

Economics 602
Macroeconomic Theory and Policy
Final Exam
Professor Sanjay Chugh
Fall 2008
December 8, 2008

NAME:

The Exam has a total of four (4) questions and pages numbered one (1) through eleven (11). Each problem's total number of points is shown below. Your solutions should consist of some appropriate combination of mathematical analysis, graphical analysis, logical analysis, and economic intuition, but in no case do solutions need to be exceptionally long. Your solutions should get straight to the point – **solutions with irrelevant discussions and derivations will be penalized.** You are to answer all questions in the spaces provided.

You may use two pages (double-sided) of notes. You may use a calculator.

Question 1	/ 15
Question 2	/ 15
Question 3	/ 28
Question 4	/ 42

TOTAL	/100
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1. **A National Service Program (15 points).** Consider the following radical policy proposal: rather than taxes being levied on individuals and the proceeds of those taxes being used by the government to fund various programs, suppose that every individual pays no taxes of any kind but must give ten hours of his time every week to national service. Here you will analyze this national service program in the context of the (one-period) consumption-leisure model we have studied. Thus, there are now **three** uses of the individual's time: work, leisure, and national service (the mandatory 10 hours). **Assume the following:**

- Instituting the national service program has no effect on any prices or wages in the economy.
- Any time spent voluntarily performing national service beyond the required 10 hours is considered leisure.

a. **(8 points)** Using the notation we developed in Chapter 2 (i.e., c to denote consumption, n to denote hours of work per week, l to denote hours of leisure per week, P to denote the nominal price of consumption, and W to denote the nominal hourly wage), construct the representative agent's (weekly) budget constraint in this model with a national service program. Recall that there are 168 hours in one calendar week. Provide brief economic justification for your work.

Solution: The individual is required to give 10 hours per week to national service. Thus he has $(168-10)=158$ hours per week left to allocate to either labor or leisure. Proceeding completely analogously as in the standard model, then, we finally arrive at the budget constraint

$$Pc + Wl = 158W .$$

(Refer to Chapter 2 of the Recitation Notes for the full algebraic derivation: the derivation here is identical except that $t = 0$ by assumption and 168 is replaced by 158).

b. **(7 points)** Now recall the standard consumption-leisure model with no national service program and suppose that both the consumption tax rate is zero and the labor tax rate is zero. How does the slope of the budget constraint in this economy compare with the slope of the budget constraint in the economy with the national service program in part a? Provide brief economic intuition.

Solution: With no taxes and no national service program, the budget constraint is $Pc + Wl = 168W$. Comparing this with the budget constraint in part a above, we see that the slopes of the two budget constraints are identical – the two constraints only differ in their intercepts. The constraint in part a has intercepts which are smaller than the constraint here in part b. The intuition is that in the consumption-leisure model the individual has **time** to allocate between working and not working. The national service program takes away some of this time but otherwise has no effect on the real (inclusive of taxes) wage W / P .

2. Monetary Policy in the MIU Model (15 points). In this question, you will analyze, using indifference curve/budget constraint diagrams, the implications of alternative nominal interest rates on the representative consumer's choices of consumption and **real** money balances.

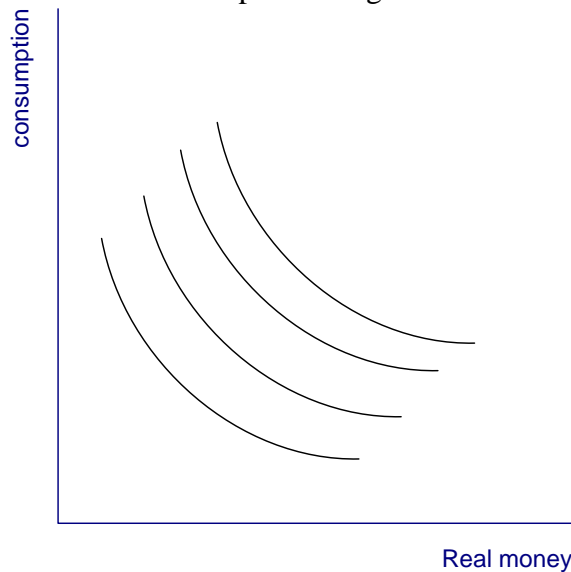
Recall that, with an instantaneous utility function $u(c_t, M_t/P_t)$ (where, as usual, c_t denotes consumption and M_t/P_t denotes **real** money balances), the consumption-money optimality condition (which we derived in Chapter 14) can be expressed as

$$\frac{u_{m_t}(c_t, M_t/P_t)}{u_{c_t}(c_t, M_t/P_t)} = \frac{i_t}{1+i_t},$$

where, again as usual, i_t is the nominal interest rate, $u_c(\cdot)$ denotes the marginal utility of consumption, and $u_m(\cdot)$ denotes the marginal utility of **real** money balances.

