Department of Applied Economics Johns Hopkins University

Economics 602

Macroeconomic Theory and Policy

Midterm Exam Suggested Solutions

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Fall 2008

NAME:

The Exam has a total of five (5) problems and pages numbered one (1) through twelve (12) (with the last three pages blank). Each problem's total number of points is shown below. Your solutions should consist of some appropriate combination of mathematical analysis, graphical analysis, logical analysis, and economic intuition, but in no case do solutions need to be exceptionally long. Your solutions should get straight to the point – **solutions with irrelevant discussions and derivations will be penalized.** You are to answer all questions in the spaces provided.

You may use one page (double-sided) of notes. You may **not** use a calculator.

Problem 1	**/ 25**
Problem 2	**/ 20**
Problem 3	**/ 10**
Problem 4	**/ 15**
Problem 5	**/ 30**
TOTAL	**/ 100**

Problem 1: Consumption and Savings in the Two-Period Economy (25 points). Consider a two-period economy (with no government), in which the representative consumer has no control over his income. The lifetime utility function of the representative consumer is $u(c_1, c_2) = \ln c_1 + \ln c_2$, where $\ln$ stands for the natural logarithm. We will work here in purely real terms: suppose the consumer's **present discounted value of ALL lifetime REAL income is 26.** Suppose that the real interest rate between period 1 and period 2 is zero (i.e., r = 0), and also suppose the consumer begins period 1 with zero net assets.

a. **(17 points)** Set up the lifetime Lagrangian formulation of the consumer's problem, in order to answer the following: i) is it possible to numerically compute the consumer's optimal choice of consumption in period 1? If so, compute it; if not, explain why not. ii) is it possible to numerically compute the consumer's optimal choice of consumption in period 2? If so, compute it; if not, explain why not. iii) is it possible to numerically compute the consumer's real asset position at the end of period 1? If so, compute it; if not, explain why not.

Solution: We know that with zero initial assets, the LBC of the consumer is

$$c_1 + \frac{c_2}{1+r_1} = y_1 + \frac{y_2}{1+r_1},$$

where the notation is standard from class. The Lagrangian is thus

$$u(c_1, c_2) + \lambda\left[y_1 + \frac{y_2}{1+r_1} - c_1 - \frac{c_2}{1+r_1}\right],$$

where λ of course is the Lagrange multiplier (note there's only one multiplier since this is the lifetime formulation of the problem not the sequential formulation of the problem). The first-order conditions with respect to c_1 and c_2 (which are the objects of choice) are, as usual:

$$u_1(c_1, c_2) - \lambda = 0$$

$$u_2(c_1, c_2) - \frac{\lambda}{1+r_1} = 0$$

(And of course the FOC with respect to the multiplier just gives back the LBC.) Also as usual, these FOCs can be combined to give the consumption-savings optimality condition, $\frac{u_1(c_1, c_2)}{u_2(c_1, c_2)} = 1 + r_1$. With the given utility function, the marginal utility functions are $u_1 = 1/c_1$ and $u_2 = 1/c_2$, so the consumption-savings optimality condition in this case becomes $c_2 / c_1 = 1 + r_1$. This can be rearranged to give $c_2 = (1+r_1)c_1$, which we can then insert in the LBC to give $c_1 + c_1 = y_1 + \frac{y_2}{1+r_1}$ (no, that's not a typo, it's $c_1 + c_1$ after the substitution…).

In this problem, you are given neither y_1 nor y_2. Instead, what you are given is $y_1 + \frac{y_2}{1+r_1} = 26$.

Thus, we have that the optimal quantity of period-1 consumption is $c_1^* = 13$ (which solves part i). We can compute c_2^*, because we are given the interest rate r_1 -- using r = 0 in the expression $c_2 = (1+r_1)c_1$ obtained above, we have $c_2 = c_1 = 13$. (This solves part ii). To compute the asset position at the end of period 1, we would need to compute $y_1 - c_1^*$, but since we don't know y_1, we cannot compute this (which solves part iii).

Problem 1 continued

b. **(8 points)** To demonstrate how important the concept of the real interest rate is in macroeconomics, an interpretation of it (in addition to the couple of different interpretations we have already discussed in class) is that it reflects the rate of consumption growth between two consecutive periods. Using the consumption-savings optimality condition for the given utility function, **briefly** describe/discuss **(rambling essays will not be rewarded)** whether the real interest rate is **positively related to, negatively related to, or not at all related to the rate of consumption growth between period one and period two.** For your reference, the definition of the rate of consumption growth rate between period 1 and period 2 is $\frac{c_2}{c_1} - 1$ (completely analogous to how we defined in class the rate of growth of prices between period 1 and period 2). (**Note:** No mathematics are especially required for this problem; also note this part can be fully completed even if you were unable to get all the way through part a).

