Department of Economics University of Maryland

Economics 325
Intermediate Macroeconomic Analysis
Problem Set 2
Professor Sanjay Chugh
Spring 2009
Due: Monday, May 11, 2009

Instructions: Written (typed is strongly preferred, but not required) solutions must be submitted no later than 11:00am on the date listed above (either in class or in the Economics Department Main Office, Tydings Hall 3105). Your solutions, which likely require some combination of mathematical derivations, economic reasoning, graphical analysis, and pure logic, should be thoroughly presented and not leave the reader (i.e., your TAs and I) guessing about what you actually meant.

You must submit your own independently-written solutions. You are permitted (in fact, encouraged) to work in groups to think through issues and ideas, but your "writing up" of solutions must be done independently of anyone else. **Under no circumstances will multiple verbatim identical solutions be considered acceptable.**

There are three problems in total, each with multiple subparts.

Problem 1: Intertemporal Consumption Leisure Framework – A Numerical Analysis (20 points). Consider the two-period intertemporal consumption-leisure economy. Suppose the representative consumer's lifetime utility function is given by $v(c_1, l_1, c_2, l_2) = \ln(c_1) + l_1 + \ln(c_2) + l_2$, where, as usual, c denotes consumption and l denotes hours of leisure (hence 168 – l is an individual's hours of labor during a week). Assume that the representative consumer begins period 1 with zero assets. The period-1, period-2, and lifetime budget constraints in this framework, expressed in real terms, are thus given, respectively, by

$$c_1 + a_1 = (1 - t_1) \cdot w_1 \cdot (168 - l_1)$$
$$c_2 + a_2 = (1 + r) \cdot a_1 + (1 - t_2) \cdot w_2 \cdot (168 - l_2)$$
$$c_1 + \frac{c_2}{1+r} = (1 - t_1) w_1 (168 - l_1) + \frac{(1 - t_2) w_2 (168 - l_2)}{1+r}$$

The tax rates in the two periods are $t_1 = t_2 = 0.5$, and the real wages in the two periods are $w_1 = 20$ and $w_2 = 22$. **Note that you are NOT given a numerical value for the real interest rate (you will solve for this in part b below).**

a. **(10 points)** Is it possible to solve numerically for the representative consumer's optimal choices of consumption in each of the two periods? If so, do so, showing any important steps in your logic/computation. If not, briefly describe the economic and/or mathematical issue(s) that prevents doing so. (**Note**: If you can solve without setting up and solving a Lagrangian, you may do so.)

b. **(5 points)** Based on your solution in part a above, is it possible to solve numerically for the real interest rate r? If so, do so, showing any important steps in your logic/computation. If not, briefly describe the economic and/or mathematical issue(s) that prevents doing so. (**Note**: if you were unable to fully solve part a, you can still receive full credit for this part by correctly and fully describing how you **would** compute r assuming you had fully solved part a).

c. **(5 points)** Is it possible to solve numerically for the optimal choices of leisure in each of the two periods? If so, do so, showing any important steps in your logic/computation. If not, briefly describe the economic and/or mathematical issue(s) that prevents doing so.

Problem 2: M1 Money and M2 Money (20 points). Consider an extended version of our infinite-period MIU framework. In addition to stocks and nominal bonds, suppose there are **two** forms of money: M1 and M2. M1 money (which we will denote by M_t^1) and M2 money (which we will denote by M_t^2) both directly affect the representative consumer's utility. The period-*t* utility function is assumed to be

$$u\left(c_t, \frac{M_t^1}{P_r}, \frac{M_t^2}{P_t}\right) = \ln c_t + \ln \frac{M_t^1}{P_t} + \kappa \ln \frac{M_t^2}{P_t},$$

which, note has three arguments. The Greek letter "kappa" (κ) in the utility function is a number between zero and one, $0 \le \kappa \le 1$, over which the representative consumer has no control. The period-t budget constraint of the consumer is

$$P_t c_t + M_t^1 + M_t^2 + B_t + S_t a_t = Y_t + M_{t-1}^1 + (1+i_{t-1}^M)M_{t-1}^2 + (1+i_{t-1})B_{t-1} + (S_t + D_t)a_{t-1},$$

where i_t denotes the nominal interest rate on bonds held between period t and $t+1$ (and hence i_{t-1} on bonds held between $t-1$ and t) and i_t^M denotes the nominal interest rate on M2 money held between period t and $t+1$ (and hence i_{t-1}^M on M2 money held between $t-1$ and t). **Thus, note that M2 money potentially pays interest, in contrast to M1 money, which pays zero interest.**

As always, assume the representative consumer maximizes lifetime utility by optimally choosing consumption and assets (i.e., in this case choosing all four assets optimally).

a. **(5 points)** Using the functional form for utility given in this problem, **what is the marginal rate of substitution between real M1 money and real M2 money?** (Hint: You do **not** need to solve a Lagrangian to answer this – all that is required is using the utility function.) Explain the important steps in your argument.

