

Economics 602
Macroeconomic Theory and Policy
Midterm Exam
Professor Sanjay Chugh
Fall 2007
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NAME: _____

The Exam has a total of four (4) questions and pages numbered one (1) through fourteen (14). Each question's total number of points is shown below. You are to answer all questions in the spaces provided.

You may use a non-programmable calculator.

Question 1	/ 20
Question 2	/ 25
Question 3	/ 25
Question 4	/ 30
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TOTAL	/100

Question 1: Consumption and Savings in the Two-Period Economy (20 points). Consider a two-period economy (with no government), in which the representative consumer has no control over his income. The lifetime utility function of the representative consumer is $u(c_1, c_2) = \ln(c_1) + \ln(c_2)$, where \ln stands for the natural logarithm. We will work here in purely real terms: suppose the consumer's **present discounted value of ALL lifetime real income is 26**. Also suppose the consumer begins period 1 with zero assets.

- a. **(13 points)** Set up the lifetime Lagrangian formulation of the consumer's problem, in order to answer the following: i) is it possible to numerically compute the consumer's optimal choice of consumption in period 1? If so, compute it; if not, explain why not. ii) is it possible to numerically compute the consumer's optimal choice of consumption in period 2? If so, compute it; if not, explain why not. iii) is it possible to numerically compute the consumer's real asset position at the end of period 1? If so, compute it; if not, explain why not.

Question 1 continued

b. (7 points) To demonstrate how important the concept of the real interest rate is in macroeconomics, an interpretation of it (in addition to the couple of different interpretations we have already discussed in class) is that it reflects the rate of consumption growth between two consecutive periods. Using the consumption-savings optimality condition for the given utility function, describe/discuss whether the real interest rate is **positively related to or negatively related to the rate of consumption growth between two periods**. (Note: No mathematics are especially required for this problem; also note this part can be fully completed even if you were unable to get all the way through part a).

Question 2: Core Inflation and Non-Core Inflation (25 points). Two distinct measures of inflation – called core inflation and non-core inflation – generally garner attention by policy-makers and the media. The core inflation rate is the rate of growth of prices of so-called “core goods” (such as food, clothing, shelter), while the non-core inflation rate is the rate of growth of prices of so-called “non-core goods” (generally energy items).

Consider our usual two-period economy (with no government), in which the representative consumer has no control over his **nominal** income. Rather than there being only one “type” of good the consumer purchases each period, however, suppose that each period there are two “types” of goods: core good and non-core goods. The lifetime utility function of the representative consumer is

$$u(c_1^{CORE}, c_1^{NONCORE}, c_2^{CORE}, c_2^{NONCORE}) = \ln(c_1^{CORE}) + \ln(c_1^{NONCORE}) + \ln(c_2^{CORE}) + \ln(c_2^{NONCORE}),$$

where \ln stands for the natural logarithm. c_1^{CORE} stands for consumption of core goods in period 1, $c_1^{NONCORE}$ stands for consumption of non-core goods in period 1, and similarly for c_2^{CORE} and $c_2^{NONCORE}$.

The representative consumer begins period one with zero assets (i.e., $A_0 = 0$). The period-by-period budget constraints of the representative consumer are thus

$$\begin{aligned} P_1^{CORE} c_1^{CORE} + P_1^{NONCORE} c_1^{NONCORE} + A_1 &= Y_1 \\ P_2^{CORE} c_2^{CORE} + P_2^{NONCORE} c_2^{NONCORE} + A_2 &= Y_2 + (1+i)A_1 \end{aligned}$$

where P_1^{CORE} stands for the **nominal** price of core goods in period 1, $P_1^{NONCORE}$ stands for the **nominal** price of non-core goods in period 1, and similarly for P_2^{CORE} and $P_2^{NONCORE}$. As usual, Y_1 and Y_2 stand for **nominal** income in periods 1 and 2, respectively.

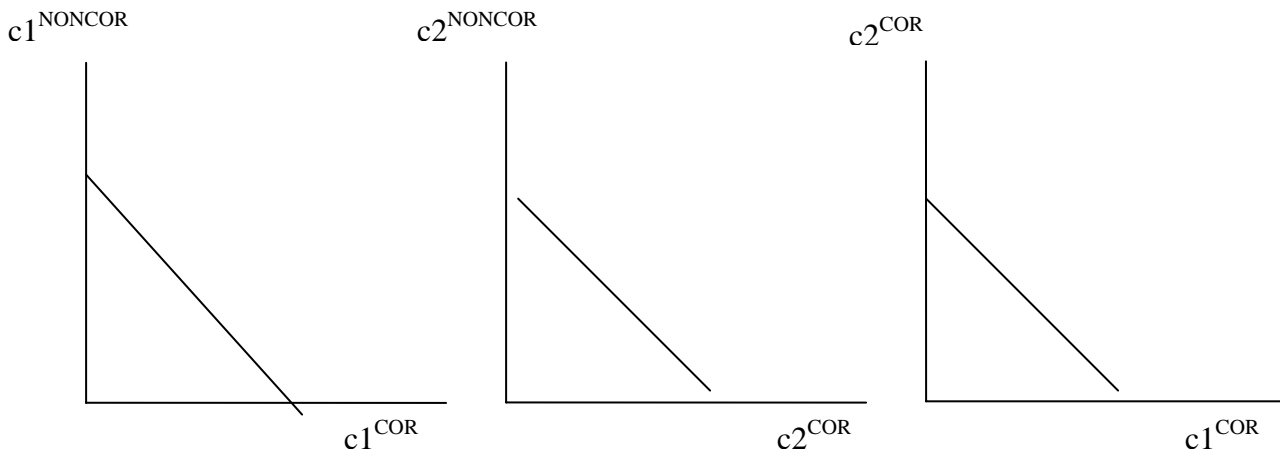
- a. (4 points) Construct, in nominal terms, the representative consumer’s lifetime budget constraint. Show all important steps. Provide **brief** (no more than one sentence) interpretation for the expression you construct.