Solution:

The familiar consumption-savings optimality condition is $\frac{u_1}{u_2} = 1 + r$. As we just saw above, for the given utility function, this becomes $\frac{1/c_1}{1/c_2} = 1 + r$, or, rewriting,

$$\frac{c_2}{c_1} = 1 + r.$$

The left-hand-side of this expression obviously measures the consumption growth rate between period 1 and period 2. That is, if $c_1 = 100$ and $c_2 = 103$, clearly the consumption growth rate is 3 percent between period 1 and period 2. Which would mean that $r = 0.03$. If the real interest rate were instead larger, clearly the left-hand-side, c_2/c_1, would be larger as well. Thus, **the higher is the real interest rate, the higher is the consumption growth rate between periods – the real interest rate and the consumption growth rate are positively related to each other.**

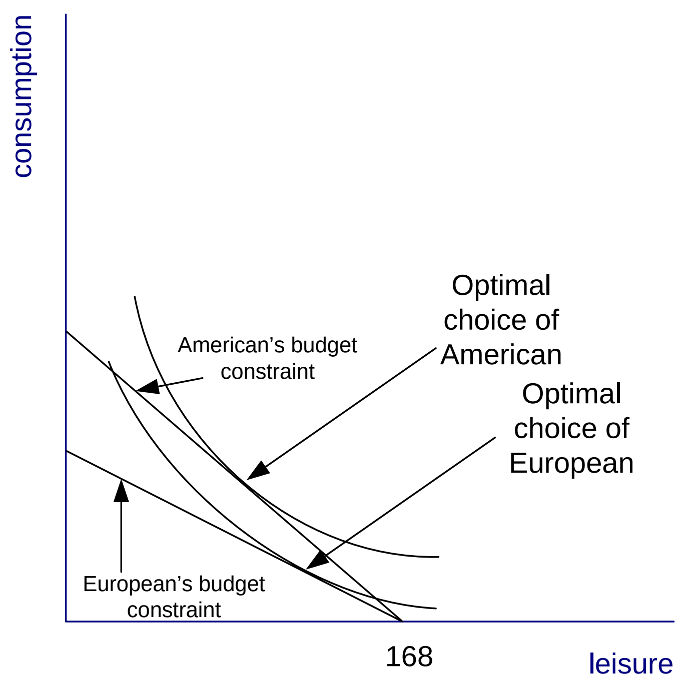
This is thus yet another way to think about the real interest rate. The two other ways we discussed in class of thinking intuitively about the real interest rate is that it measures the price of current (period-1) consumption in terms of future (period-2) consumption; and as reflecting the fundamental degree of (human) impatience of individuals in the economy. All of these various (and ultimately inter-related) ways of thinking about the real interest underline its fundamental importance in macroeconomic theory.

Note that simply arguing/explaining here that a rise in the real interest rate leads to a fall in period-1 consumption does not address the question – the question is about the **rate of change of consumption between period 1 and period 2,** not about the **level** of consumption in period 1 by itself.

Problem 2: European and U.S. Consumption-Leisure Choices (20 points). Europeans work fewer hours than Americans. There are likely very many possible reasons for this, and indeed in reality this fact arises from a combination of many reasons. In this question, you will consider two reasons using the simple (one-period) consumption-leisure model.

a. **(10 points)** Suppose that both the utility functions and pre-tax real wages W/P of American and European individuals are identical. However, the labor income tax rate in Europe is higher than in America. In a **single** carefully-labeled indifference-curve/budget constraint diagram (with consumption on the vertical axis and leisure on the horizontal axis), show how it can be the case that Europeans work fewer hours than Americans. Provide any explanation of your diagram that is needed.

