

MONEY IN THE INFINITE-PERIOD ECONOMY

APRIL 13, 2009

Introduction

BASICS

- **Extend our infinite-period model**
 - Introduce money and bonds into the Chapter 8 model
 - So now three types of assets (stocks, bonds, money) for representative consumer to use for savings purposes

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- ❑ Will allow us to think further about what the “pricing kernel” is
- ❑ Will allow us to think about connection between bond prices and stock prices
- ❑ Will allow us to think about issue of monetary neutrality (the main issue in the RBC vs. New Keynesian debate)
 - ❑ i.e., does money (and thus monetary policy) have important consequences for *real* (i.e., consumption and real GDP) variables?

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 - ❑ i.e., does money (and thus monetary policy) have important consequences for *real* (i.e., consumption and real GDP) variables?
- ❑ **Index time periods by arbitrary indexes t , $t+1$, $t+2$, etc.**
 - ❑ **Important: all of our analysis will be conducted from the perspective of the very beginning of period t ...**
- ❑ **Sequential Lagrangian analysis** (with money in the utility function)

BASICS

□ Timeline of events



□ Notation

- c_t : consumption in period t
- P_t : nominal price of consumption in period t
- Y_t : nominal income in period t ("falls from the sky")
- a_{t-1} : real stock holdings at beginning of period t /end of period $t-1$

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 a_{t-1} Economic events during
period t : income,
consumption, savings a_t Economic
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consumpPeriod t

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Now three types
of assets
consumers can
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purposes

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- B_{t-1} : nominal bond holdings at beginning of period t /end of period $t-1$
- S_t : nominal price of a unit of stock in period t
- D_t : nominal dividend paid in period t by each unit of stock held at the start of t
- P_t^b : nominal price of a bond in period t
- i_t : nominal interest rate on a bond purchased in t and which pays off in $t+1$

Now three types of assets consumers can use for savings purposes

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a_{t-1}
 B_{t-1}
 M_{t-1}

Economic events during period t : income, consumption, savings

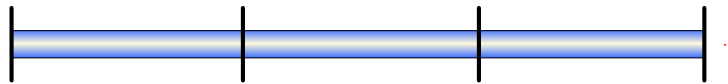
a_t
 B_t
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Economic period consumption

Period t

BASICS

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- n_{t+1} : net inflation rate between period t and period $t+1$
- y_t : real income in period t ($= Y_t/P_t$)

Now three types of assets consumers can use for savings purposes

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Period

BASICS

Timeline of events



Notation

Now three types of assets consumers can use for savings purposes

- c_{t+1} : consumption in period $t+1$
- P_{t+1} : nominal price of consumption in period $t+1$
- Y_{t+1} : nominal income in period $t+1$ ("falls from the sky")
- a_t : real stock holdings at beginning of period $t+1$ /end of period t
- M_t : nominal money holdings at beginning of period $t+1$ /end of period t
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- P_{t+1}^b : nominal price of a bond in period $t+1$
- i_{t+1} : nominal interest rate on a bond purchased in $t+1$ and which pays off in $t+2$
- π_{t+2} : net inflation rate between period $t+1$ and period $t+2$
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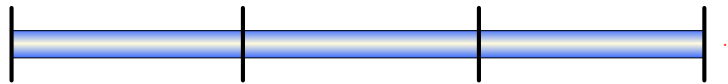
Economic events during period t : income, consumption, savings

a_t
 B_t
 M_t

Economic period consumption

BASICS

Timeline of events



Notation

- And so on for period $t+2$, $t+3$, etc...

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BUDGET CONSTRAINT(S)

- Extend budget constraints from Chapter 8 stock-pricing model to now include the three distinct types of assets: stocks, money, and bonds
- Need **infinite** budget constraints to describe economic opportunities and possibilities
 - One for each period

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 - In period t

$$P_t c_t + P_t^b B_t + M_t + S_t a_t = Y_t + M_{t-1} + B_{t-1} + S_t a_{t-1} + D_t a_{t-1}$$

Total outlays in period t : period- t consumption + stocks to *carry into period $t+1$* + money to *carry into period $t+1$* + bond purchases

Total income in period t : period- t Y + income from stock-holdings *carried into period t* (has value S_t and pays dividend D_t) + money-holdings *carried into period t* + bond-holdings *carried into period t* (each unit repays $FV = 1$)

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- In period $t+1$

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- And identical-looking budget constraints in period $t+2$, $t+3$, $t+4$, etc.