- a. **(5 points)** Suppose the central bank is considering setting one of two (and only two) nominal interest rates: i_t^1 and i_t^2 , with $i_t^2 > i_t^1$. On the indifference map below, qualitatively (and clearly) sketch relevant budget lines and show the consumer's optimal choices of consumption and real money under the two alternative policies. **On the diagram below, note the point on the vertical axis marked "FIXED" – this denotes a point that must lie on ANY budget constraint. Clearly label your diagram, including the slopes of the budget lines.**

Solution: Examining the right-hand-side of the above, it is clear that the smaller is i , the flatter is the budget line. Starting from the FIXED point, draw two budget lines, with the budget line with slope i_1 flatter than the budget line with slope i_2 . On the flatter budget line, the consumer's optimal choice of money balances and consumption is higher than on the steeper budget line.



Question 2 continued

- b. **(5 points)** You are a policy adviser to the central bank, and any advice you give is based on the goal of maximizing the utility of the representative consumer. The central bank asks you to help it choose between the two nominal interest rates i_t^1 and i_t^2 (and only these two). Which nominal interest rate would you recommend implementing? **Briefly** explain.

Solution: Again as is clear from the diagram, choosing the smaller value of i allows the representative consumer to attain a higher level of utility (a higher indifference curve), so i^1 is preferred to i^2 .

- c. **(5 points)** Suppose instead the central bank is open to setting any nominal interest rate, not just either i_t^1 or i_t^2 . What would your policy recommendation be? **Briefly** justify your recommendation, **and also in the diagram in part (a) sketch and clearly label a new budget line consistent with your policy recommendation.**

Solution: Setting $i = 0$ (or, technically speaking, very very very close to zero) would make the budget line completely flat, and allow the consumer to obtain the highest possible utility. Note that, because indifference curves are downward sloping, if $i < 0$, then there would not be a point of tangency between the budget line and an indifference curve – there would no equilibrium. (Indeed, $i = 0$ is the lowest that nominal interest rates can ever go (something known as the “zero-lower-bound” on interest rates – were they to go lower, a monetary economy (ie, one in which money is used as a medium of exchange) would not exist. A topic for a more advanced course in monetary economics.)

3. **The Fiscal Theory of Exchange Rates (28 points).** In this question, you will use the fiscal theory of exchange rates to analyze some consequences of a fixed exchange rate system. The model is just as we have studied in class – in particular, consumption is constant at $\bar{c} = 11$ in every period, real money demand is described by the function, $M_t / P_t = \phi(\bar{c}, i_t)$, PPP holds, and the foreign price level is equal to one in every period (i.e., $P_t^* = 1$ in every period t). The domestic country runs a fiscal deficit of $DEF = -5.5$ (a **negative** deficit is a **surplus...**) every period, and there is no political will to ever change this deficit. The real money demand function is given by $\phi(\bar{c}, i_t) = \bar{c} - 10 \cdot i_t$, and the exchange rate that the country is pegging (for as long as it can) is $E = 2$ units of domestic currency per unit of foreign currency. Finally, the foreign real interest rate is $r^* = 0.10$, the government starts period 1 with foreign reserves of $B_0^G = 22$, and foreign reserves can never go below zero.

- a. **(3 points)** As long as the fixed exchange rate is in place and is expected to remain in place, what is the numerical value of the domestic nominal interest rate? Briefly justify your answer.

Solution: Use the interest parity condition. If the peg is expected to remain in place, that means $E_{t+1}^e = E_t$, so the interest parity condition tells us that the domestic nominal interest rate equals the foreign real interest, so $i_t = r^* = 0.10$ during this time.

- b. **(3 points)** As long as the fixed exchange rate is in place and is expected to remain in place, what is the numerical value of the domestic country's BOP surplus or BOP deficit? Briefly justify your answer.

Solution: Recall the government budget constraint is $B_t^G - B_{t-1}^G = \frac{M_t - M_{t-1}}{E_t} - DEF_t$.