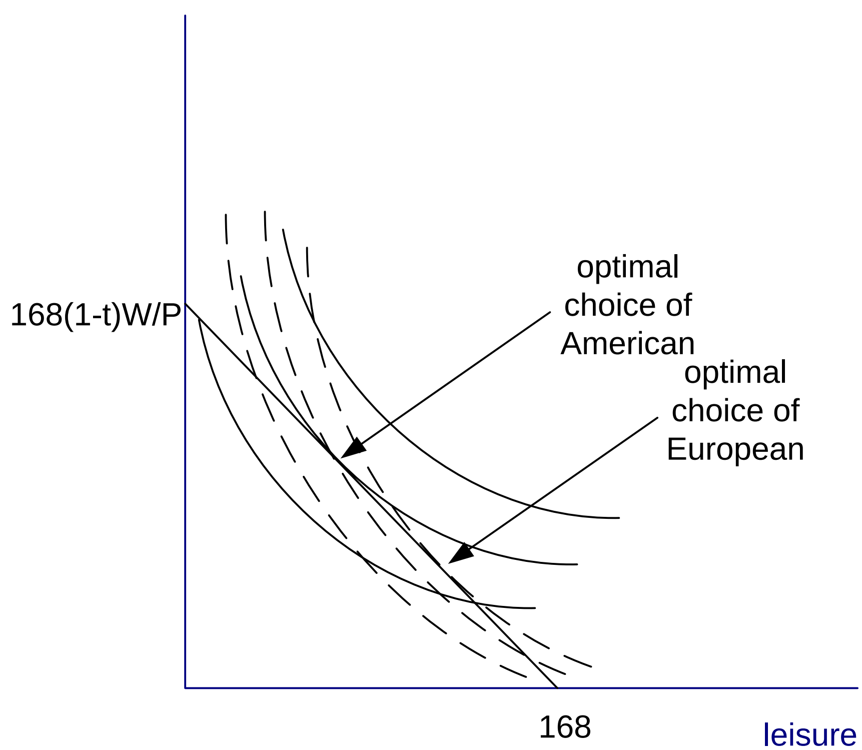
Solution: If Europeans work fewer hours than Americans, then Europeans have more leisure time than Americans, simply because (in our weekly model) $n + l = 168$. Europeans and Americans have identical utility functions, which means that their indifference maps are identical. This means that the difference in hours worked must arise completely from differences in their budget constraints. With a higher labor income tax in Europe, the budget constraint of the European consumer is less steep than the budget constraint of the American, as the diagram below shows (because the slope of the budget constraint is $(1-t)W/P$, and you are given that W/P is the same in the two countries). The diagram shows that the European optimally chooses more leisure (hence less labor) and less consumption than the American. Here, the difference between Europeans and Americans is solely in the relative prices (embodied by the slope of the budget constraint) they face. (For full credit here, you had to somehow make clear that the indifference maps of the representative European and the representative American are identical.)



Problem 2 continued.

b. **(10 points)** Suppose that both the pre-tax real wages W/P and the labor tax rates imposed on American and European individuals are identical. However, the utility function $u^{AMER}(c,l)$ of Americans differs from that of Europeans $u^{EUR}(c,l)$. In a **single** carefully-labeled indifference-curve/budget constraint diagram (with consumption on the vertical axis and leisure on the horizontal axis), show how it can be the case that Europeans work fewer hours than Americans. Provide any explanation of your diagram that is needed.

Solution: In this case, the budget constraints of the European consumer and American consumer are identical, so the difference in hours worked must arise completely from differences in their utility functions. Graphically, this means that the two types of consumers have different indifference maps (i.e., a different set of indifference curves). In the diagram below, the budget line is the common budget line of the European and the American. The solid indifference curves are the American's, while the dashed indifference curves are the European's. With steeper indifference curves, the European's optimal choice along the same budget line must occur at a point that features more leisure (hence less labor) and less consumption than the American's optimal choice. Here, the difference between Europeans and Americans is solely in their preferences.



Problem 3: Government Budgets and Government Asset Positions (10 points). Just as we can analyze the economic behavior of consumers over many time periods, we can analyze the economic behavior of the government over many time periods. **Suppose that at the beginning of period *t*, the government has zero net assets**. Also assume that the real interest rate is **always *r* = 0.** The following table describes the **real** quantities of government spending and **real** tax revenue the government collects starting in period *t* and for several periods thereafter.

Period	**Real government expenditure (*g*) during the period**	**Real tax collections during the period**	**Quantity of net government assets at the END of the period**
t	10	12	
***t*+1**	8	14	
***t*+2**	15	10	
***t*+3**	10	10	
***t*+4**	8	12	

a. **(6 points)** Complete the last column of the table based on the information given. **Briefly** explain the logic behind how you calculate these values.