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Step 1:** Construct Lagrange function (starting from t)

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$$u(c_t, M_t / P_t) + \beta u(c_{t+1}, M_{t+1} / P_{t+1}) + \beta^2 u(c_{t+2}, M_{t+2} / P_{t+2}) + \dots$$

First the lifetime utility function....(no different than Chapter 8)

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

□ **Step 1: Construct Lagrange function (starting from t)**

$$u(c_t, M_t / P_t) + \beta u(c_{t+1}, M_{t+1} / P_{t+1}) + \beta^2 u(c_{t+2}, M_{t+2} / P_{t+2}) + \dots$$

$$+ \lambda_t [Y_t + (S_t + D_t)a_{t-1} + M_{t-1} + B_{t-1} - P_t c_t - S_t a_t - M_t - P_t^b B_t]$$

First the lifetime utility function....(no different than Chapter 8)
 ...then the period t constraint...

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$$+ \beta \lambda_{t+1} [Y_{t+1} + (S_{t+1} + D_{t+1})a_t + M_t + B_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1} - M_{t+1} - P_{t+1}^b B_{t+1}]$$

First the lifetime utility function....(no different than Chapter 8)
 ...then the period t constraint...

...then the period $t+1$ constraint...

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

□ **Step 1: Construct Lagrange function (starting from t)**

$$\begin{aligned}
 & u(c_t, M_t / P_t) + \beta u(c_{t+1}, M_{t+1} / P_{t+1}) + \beta^2 u(c_{t+2}, M_{t+2} / P_{t+2}) + \dots && \text{First the lifetime utility function....(no} \\
 & + \lambda_t [Y_t + (S_t + D_t)a_{t-1} + M_{t-1} + B_{t-1} - P_t c_t - S_t a_t - M_t - P_t^b B_t] && \text{different than Chapter 8)} \\
 & && \dots \text{then the period } t \text{ constraint...} \\
 & + \beta \lambda_{t+1} [Y_{t+1} + (S_{t+1} + D_{t+1})a_t + M_t + B_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1} - M_{t+1} - P_{t+1}^b B_{t+1}] && \dots \text{then the period } t+1 \text{ constraint...} \\
 & + \beta^2 \lambda_{t+2} [Y_{t+2} + (S_{t+2} + D_{t+2})a_{t+1} + M_{t+1} + B_{t+1} - P_{t+2} c_{t+2} - S_{t+2} a_{t+2} - M_{t+2} - P_{t+2}^b B_{t+2}] && \dots \text{then the period } t+2 \\
 & && \text{constraint...}
 \end{aligned}$$

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$$\begin{aligned}
 & u(c_t, M_t / P_t) + \beta u(c_{t+1}, M_{t+1} / P_{t+1}) + \beta^2 u(c_{t+2}, M_{t+2} / P_{t+2}) + \dots && \text{First the lifetime utility function....(no} \\
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 & && \dots \text{then the period } t \text{ constraint...} \\
 & + \beta \lambda_{t+1} [Y_{t+1} + (S_{t+1} + D_{t+1})a_t + M_t + B_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1} - M_{t+1} - P_{t+1}^b B_{t+1}] && \dots \text{then the period } t+1 \text{ constraint...} \\
 & + \beta^2 \lambda_{t+2} [Y_{t+2} + (S_{t+2} + D_{t+2})a_{t+1} + M_{t+1} + B_{t+1} - P_{t+2} c_{t+2} - S_{t+2} a_{t+2} - M_{t+2} - P_{t+2}^b B_{t+2}] && \dots \text{then the period } t+2 \\
 & && \text{constraint...} \\
 & + \beta^3 \lambda_{t+3} [Y_{t+3} + (S_{t+3} + D_{t+3})a_{t+2} + M_{t+2} + B_{t+2} - P_{t+3} c_{t+3} - S_{t+3} a_{t+3} - M_{t+3} - P_{t+3}^b B_{t+3}] && \dots \text{then the period } t+3 \\
 & && \text{constraint...} \\
 & + \dots
 \end{aligned}$$

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 & u(c_t, M_t / P_t) + \beta u(c_{t+1}, M_{t+1} / P_{t+1}) + \beta^2 u(c_{t+2}, M_{t+2} / P_{t+2}) + \dots \\
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 & \dots
 \end{aligned}$$