b. **(5 points)** A sudden, unexplained change in the value of κ would be interpretable as which of the following: a preference shock, a technology shock, or a monetary policy shock? Briefly explain.

c. **(10 points)** Let $\phi^2(c_t, i_t, i_t^M)$ denote the real money demand function for M2 money. Note the three arguments to the function $\phi^2(.)$. Using the first-order conditions of the representative consumer's Lagrangian, generate the function $\phi^2(c_t, i_t, i_t^M)$ (i.e., solve for real M2 money demand as a function of c_t, i_t, and i_t^M). Briefly explain (economically) why i_t^M appears in this money demand function. (**Note:** you must determine yourself which are the relevant first-order conditions needed to create this money demand function – draw on our approach from Chapter 14.)

Problem 3: Stock, Bonds, "Bills," and the Financial Accelerator (60 points). In this problem, you will study an enriched version of the accelerator framework we studied in class. As in our basic analysis, we continue to use the two-period theory of firm profit maximization as our vehicle for studying the effects of financial-market developments on macroeconomic activity. However, rather than supposing it is just "stock" that is the financial asset at firms' disposal for facilitating physical capital purchases, we will now suppose that **both "stock" and "bonds" are at firms' disposal for facilitating physical capital purchases.**

Before describing more precisely the analysis you are to conduct, a deeper understanding of "bond markets" is required. In "normal economic conditions," (i.e, in or near a "steady state," in the sense we first discussed in Chapter 8), it is usually sufficient to think of all bonds of various maturity lengths in a highly simplified way: by supposing that they are all simply one-period face-value = 1 bonds with the same nominal interest rate. Recall, in fact, that this is how our basic discussion of monetary policy proceeded. In "unusual" (i.e., far away from steady state) financial market conditions, however, it can become important to distinguish between different types of bonds and hence different types of nominal interest rates on those bonds.

You may have seen discussion in the press about central banks, such as the U.S. Federal Reserve, considering whether or not to "begin buying bonds" as a way of conducting policy; for example, see News Supplement 18. Viewed through the standard lens of how to understand open-market operations, this discussion makes no sense because in the standard view, central banks **already do** buy (and sell) "bonds" as the mechanism by which they conduct open-market operations!

A difference that becomes important to understand during unusual financial market conditions is that open-market operations are conducted using the **shortest-maturity** "bonds" that the Treasury sells, of duration one month or shorter. In the lingo of finance, this type of "bond" is called a "Treasury bill." The term "Treasury bond" is usually used to refer to **longer-maturity** Treasury securities – those that have maturities of one, two, five, or more years. These longer-maturity Treasury "bonds" have typically **not** been assets that the Federal Reserve buys and sells as regular practice; buying such longer-maturity bonds is/has not been the usual way of conducting monetary policy. But the Federal Reserve has recently been considering this option.

In the ensuing analysis, part of the goal will be to understand/explain why policy-makers are currently considering this option. Before beginning this analysis, though, there is more to understand.

Problem 3 continued

In private-market borrower/lender relationships, longer-maturity Treasury bonds ("bonds") are typically allowed to be used just like stocks in financing firms' physical capital purchases.[1] We can capture this idea by enriching the financing constraint in our financial accelerator framework to read:

$$P_1 \cdot (k_2 - k_1) = R^S \cdot S_1 \cdot a_1 + R^B \cdot P_1^b \cdot B_1 .$$

The left hand side of this richer financing constraint is the same as the left hand side of the financing constraint we considered in our basic theory (and the notation is identical, as well – refer to your notes for the notational definitions).

The right hand side of the financing constraint is richer than in our basic theory, however. The market value of "stock," S_1a_1, still affects how much physical investment firms can do, scaled by the government regulation R^S. **In addition, now the market value of a firm's "bond-holdings" (which, again, means long-maturity government bonds) also affects how much physical investment firms can do,** scaled by the government regulation R^B. The notation here is that B_1 is a firm's holdings of nominal bonds ("long-maturity") at the end of period 1, and P_1^b is the nominal price of that bond during period 1. Note that R^B and R^S need not be equal to each other.

In the context of the two-period framework, the firm's two-period discounted profit function now reads:

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 + B_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 - P_1^b B_1$$
$$+ \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} + \frac{B_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i} - \frac{P_2^b B_2}{1+i}$$

The new notation compared to our study of the basic accelerator mechanism is the following: B_0 is the firm's holdings of nominal bonds (**which have face value = 1)** at the start of period one, B_1 is the firm's holdings of nominal bonds (**which have face value = 1**) at the end of period one, and B_2 is the firm's holdings of nominal bonds (**which have face value = 1)** at the end of period two.

