run (period 1) vs. long run (period 2) consumption. The argument you had to make was essentially just that we covered in class about how/why proportional (aka non-lump-sum) taxes lead to changes in consumption behavior by changing relative prices.)

g. **(4 points)** You are an adviser to the Obama administration's fiscal policy makers. Suppose that any advice you offer is based solely on the goal of raising total GDP in the years 2009-2010. Which of the two scenarios outlined in part e or part f would you advise the government to follow? Briefly explain.

Solution: Based solely on the stark results of parts e and f above, clearly the best advice would be to raise government spending rather than provide tax cuts.