Seignorage revenue is zero during this period, so the government budget constraint reduces to $B_t^G - B_{t-1}^G = -DEF_t$. The left-hand side is the change in foreign reserves during period t , which is **our definition of the balance of payments**. That is, a country's BOP during a particular period equals the change in its foreign reserves during that period. With $DEF = -5.5$, there is a BOP **surplus** of 5.5 every period.

- c. **(3 points)** Based on your answer in part b, is the **floating** exchange rate higher than, lower than, or equal to $E = 2$? Briefly justify your answer.

Solution: Because the country is running a BOP surplus, it is accumulating foreign reserves, meaning the domestic currency is being held weaker than at the floating rate. The floating rate is thus smaller than $E = 2$.

Question 3 continued

- d. (4 points) If markets/investors for some reason **never expect** a change in the nominal exchange rate, how many periods will the fixed exchange rate last? Briefly justify your answer.

Solution: With a BOP surplus, the domestic central bank is **accumulating** foreign reserves. There is no economic upper limit on how many foreign reserves a central bank can hold, so in principle, the fixed exchange rate could last forever.

Suppose the government of the domestic country **announces in period T-1 that in period T** the nominal exchange rate will be $E_T = 1.9$, and markets/investors believe this announcement.

- e. (6 points) What is the numerical value of the nominal interest rate in period T-1 (i.e., compute i_{T-1})? Briefly justify your answer, and provide economic intuition for what you find, **including a brief economic explanation for why i_{T-1} differs from r^* if it does.**

Solution: Use the interest parity in period T-1 to compute the nominal interest rate:

$$1 + i_{T-1} = (1 + r^*) \frac{E_T^e}{E_{T-1}}$$

Because markets/investors believe the government's announcement, $E_T^e = 1.9$, in which the above tells us that $i_{T-1} = 0.045$. Thus the nominal interest rate **falls below** the world interest rate when markets expect an **appreciation** of the currency in the near future (going from $E = 2$ to $E = 1.9$ is an **appreciation** of the domestic currency). Because the domestic currency is expected to become **stronger** in the near future, markets/investors require/demand **less** interest compensation in order to hold bonds denominated in the domestic currency.

Question 3 continued

- f. (5 points) What is the numerical value of seignorage revenue for the domestic government in period T-1? Briefly justify your answer, and provide economic intuition for what you find, **including a brief economic explanation for why seignorage revenue differs from zero if it does.**

Solution: By definition, seignorage revenue in period T-1 is given by

$$SR_{T-1} = \frac{M_{T-1} - M_{T-2}}{P_{T-1}},$$

which in turn, because of the money demand function, can be expressed as

$$SR_{T-1} = \frac{P_{T-1}L(\bar{c}, i_{T-1}) - P_{T-2}L(\bar{c}, i_{T-2})}{P_{T-1}}.$$

Because $P = E$ in every period and the peg doesn't change until period T, the P terms cancel in the above expression, meaning $SR_{T-1} = L(\bar{c}, i_{T-1}) - L(\bar{c}, i_{T-2})$. We computed in part e that $i_{T-1} = 0.045$. We know from part a above that $i_{T-2} = r^* = 0.1$. Inserting these values in the given money demand function,

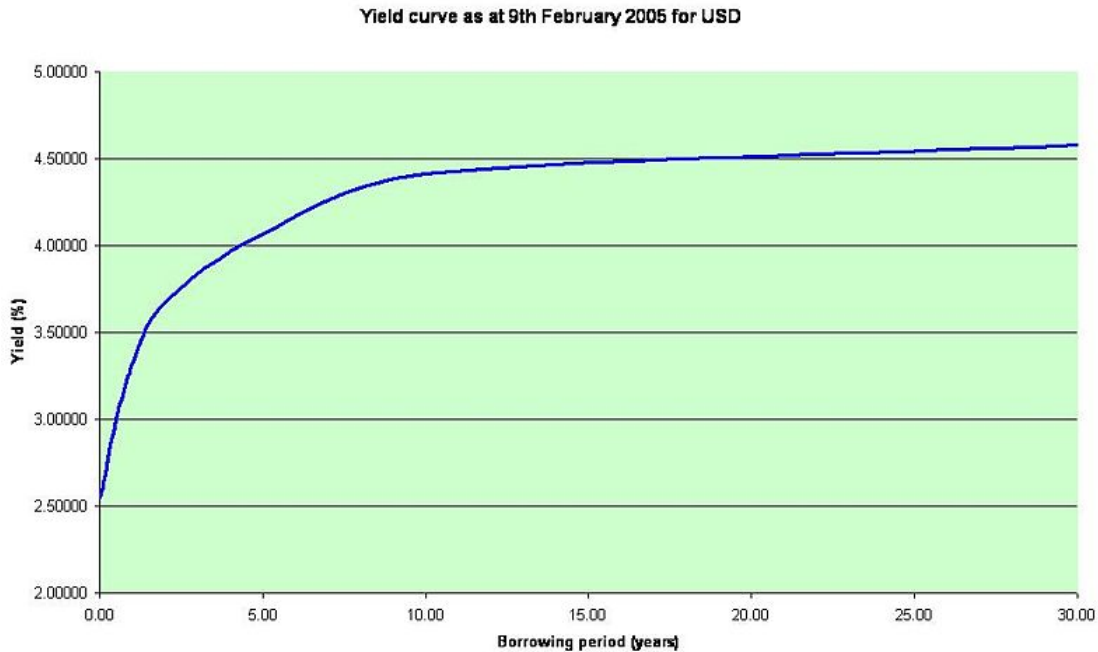
$$SR_{T-1} = 11 - 10(1 + 0.045) - (11 - 10(1 + 0.1)) = 10.55 - 10 = 0.55.$$