Note that period-2 profits are being discounted by the nominal interest rate i: in this problem, we will consider i to be the "Treasury bill" interest rate (as opposed to the "Treasury bond" interest rate). The Treasury-**bill** interest rate is the one the Federal Reserve usually (i.e., in "normal times") controls. We can **define** the nominal interest rate on Treasury **bonds** as

$$i^{BOND} = \frac{1}{P_1^b} - 1 \left(\Leftrightarrow P_1^b = \frac{1}{1 + i^{BOND}} \right)$$

Thus, note that i^{BOND} and i need not equal each other.

[1] Whereas, for various institutional and regulatory reasons, very short-term Treasury assets ("T-bills") are typically not allowed to be used in financing firms' physical capital purchases.

Problem 3 continued

The rest of the notation above is just as in our study of the basic financial accelerator framework. Finally, because the economy ends at the end of period 2, we can conclude (as usual) that $k_3 = 0$, $a_2 = 0$, and $B_2 = 0$.

With this background in place, you are to analyze a number of issues.

a. **(5 points)** Using λ as your notation for the Lagrange multiplier on the financing constraint, construct the Lagrangian for the representative firm's (two-period) profit-maximization problem.

b. **(5 points**) Based on this Lagrangian, compute the first-order condition with respect to nominal bond holdings at the end of period 1 (i.e., compute the FOC with respect to B_1). **(Note:** This FOC is critical for much of the analysis that follows, so you should make sure that your work here is absolutely correct! If your FOC here is incorrect, we will **not** necessarily "carry through the error" all the way through the remainder of your analysis when reviewing solutions.)

c. **(10 points**) Recall that in this enriched version of the accelerator framework, the nominal interest rate on "Treasury bills," i, and the nominal interest rate on "Treasury bonds," i^{BOND}, are potentially different from each other. If financing constraints do NOT at all affect firms' investment in physical capital, how does i^{BOND} compare to i? Specifically, is i^{BOND} equal to i, is i^{BOND} smaller than i, is i^{BOND} larger than i, or is it impossible to determine? Be as thorough in your analysis and conclusions as possible (i.e., tell us as much about this issue as you can!). Your analysis here should be based on the FOC on B_1 computed in part b above. (**Hint:** if financing constraints "don't matter," what is the value of the Lagrange multiplier λ?)

d. **(10 points)** If financing constraints DO affect firms' investment in physical capital, how does i^{BOND} compare to i? Specifically, is i^{BOND} equal to i, is i^{BOND} smaller than i, is i^{BOND} larger than i, or is it impossible to determine? Furthermore, if possible, use your solution here as a basis for justifying whether or not it is appropriate in "normal economic conditions" to consider both "Treasury bills" and "Treasury bonds" as the "same" asset. Be as thorough in your analysis and conclusions as possible (i.e., tell us as much about this issue as you can!). Once again, your analysis here should be based on the FOC on B_1 computed in part b above. **(Note:** the government regulatory variables R^S and R^B are both strictly positive **–** that is, neither can be zero or less than zero).

The above analysis was framed in terms of nominal interest rates; the remainder of the analysis is framed in terms of real interest rates.

Problem 3 continued

e. **(10 points)** By computing the first-order condition on firms' stock-holdings at the end of period 1, a_1, and following exactly the same algebra as presented in class, we can express the Lagrange multiplier λ as

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r} \right] \cdot \frac{1}{R^S} . \tag{1.1}$$

Use the first-order condition on B_1 you computed in part b above to derive an analogous expression for λ except in terms of the real interest rate on bonds (i.e., r^{BOND}) and R^B (rather than R^S). (**Hint:** Use the FOC on B_1 you computed in part b above and follow a very similar set of algebraic manipulations as we followed in class.)

f. **(5 points)** Compare the expression you just derived in part e with expression (1.1). Suppose $r = r^{STOCK}$. If this is the case, is r^{BOND} equal to r, is r^{BOND} smaller than r, is r^{BOND} larger than r, or is it impossible to determine? Furthermore, in this case, does the financing constraint affect firms' physical investment decisions? Briefly justify your conclusions and provide brief explanation.

g. **(15 points)** Through late 2008, suppose that $r = r^{STOCK}$ was a reasonable description of the U.S. economy for the preceding 20+ years. In late 2008, r^{STOCK} fell dramatically below r, which, as we studied in class, would cause the financial accelerator effect to begin. Suppose government policy-makers, both fiscal policy-makers and monetary policy-makers, decide that they need to intervene in order to try to choke off the accelerator effect. Furthermore, suppose that there is no way to change either R^S or R^B (because of coordination delays amongst various government agencies, perhaps). Using all of your preceding analysis as well as drawing on what we studied in class, explain why "buying bonds" (which, again, means long-maturity bonds in the sense described above) might be a sound strategy to pursue. (**Note:** The analysis here is largely **not** mathematical. Rather, what is required is an **extremely careful** logical progression of thought that explains why buying bonds might be a good idea. You should treat this as a (brief!) "memo" you are writing to, say, President Obama (who likely does not understand all the technical details of the accelerator mechanism but could follow the ideas behind it if presented clearly) logically justifying this proposed policy action.)