Solution: If this were the two-period model, we could compute the government asset position at the end of period, say, one, by rearranging the period-1 government budget constraint: $b_1 = t_1 - g_1 + b_0$ -- in this expression we have used the assumption that $r = 0$. Furthermore (and again with $r = 0$), we can compute the government asset position at the end of period two as: $b_2 = t_2 - g_2 + b_1$ (In the simple two-period model, we assumed $b_2 = 0$, but if we want to extend past two periods, we of course would not make this assumption.) Directly extending this logic to an infinite-period setting, then, the government's asset position at the **end** of any particular period *t* is given by: $b_t = t_t - g_t + b_{t-1}$. Successively applying this rule then gives rise to the net asset positions presented in the table above.

b. **(4 points)** Suppose instead the government ran a balanced budget every period (i.e., every period it collected in taxes exactly the amount of its expenditures that period). In this balanced-budget scenario, what would be the government's net assets at the end of period *t*+4? **Briefly** explain/justify.

Solution: A balanced budget means *g* equals tax collections every period. If this were true in the above table, and applying the logic of part a above, the government net assets at the end of **every** period would always be zero; thus at the end of period *t*+4 they are zero as well.

Problem 4: A Contraction in Credit Availability (15 points). The graph below shows our usual two-period indifference-curve/budget constraint diagram, with period-1 consumption plotted on the horizontal axis, period-2 consumption plotted on the vertical axis, and the downward-sloping line representing, as always, the consumer's LBC. Throughout all of the analysis here, assume that ***r* = 0 always.** Furthermore, there is no government, hence never any taxes.

Suppose that the representative consumer has lifetime utility function $u(c_1, c_2) = \ln c_1 + \ln c_2$, and that the **real** income of the consumer in period 1 and period 2 is $y_1 = 12$ and $y_2 = 8$. Finally, suppose that the initial amount of net assets the consumer has is $a_0 = 0$. **EVERY** consumer in the economy is described by this utility function and these values of y_1, y_2, and a_0.

a. **(6 points)** If there are no problems in credit markets whatsoever (so that consumers can borrow or save as much or as little as they want), compute the numerical value of the optimal quantity of period-1 consumption. (**Note:** if you can solve this problem without setting up a Lagrangian, you are free to do so as long as you explain your logic.)

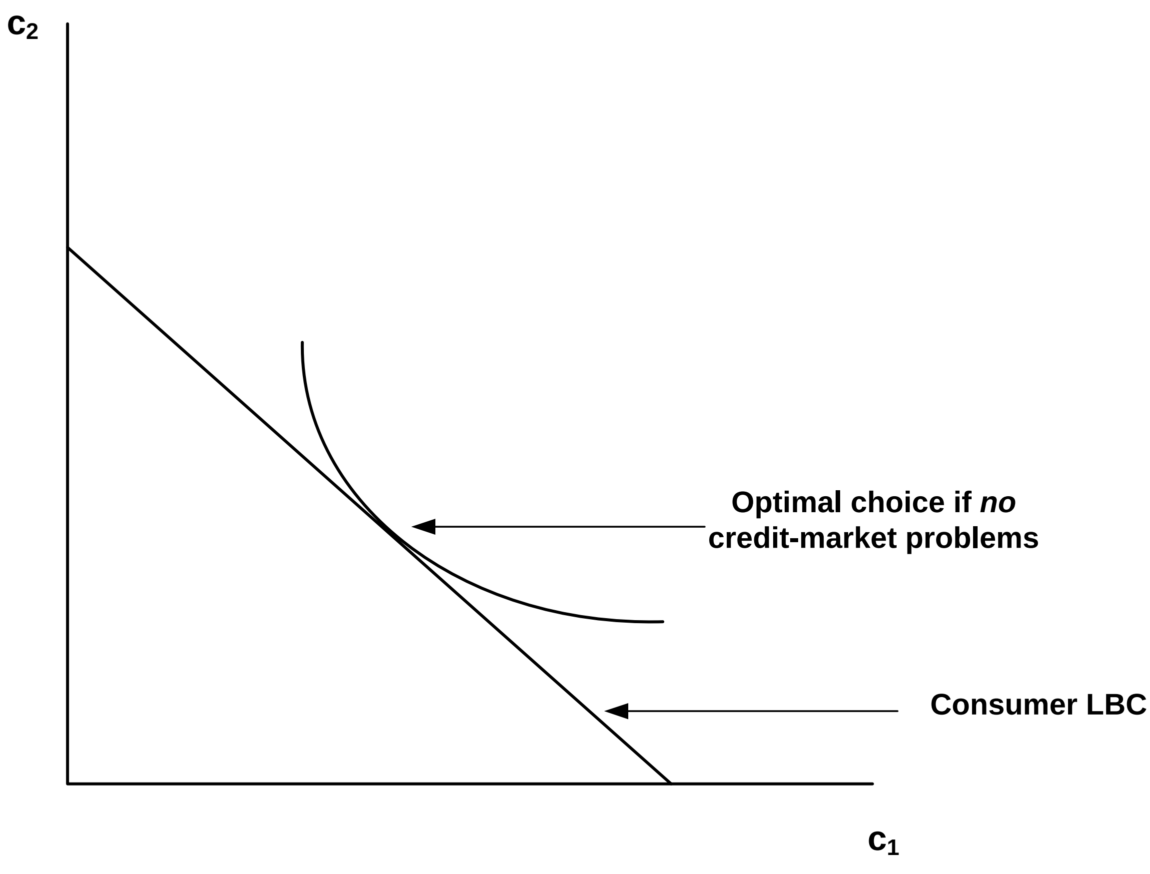
Solution: The consumption-savings optimality condition (given the natural-log utility function) is given by $c_2/c_1 = 1+r = 1$ (the second equality follows because $r = 0$ here). Thus, at the optimal choice, it is the case that $c_1 = c_2$. Using this relationship (and again using the fact that $r = 0$ here), we can express the consumer's LBC as $c_1 + c_1 = y_1 + y_2 = 20$, which obviously implies the optimal choice of period-1 consumption is $c_1 = 10$.