Seignorage revenue is thus **positive** in period T-1. The reason for this is fundamentally the same as in part e above: the domestic currency is expected to become **stronger** in period T, hence markets/investors' demand for it **rises**, which allows the domestic government to print more of it and earn **positive** seignorage revenue.

- g. (6 points) How does the domestic country's BOP in period T-1 compare to its BOP in period T-2? Does it rise, fall, or stay the same? **Explain precisely, including why.**

Solution: In period T-2, because seignorage revenue was zero, the BOP **surplus** was 5.5, as computed in part c above. In period T-1, the fiscal **surplus** (i.e., a negative DEF) is still in place, but the government also collects **positive** seignorage revenue, hence the government budget constraint shows us that the BOP **surplus** becomes even larger than it was in period T-2 (specifically, BOP surplus = $0.55 - (-5.5) = 6.05$, but you did not have to make this calculation because it was not asked).

4. The Yield Curve (42 points). An important indicator of markets' beliefs/expectations about the future path of the macroeconomy is the “yield curve,” which, simply put, describes the relationship between the maturity length of a particular bond (recall that bonds come in various maturity lengths) and the per-year interest rate on that bond. A bond's “yield” is alternative terminology for its (annual) interest rate. A sample yield curve is shown in the following diagram:



This diagram plots the yield curve for U.S. Treasury bonds that existed in markets on February 9, 2005: as it shows, a 5-year Treasury bond on that date carried an interest rate of about 4 percent, a 10-year Treasury bond on that date carried an interest rate of about 4.4 percent, and a 30-year Treasury bond on that date carried an interest rate of about 4.52 percent.

Recall from our study of bond markets that prices of bonds and nominal interest rates on bonds are negatively related to each other. The yield curve is typically discussed in terms of nominal interest rates (as in the graph above). However, because of the inverse relationship between interest rates on bonds and prices of bonds, the yield curve could equivalently be discussed in terms of the prices of bonds.

In this problem, you will use an enriched version of our infinite-period monetary framework from Chapter 14 to study how the yield curve is determined. Specifically, rather than assuming the representative consumer has only one type of bond (a one-period bond) he can purchase, we will assume the representative consumer has several types of bonds he can purchase – a one-period bond, a two-period bond, and, in the later parts of the problem, a three-period bond.

Let's start just with two-period bonds. We will model the two-period bond in the simplest possible way: in period t , the consumer purchases B_t^{TWO} units of two-period bonds, each of which has a market price $P_t^{b,TWO}$ and a face value of one (i.e., when the two-period bond pays

off, it pays back one dollar). **The defining feature of a two-period bond is that it pays back its face value *two* periods after purchase** (indeed, hence the term “two-period bond”...). The one-period bond is just as we have discussed in class and in Chapter 14.