First the lifetime utility function....(no different than Chapter 8)
 ...then the period t constraint...
 ...then the period t+1 constraint...
 ...then the period t+2 constraint...
 ...then the period t+3 constraint...
 Infinite number of terms

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 ...then the period t constraint...
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 ...then the period t+2 constraint...
 ...then the period t+3 constraint...
 Infinite number of terms

□ **Step 2: Compute FOCs with respect to $c_t, a_t, B_t, M_t, \dots$**

with respect to c_t :

with respect to a_t :

with respect to B_t :

with respect to M_t :

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 ...then the period t constraint...
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□ **Step 2: Compute FOCs with respect to $c_t, a_t, B_t, M_t, \dots$**

with respect to c_t : $u_1(c_t, M_t / P_t) - \lambda_t P_t = 0$ Equation 1

with respect to a_t : $-\lambda_t S_t + \beta \lambda_{t+1} (S_{t+1} + D_{t+1}) = 0$ Equation 2

with respect to B_t : $-\lambda_t P_t^b + \beta \lambda_{t+1} = 0$ Equation 3

with respect to M_t : $\frac{u_2(c_t, M_t / P_t)}{P_t} - \lambda_t + \beta \lambda_{t+1} = 0$ Equation 4 (need chain rule of calculus to derive this)

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□ Equation 2 → $S_t = \left(\frac{\beta \lambda_{t+1}}{\lambda_t} \right) (S_{t+1} + D_{t+1})$ STOCK-PRICING EQUATION

Period-t stock price = Pricing kernel × Future return

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$$u_1(c_t, M_t / P_t) - \lambda_t P_t = 0 \quad \text{Equation 1}$$

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$$\text{Period-}t \text{ stock price} = \text{Pricing kernel} \times \text{Future return}$$

- Much of finance theory concerned with pricing kernel
 - Theoretical properties
 - Empirical models of kernels
- Pricing kernel where macro theory and finance theory intersect

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□ Equation 3 → $P_t^b = \frac{\beta \lambda_{t+1}}{\lambda_t}$ BOND-PRICING EQUATION

ASSET PRICING REVISITED

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□ Equation 3 →
$$P_t^b = \frac{\beta \lambda_{t+1}}{\lambda_t}$$
 BOND-PRICING EQUATION

- Price of a bond is the pricing kernel
- Stock prices and bond prices are connected
 - Most (all?) asset prices fundamentally connected to bond prices
 - (Advanced finance course: pricing kernel reflects the price/return of the least risky asset in the economy – U.S. Treasury bonds)

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ASSET PRICING REVISITED

$$u_1(c_t, M_t / P_t) - \lambda_t P_t = 0 \quad \text{Equation 1}$$

$$-\lambda_t S_t + \beta \lambda_{t+1} (S_{t+1} + D_{t+1}) = 0 \quad \text{Equation 2}$$

$$-\lambda_t P_t^b + \beta \lambda_{t+1} = 0 \quad \text{Equation 3}$$

$$\frac{u_2(c_t, M_t / P_t)}{P_t} - \lambda_t + \beta \lambda_{t+1} = 0 \quad \text{Equation 4}$$

□ Equation 2 →
$$S_t = \left(\frac{\beta \lambda_{t+1}}{\lambda_t} \right) (S_{t+1} + D_{t+1})$$
 STOCK-PRICING EQUATION

Period- t stock price = Pricing kernel × Future return

□ Equation 3 →
$$P_t^b = \frac{\beta \lambda_{t+1}}{\lambda_t}$$
 BOND-PRICING EQUATION

□ Recall
$$P_t^b = \frac{1}{1+i_t}$$

□ → can express pricing kernel as
$$\frac{\beta \lambda_{t+1}}{\lambda_t} = \frac{1}{1+i_t}$$

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FISHER EQUATION

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$$\frac{u_2(c_t, M_t / P_t)}{P_t} - \lambda_t + \beta \lambda_{t+1} = 0 \quad \text{Equation 4}$$

- Combining stock-pricing equation with bond-pricing equation →

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}} \quad \text{FISHER EQUATION}$$

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- Combining stock-pricing equation with bond-pricing equation →

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}} \quad \text{FISHER EQUATION}$$

- Fisher equation a relationship between returns on nominal bonds and returns on stock (finance theory: "no-arbitrage" condition)
- (See derivation in Chapter 14)
- Was a building block of two-period model
- Recall approximate form: $r \approx i - \pi$