Question 2a continued (if you need more space)

- b. **(10 points)** Using an appropriate Lagrange formulation (you are free to choose either a lifetime formulation or a sequential formulation), derive **four** optimality conditions: one between period-1 core consumption and period-1 non-core consumption; one between period-2 core consumption and period-2 non-core consumption; one between period-1 core consumption and period-2 core consumption; and one between period-1 non-core consumption and period-2 non-core consumption. Show all important steps.

Question 2b continued (if you need more space)

- c. **(6 points)** In each of the following three diagrams, appropriately label the slope of the budget line. **Briefly** (in no more than one or two sentences) describe how you determined the relevant slopes (you may refer to your work in part b if you need).



Question 2 continued

- d. (5 points) Suppose that the Fed changes the nominal interest rate, and suppose that this change in policy does not at all affect P_1^{CORE} , $P_1^{NONCORE}$, P_2^{CORE} , or $P_2^{NONCORE}$, nor does it affect Y_1 or Y_2 . Will this change in monetary policy affect consumers' MRS between period-1 core and period-1 non-core consumption? Will this change in monetary policy affect consumers' MRS between period-2 core and period-2 non-core consumption? For each, **briefly** explain why or why not. You may refer to the diagrams in part c if you need.

Question 3: International Financial and Macroeconomic Linkages (25 points). Consider a variation of our usual infinite-period “stock-pricing” model. The variation here is that there are two “types” of stock that the representative U.S. consumer can buy: “U.S.” stock and “Foreign” stock. Denote by a_{t-1}^{US} the representative consumer’s holdings of U.S. stock at the beginning of period t and by a_{t-1}^{FOR} the representative consumer’s holdings of foreign stock at the beginning of period t . Likewise, let P_t^{US} and P_t^{FOR} denote, respectively, the nominal price of U.S. and foreign stock in period t , and D_t^{US} and D_t^{FOR} denote, respectively, the per-share nominal dividend that U.S. and foreign stock pay in period t . All of these prices and dividends are in dollar terms. The period- t budget constraint of the representative U.S. consumer is thus

$$P_t c_t + S_t^{FOR} a_t^{FOR} + S_t^{US} a_t^{US} = Y_t + (S_t^{FOR} + D_t^{FOR}) a_{t-1}^{FOR} + (S_t^{US} + D_t^{US}) a_{t-1}^{US},$$

in which all of the other notation is standard: Y_t denotes nominal income (over which the consumer has no control) in period t , c_t is real units of consumption, and P_t is the nominal price of each unit of consumption. To be clear, all of these latter variables refer to U.S.-economy variables. Also as usual, the lifetime utility of the U.S. consumer starting from period t onwards is $u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \dots$, where $\beta \in (0,1]$ is the usual measure of consumer impatience.

- a. **(8 points)** Setting up an appropriate Lagrangian, derive period- t stock-pricing expressions for both U.S. stock and Foreign stock. Your expressions should be of the form $S_t^{US} = \dots$ and $S_t^{FOR} = \dots$

Question 3 continued

Throughout the rest of this question, suppose that NO stocks (neither U.S. nor Foreign) EVER pay any dividends.

- b. **(5 points)** Using the two expressions you developed in part a, describe/show how the rate of change (between any two consecutive periods) of U.S. stock prices compares to the rate of change (between the same two consecutive periods) of Foreign stock prices. Clearly show/state all important steps in your argument.
- c. **(5 points)** Suppose for simplicity that $\beta = 1$ and suppose the economy eventually reaches a steady state. Further suppose that in the steady state, the U.S. inflation rate of consumer goods prices between one period and the next is 5 percent (i.e., $\pi = 0.05$). In the steady state, how quickly do nominal prices of **both** U.S. and Foreign stock grow? Justify your answer with any appropriate combination of mathematical, graphical, or logical arguments.