Mathematically, then, suppose (just as in Chapter 14) that the representative consumer has a lifetime utility function starting from period t

$$\ln c_t + \ln \left(\frac{M_t}{P_t} \right) + \beta \ln c_{t+1} + \beta \ln \left(\frac{M_{t+1}}{P_{t+1}} \right) + \beta^2 \ln c_{t+2} + \beta^2 \ln \left(\frac{M_{t+2}}{P_{t+2}} \right) + \beta^3 \ln c_{t+3} + \beta^3 \ln \left(\frac{M_{t+3}}{P_{t+3}} \right) \dots,$$

and his period- t budget constraint is given by

$$P_t c_t + P_t^b B_t + P_t^{b,TWO} B_t^{TWO} + M_t + S_t a_t = Y_t + M_{t-1} + B_{t-1} + B_{t-2}^{TWO} + (S_t + D_t) a_{t-1}.$$

(Based on this, you should know what the period $t+1$ and period $t+2$ and period $t+3$, etc. budget constraints look like). This budget constraint is identical to that in Chapter 14, except of course for the terms regarding two-period bonds. **Note carefully the timing on the right hand side – in accordance with the defining feature of a two-period bond, in period t , it is B_{t-2}^{TWO} that pays back its face value.** The rest of the notation is just as in Chapter 14, including the fact that the subjective discount factor (i.e., the measure of impatience) is $\beta < 1$.

- a. **(5 points)** Qualitatively represent (using the axes below) the yield curve shown in the diagram above in terms of **prices of bonds** rather than interest rates on bonds. That is, in the empty set of axes below, plot (qualitatively) on the vertical axis the prices associated with the bonds of various maturity lengths show in the diagram above.

Solution: With maturity lengths plotted on the horizontal axis, the yield curve **in terms of bond prices** is downward-sloping. This follows simply because of the inverse relationship between bond prices and interest rates. The yield curve shown above is in terms of interest rates and is strictly increasing; hence the associated yield curve in terms of prices must be strictly decreasing.

Question 4 continued

- b. (10 points) Based on the utility function and budget constraint given above, set up an appropriate Lagrangian, and use it to derive the representative consumer's first-order conditions with respect to **both** B_t and B_t^{TWO} (as usual, the analysis is being conducted from the perspective of the very beginning of period t). Define any auxiliary notation that you need in order to conduct your analysis.

Solution: The only two first-order conditions that you needed here are those on B_t and B_t^{TWO} .

Denoting by λ_t the Lagrange multiplier on the period- t budget constraint and by λ_{t+1} the Lagrange multiplier on the period- $t+1$ budget constraint, the two first-order conditions, respectively, are

$$-\lambda_t P_t^b + \beta \lambda_{t+1} = 0$$

and

$$-\lambda_t P_t^{b,TWO} + \beta^2 \lambda_{t+2} = 0.$$

Note well the $t+2$ time subscripts in the second expression; this follows from the fact that a two-period bond purchased in period t does not repay its promised face value until period $t+2$. (Refer back to Problem Set 4 for an analogous stock-pricing model in which stocks took two periods to pay off their capital gains and dividends.)

- c. (10 points) Using the two first-order conditions you obtained in part b, construct a relationship between the price of a two-period bond and the price of a one-period bond. Your final relationship should be of the form $P_t^{b,TWO} = \dots$, and **on the right-hand-side of this expression should appear** (potentially among other things), P_t^b . (Hint: in order to get P_t^b into this expression, you may have to multiply and/or divide your first-order conditions by appropriately-chosen variables.)

Solution: From the first expression above, we have, as usual that $P_t^b = \frac{\beta \lambda_{t+1}}{\lambda_t}$. From the second

expression above, we analogously can obtain $P_t^{b,TWO} = \frac{\beta^2 \lambda_{t+2}}{\lambda_t}$. We can rewrite this expression

for the price of a two-period bond as

$$P_t^{b,TWO} = \frac{\beta \lambda_{t+2}}{\lambda_{t+1}} \frac{\beta \lambda_{t+1}}{\lambda_t},$$

in which we have simply multiplied and divided the preceding expression by λ_{t+1} (i.e., we have multiplied by one, always a valid mathematical operation). The final term on the far right-hand-side is nothing more than the price of a one-period bond, so we can write

Question 4c continued (if you need more space)

$$P_t^{b,TWO} = \frac{\beta\lambda_{t+2}}{\lambda_{t+1}} P_t^b,$$

which satisfies the form of the relationship you were asked to derive. We can actually boil this down further, though. Note that **the price of one-period bond purchased in period $t+2$ would be given by** $P_{t+1}^b = \frac{\beta\lambda_{t+2}}{\lambda_{t+1}}$, which follows from optimization with respect to period $t+1$ **one-period** bond holdings. Using this expression in the period- t price of a **two-period** bond, we thus obtain

$$P_t^{b,TWO} = P_{t+1}^b P_t^b,$$

which is a key idea in finance theory: the price of a multi-period asset (bond) is nothing more than the **product of the prices of two consecutive one-period assets** (bond).