CONSUMPTION-MONEY OPTIMALITY CONDITION

Begin with equation 4:

$$\frac{u_2(c_t, M_t / P_t)}{P_t} - \lambda_t = -\beta \lambda_{t+1}$$

↓ Use $\beta \lambda_{t+1} = \lambda_t P_t^b$ from equation 2

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$$\frac{u_2(c_t, M_t / P_t)}{P_t} - \lambda_t = -\lambda_t P_t^b$$

↓ Divide through by λ_t

$$\frac{u_2(c_t, M_t / P_t)}{\lambda_t P_t} - 1 = -P_t^b$$

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↓ Use $\lambda_t P_t = u_{1t}$ from equation 1

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↓ Use $P_t^b = 1 / (1 + i_t)$

$$\frac{u_2(c_t, M_t / P_t)}{u_1(c_t, M_t / P_t)} = \frac{i_t}{1 + i_t}$$

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CONSUMPTION-MONEY OPTIMALITY CONDITION

MRS (between consumption and real money holdings) price ratio (between consumption and money)

MONEY DEMAND

- Consumption-money optimality condition the foundation of money demand function
- Example: suppose $u\left(c_t, \frac{M_t}{P_t}\right) = \ln c_t + \ln\left(\frac{M_t}{P_t}\right)$

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- Consumption-money optimality condition (for this utility function...) is

$$\frac{P_t c_t}{M_t} = \frac{i_t}{1+i_t}$$

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↓ Isolate the M_t/P_t term

$$\frac{M_t}{P_t} = c_t \cdot \left(\frac{1+i_t}{i_t}\right)$$

REAL MONEY DEMAND FUNCTION: depends positively on c_t and negatively on i_t (i_t is the opportunity cost of money)

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REAL MONEY DEMAND FUNCTION: depends positively on c_t and negatively on i_t (i_t is the opportunity cost of money)

- Will use this money demand function to analyze
 - The monetary neutrality debate
 - The long-run (aka steady-state) connection between monetary policy and inflation

MONEY IN THE INFINITE-PERIOD ECONOMY: THE NEUTRALITY DEBATE AND THE STEADY STATE

APRIL 13, 2009

Monetary Policy Analysis: Short-Run Effects

IS MONETARY POLICY NEUTRAL?

- ❑ An enduring question in macroeconomics: does monetary policy have any important effects on the real (i.e., *real* GDP, consumption, etc) economy?
- ❑ **Definition:** Money (and hence monetary policy) is neutral if changes in the money supply (i.e., changes in monetary policy) have no effect on the real economy
 - ❑ Monetary policy is non-neutral if it does have effects on the real economy
- ❑ New Keynesian view: money is non-neutral (because prices are rigid/sticky, often for long periods of time)
- ❑ RBC view: money is neutral (because prices are not rigid/sticky in any important way)
- ❑ **MIU model allows us to consider how/why money is or is not neutral**

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MONEY DEMAND

CONSUMPTION-MONEY OPTIMALITY CONDITION

$$\frac{u_2(c_t, M_t / P_t)}{u_1(c_t, M_t / P_t)} = \frac{i_t}{1+i_t}$$

MRS (between consumption and real money holdings) price ratio (between consumption and money)

NOTE: consumption-money optimality condition and money demand function are the same thing, just viewed from different points of view

Using utility function $u\left(c_t, \frac{M_t}{P_t}\right) = \ln c_t + \ln\left(\frac{M_t}{P_t}\right)$, generate money demand function

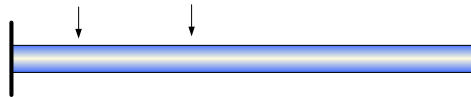
REAL MONEY DEMAND FUNCTION: depends positively on c_t and negatively on i_t (i_t is the opportunity cost of money)

$$\frac{M_t}{P_t} = c_t \cdot \left(\frac{1+i_t}{i_t}\right)$$

- Use money demand function to illustrate effects of **money shocks**
- Gets at core of neutrality debate
- Let's be even more precise about the timing of events...