Question 3c continued (if you need more space)

- d. **(7 points – Harder)** There is also a representative consumer and a central bank in the Foreign country. Continue to focus on just the steady state (of both the U.S. and now also the Foreign economies). If steady-state inflation in the Foreign country is 10 percent (while steady-state inflation in the U.S. remains at 5 percent), how will the growth rates of U.S. stock prices and Foreign stock prices compare with each other? **Here your answer need only be qualitative, rather than a precise numerical value.** Justify your answer with any appropriate combination of mathematical, graphical, or logical arguments. **(Hint:** Your answer should take into account **both** what you found in part c above along with the economic events that must be happening in the Foreign country. Also note there isn't necessarily a single correct analysis here.)

Question 4: Welfare Losses from Distortionary Taxation (30 points). A central question behind many policy proposals in macroeconomics and public finance is the potential welfare consequences of distortionary tax systems. Here you will consider the welfare consequences associated with labor income taxation in the context of the **static consumption-leisure model**. Specifically, we will consider how much consumers “lose” due to a labor income tax rate of 20 percent as compared to a labor income tax rate of 0 (zero) percent. (To connect with ideas you know from basic economics, what you will formally compute is the “deadweight loss” of a system with 20-percent labor income taxation compared to a system with zero-percent labor income taxation.)

A typical welfare analysis proceeds in the following three steps:

1. Solve for the representative consumer’s optimal choices under the proposed tax policy (in our case, a labor income tax rate of 20 percent).
2. Solve for the representative consumer’s optimal choices under the alternative tax policy (in our case, a labor income tax rate of zero percent).
3. **Compute the ADDITIONAL PERCENTAGE OF CONSUMPTION the representative consumer would need under the proposed tax policy to be EQUALLY HAPPY IN TERMS OF UTILITY as under the alternative tax policy.**

How to implement step 3 is described in detail after part b below.

Recall that, in the static consumption-leisure model, the representative consumer’s budget constraint can be expressed in real terms as $c + (1-t)wl = 168(1-t)w$; for simplicity, suppose throughout the analysis that the real wage is $w = 1$ no matter what tax policy is in place.

A critical component to the answer to welfare questions can be what the precise functional form of the (representative consumer’s) utility function is.

- a. **(7 points)** Suppose the representative consumer’s utility function over consumption and leisure is $u(c, l) = \ln c + \ln l$. Numerically solve for the optimal choices of consumption and leisure under **both** the proposed policy (20 percent labor tax) and the alternative policy (zero labor tax) – that is, conduct step 1 and step 2 of the analysis for this utility function.

Question 4a continued (if you need more space)

- b. **(8 points)** Suppose the representative consumer's utility function over consumption and leisure is $u(c, l) = 2\sqrt{c} + 2\sqrt{l}$. Numerically solve for the optimal choices of consumption and leisure under **both** the proposed policy (20 percent labor tax) and the alternative policy (zero labor tax) – that is, conduct step 1 and step 2 of the analysis for this utility function.

Question 4 continued

(NOTE: The remainder of this question is more difficult.)

Implementing step 3 of the welfare analysis means first comparing the total number of **utils** under the proposed policy with the total number of **utils** under the alternative tax policy and then determining how much **additional consumption (measured in percent)** would be required under the proposed policy to make the total number of utils the **same** under both policies.

For example, suppose the total number of utils (when consumers are making their optimal consumption-leisure choices) under the proposed policy were 5, while the total number of utils (when consumers are making their optimal consumption-leisure choices) under the alternative policy were 7. If policy-makers implemented the proposed policy and also could somehow just “hand” to consumers an extra X percent of consumption that would make them equally well off (in utility terms) as under the alternative policy, the number X would measure the welfare loss of the proposed policy compared to the alternative policy. The reason why welfare losses are computed this way is because, as you recall from basic economics, utils in and of themselves are meaningless numbers.

Mathematically, the welfare measure X solves the equation

$$u\left((1+X)c_{PROPOSED\ POLICY}^*, l_{PROPOSED\ POLICY}^*\right) = u\left(c_{ALTERNATIVE\ POLICY}^*, l_{ALTERNATIVE\ POLICY}^*\right);$$

note carefully how the variable X enters this expression.

- c. **(7 points)** Based on your work in part a, compute, for the utility function $u(c, l) = \ln c + \ln l$, how much extra consumption X (as defined above) is required under the tax policy of 20 percent as compared to the tax policy of zero percent? Show all important steps/logic in your work. (**Note:** the number X will not necessarily be a nice “round” number and may require calculator-work.)

Question 4c continued (if you need more space)

- d. **(8 points)** Based on your work in part a, compute, for the utility function $u(c, l) = 2\sqrt{c} + 2\sqrt{l}$, how much extra consumption X (as defined above) is required under the tax policy of 20 percent as compared to the tax policy of zero percent? Show all important steps/logic in your work. (**Note:** the number X will not necessarily be a nice “round” number and may require calculator-work.)

Question 4d continued (if you need more space)

END OF EXAM