Note: although you were not asked to compute it, you could have computed the implied value of the consumer's asset position at the end of period one. Because $a_0 = 0$, $y_1 = 12$, and we just computed $c_1 = 10$, the asset position at the end of period one is $a_1 = y_1 - c_1 = 2$ (i.e., **positive** 2).

b. **(9 points)** Now suppose that because of problems in the financial sector, no consumers are allowed to be in debt at the end of period 1. With this credit restriction in place, compute the numerical value of the optimal quantity of period-1 consumption. **ALSO,** on the diagram on the next page, qualitatively and **clearly** sketch the optimal choice with this credit restriction in place (qualitatively sketched already for you is the optimal choice if there are no problems in credit markets). Your sketch should indicate **both** the new optimal choice **and** an appropriately-drawn and labeled indifference curve that contains the new optimal choice. (**Note:** if you can solve this problem without setting up a Lagrangian, you are free to do so as long as you explain your logic.)

Solution: Because in part a (ie, without any credit restrictions), the representative consumer was choosing to NOT be in debt at the end of period 1 (ie, $a_1 > 0$ under the optimal choice in part a), the imposition of the credit restriction, **nothing changes compared to part a.** That is, the optimal choice of period-1 consumption is still 10. Hence, in the diagram below, the optimal choice in the presence of credit constraints is **exactly the same as the optimal choice without credit constraints.** The general lesson to draw from this example and our analysis in class is that it is not *necessarily* the case that financial market problems *must* and *always* spill over into real economic activity (i.e., consumption in this case).

Problem 4b continued



Problem 5: Effects of Tax Policy on Stock Prices (30 points). Consider our infinite-period model with stocks as the only asset. Stocks held at the beginning of period t pay a nominal dividend D_t at the very beginning of period t. Suppose that dividend payments are subject to a proportional tax rate t_t^D in period t, where t_t^D is a number between zero and one. For example, if $t_t^D = 0.20$, then 20 percent of all dividends received by the representative consumer in period t must be paid to the government (we'll disregard here any issues related to what the government does with those revenues).

a. **(5 points)** Set up the period-t flow budget constraint, briefly explaining how the dividend tax enters the expression.

Solution: In period t, the flow budget constraint reads

$$P_t c_t + S_t a_t = Y_t + S_t a_{t-1} + (1 - t_t^D) D_t a_{t-1},$$

in which the term on the far-right-side of the equation says that the consumer only keeps the fraction $1 - t_t^D$ of dividend payments after paying the dividend tax.

b. **(10 points)** Using the flow budget constraint you set up above, show algebraically (i.e., using a Lagrangian) how the nominal stock price in period t, denoted as usual by S_t, depends on the dividend tax when the representative consumer is maximizing lifetime utility from period t onwards. **Also, the dividend tax rate in WHICH period affects the period-t stock price?** Provide brief economic interpretation/logic.