- d. **(5 points)** Suppose that the optimal **nominal expenditure on consumption** (Pc) is equal to 1 in every period. Using this fact, is the price of a two-period bond greater than, smaller than, or equal to the price of a one-year bond? If it is impossible to tell, explain why; if you can tell, be as precise as you can be about the relationship between the prices of the two bonds. (Hint: you may need to invoke the consumer's first-order condition on consumption)

Solution: Start with the relationship $P_t^{b,TWO} = \frac{\beta\lambda_{t+2}}{\lambda_{t+1}} P_t^b$ derived above. If nominal consumption expenditures are constant (and equal to one) every period, this means that $\lambda = 1$ every period. (This conclusion follows from the fact that the FOC with respect to consumption is $1/c_t - \lambda_t P_t = 0$ in every period, which can be rearranged to $\lambda_t = \frac{1}{P_t c_t}$). If the multiplier is one every period, we immediately have

$$P_t^{b,TWO} = \beta P_t^b.$$

Because $\beta < 1$, we conclude $P_t^{b,TWO} < P_t^b$.

- e. **(5 points)** Now suppose there is also a three-period bond. A three-period bond purchased in any given period does not repay its face value (also assumed to be 1) until *three* periods after it is purchased. The period- t budget constraint, now including one-, two-, and three-period bonds, is given by

$$P_t c_t + P_t^b B_t + P_t^{b,TWO} B_t^{TWO} + P_t^{b,THREE} B_t^{THREE} + M_t + S_t a_t = Y_t + M_{t-1} + B_{t-1} + B_{t-2}^{TWO} + B_{t-3}^{THREE} + (S_t + D_t) a_{t-1},$$

where B_t^{THREE} is the quantity of three-period bonds purchased in period t and $P_t^{b,THREE}$ its associated price. Following the same logical steps as in parts b, c, and d above (and

Question 4e continued

continuing to assume that nominal expenditure on consumption (Pc) is equal to one in period every period), is the price of a three-year bond greater than, smaller than, or equal to the price of a two-year bond? If it is impossible to tell, explain why; if you can tell, be as precise as you can be about the relationship between the prices of the two bonds. (**Note: if you can answer this question without setting up a Lagrangian, you may do so.**)

Solution: Extending the Lagrangian from above, the first-order condition with respect to B_t^{THREE} is

$$-\lambda_t P_t^{b,THREE} + \beta^3 \lambda_{t+3} = 0,$$

which can be rearranged to yield $P_t^{b,THREE} = \frac{\beta^3 \lambda_{t+3}}{\lambda_t}$. Just like in part c above, by cleverly multiplying by one, we can express this as

$$P_t^{b,THREE} = \frac{\beta^2 \lambda_{t+3}}{\lambda_{t+2}} \frac{\beta \lambda_{t+2}}{\lambda_{t+1}} \frac{\beta \lambda_{t+1}}{\lambda_t},$$

which, in exactly the same way as in part c, we can express in terms of chained one-period bond prices,

$$P_t^{b,THREE} = \frac{\beta \lambda_{t+3}}{\lambda_{t+2}} P_{t+1}^b P_t^b.$$

If the Lagrange multiplier λ is constant every period, we can conclude the price of a three-period bond is smaller than the price of a two-period bond (which in turn, from part c, is smaller than the price of a one-period bond). This again follows because $\beta < 1$.

- f. **(5 points)** Suppose that $\beta = 0.95$. Using your conclusions from parts d and e, **qualitatively** plot a yield curve in terms of bond prices in the set of axes below (obviously, you can plot only three different maturity lengths here).

Solution: Based on the analyses in parts d and e, the price of bonds is clearly negatively-related to its maturity length, hence the yield curve in terms of prices is strictly decreasing. This is just as your sketch of the empirical yield curve in part a.

- g. **(5 points)** What is the single most important reason (economically, that is) for the shape of the yield curve you found in part f? (This requires only a brief, qualitative/conceptual response.)

Solution: Re-examining our conclusions/analyses in parts d, e, and f, the sole reason we were able to reach the conclusions we reached in each of those parts was the fact that $\beta < 1$. Thus, the idea of impatience and its effects on the macroeconomy rears its head again, this time with respect to bond prices of different maturities. The conceptual idea is simple: because of impatience, the longer a bond purchaser must wait to receive a given face value, the less he will be willing to pay for it today (and this is reflected in bond market prices through the bond demand function for different maturity bonds).

END OF EXAM