Solution: We only need the period-t and and period-(t+1) terms of the Lagrangian (since there is no habit persistence and the holding period is just one period here), which are:

$$u(c_t) + \lambda_t \left[Y_t + S_t a_{t-1} + (1 - t_t^D) D_t a_{t-1} - P_t c_t - S_t a_t \right]$$
$$+ \beta u(c_{t+1}) + \beta \lambda_{t+1} \left[Y_{t+1} + S_{t+1} a_t + (1 - t_{t+1}^D) D_{t+1} a_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1} \right] + \ldots$$

where as usual λ_t is the multiplier on the period-t budget constraint and λ_{t+1} is the multiplier on the period t+1 budget constraint. The first-order condition with respect to a_t (which is one of the objects of choice in period t) is (after a slight algebraic rearrangement)

$$S_t = \frac{\beta \lambda_{t+1}}{\lambda_t} \left(S_{t+1} + (1 - t_{t+1}^D) D_{t+1} \right),$$

which shows that it is the **period-(t+1) dividend tax rate that affects the period-t stock price.** This should make sense because it's the period-t+1 return that matters for the period-t stock price – i.e., when markets (represented by our representative consumer) determine prices in the current period, the future **after-tax** return is what matters. As the above expression shows, the lower is t_{t+1}^D (all else equal), the higher is S_t. Thus, if it's announced that there will be a lowering of the dividend tax rate in the future, that might be expected to boost stock prices in the present. Note that here you did not even need to compute the FOC with respect to c_t (nor proceed to construct the pricing kernel), since that wasn't essential to the argument.

Problem 5b continued (if you need more space)

c. **(15 points – Harder)** Suppose in addition to the dividend tax described above, there is also a proportional tax on consumption (a sales tax). The consumption tax rate in period t is t_t^C. Suppose that t_t^C rises, but **all** other tax rates (including those beyond period t) remain unchanged. Show algebraically (i.e., using a Lagrangian) how this one-time consumption-tax hike policy change affects the period-t stock price, assuming all else is equal? (i.e., this is a usual *ceteris paribus* exercise) Also provide brief economic interpretation for your finding.

Solution: The flow budget constraint here modifies to

$$(1+t_t^C)P_t c_t + S_t a_t = Y_t + S_t a_{t-1} + (1-t_t^D)D_t a_{t-1},$$

and the Lagrangian modifies to

$$u(c_t) + \lambda_t \left[Y_t + S_t a_{t-1} + (1-t_t^D)D_t a_{t-1} - (1+t_t^C)P_t c_t - S_t a_t \right]$$
$$+\beta u(c_{t+1}) + \beta\lambda_{t+1}\left[Y_{t+1} + S_{t+1}a_t + (1-t_{t+1}^D)D_{t+1}a_t - (1+t_{t+1}^C)P_{t+1}c_{t+1} - S_{t+1}a_{t+1} \right] + \ldots$$

The first-order condition with respect to a_t is unchanged from above, but here you **did** need to explicitly consider the FOC with respect to c_t (and proceed to construct the kernel) in order to see the effect of the consumption tax on stock prices. The FOC with respect to consumption in period t is $u'(c_t) - \lambda_t(1+t_t^C)P_t = 0$, which can be solved for the multiplier, $\lambda_t = \dfrac{u'(c_t)}{(1+t_t^C)P_t}$.

Similarly, in period t+1 we'd have $\lambda_{t+1} = \dfrac{u'(c_{t+1})}{(1+t_{t+1}^C)P_{t+1}}$. Inserting these expressions for the multipliers into the stock price equation,

$$S_t = \frac{\beta u'(c_{t+1})}{u'(c_t)} \cdot \frac{(1+t_t^C)P_t}{(1+t_{t+1}^C)P_{t+1}} \left(S_{t+1} + (1-t_{t+1}^D)D_{t+1} \right).$$

If the period-t sales tax t_t^C rises (and all else remains unchanged), the above expression shows that S_t rises. The economics behind this is that if the period-t price of consumption (inclusive of taxes) rises, consumers will desire less period-t consumption and substitute into period t+1 consumption. Doing so means increased demand for assets (stocks) in period t as a means by which to transfer resources from the present (period t) to the future (period t+1). The increased demand for stocks in period t bids up the price S_t.