MONETARY NEUTRALITY DEBATE

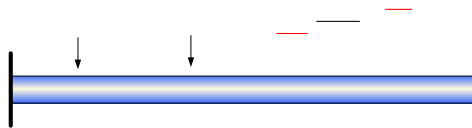
- Precise timing of events **within period t**



- Fed sets "**actual M_t** " after consumers makes their choices of c_t and "**planned M_t** " (and other choices, too...)
 - If actual M_t differs from planned M_t , **money shock** has occurred

MONETARY NEUTRALITY DEBATE

- Precise timing of events **within period t**



- Fed sets **"actual M_t "** after consumers makes their choices of c_t and **"planned M_t "** (and other choices, too...)
 - If actual M_t differs from planned M_t , **money shock** has occurred
- **Question: which adjusts (P_t or c_t) to ensure consumption-money optimality condition holds?** (simplify by assuming i_t doesn't adjust)

$$\frac{M_t}{P_t} = c_t \cdot \left(\frac{1+i_t}{i_t} \right)$$

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45 Consumers make optimal choices: in particular, of c_t and "planned M_t "

Fed comes along and determines **actual M_t** of economy – if different from planned M_t , a "money shock" has occurred

a_{t-1}
 B_{t-1}
 M_{t-1}

MONETARY NEUTRALITY DEBATE

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$$\frac{M_t}{P_t} = c_t \cdot \left(\frac{1+i_t}{i_t} \right)$$

- **Keynesian/New Keynesian view**
 - P_t cannot adjust because prices are sticky
 - (Prices will adjust **later** (i.e. in period $t+1$ or later), just not in period t)
 - A positive (negative) money shock leads to a rise (fall) in c_t
 - Money (and hence monetary policy) is not neutral

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Period

MONETARY NEUTRALITY DEBATE

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- RBC view
 - P_t can adjust because prices are not sticky
 - No reason for c_t to adjust (they do reflect optimal choices, after all...)
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- Empirical evidence for "how sticky" are prices is very mixed...

MONETARY NEUTRALITY DEBATE: EXAMPLE

- Assume $i_t = 0.1$ is fixed
- Consumers' "planned" choices are $c_t = 2$ and $M_t = 180$
- This plan was made with $P_t = 10$ in mind

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- This plan was made with $P_t = 10$ in mind
- Fed sets actual $M_t = 270$ (a positive money shock because actual M_t greater than planned M_t)
- Keynesian/New Keynesian view
 - $P_t = 10$ won't change (sticky prices)
 - c_t will rise (to $c_t = 3$) to make consumption-money optimality condition hold
 - **Monetary policy is non-neutral**

$$\frac{M_t}{P_t} = c_t \cdot \left(\frac{1+i_t}{i_t} \right)$$

MONETARY NEUTRALITY DEBATE: EXAMPLE

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 - ❑ $P_t = 10$ won't change (sticky prices)
 - ❑ c_t will rise (to $c_t = 3$) to make consumption-money optimality condition hold
 - ❑ **Monetary policy is non-neutral**
- ❑ RBC view
 - ❑ Consumers' plan of $c_t = 2$ is what the economy really wants
 - ❑ P_t can fully and quickly adjust to accommodate this $\rightarrow P_t = 15$
 - ❑ **Monetary policy is neutral; only effect of monetary policy is on inflation**

$$\frac{M_t}{P_t} = c_t \cdot \left(\frac{1+i_t}{i_t} \right)$$

MONEY AND INFLATION IN THE LONG-RUN

- ❑ **Question: what determines inflation in the long run (i.e., in steady-state)?**
- ❑ Use both period-($t-1$) and period- t money demand functions to analyze

Money demand function in $t-1$

$$\frac{M_{t-1}}{P_{t-1}} = c_{t-1} \cdot \left(\frac{1+i_{t-1}}{i_{t-1}} \right)$$

Money demand function in t

$$\frac{M_t}{P_t} = c_t \cdot \left(\frac{1+i_t}{i_t} \right)$$

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Divide period t money demand by period $t-1$ money demand

$$\frac{M_t/P_t}{M_{t-1}/P_{t-1}} = \frac{c_t}{c_{t-1}} \cdot \left(\frac{1+i_t}{i_t} \right) \left(\frac{i_{t-1}}{1+i_{t-1}} \right)$$

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Recall definition of inflation $\pi_t = \frac{P_t}{P_{t-1}} - 1$
And now define the money growth rate $\mu_t = \frac{M_t}{M_{t-1}} - 1$ in an analogous way:

$$\frac{1+\mu_t}{1+\pi_t} = \frac{c_t}{c_{t-1}} \cdot \left(\frac{1+i_t}{i_t} \right) \left(\frac{i_{t-1}}{1+i_{t-1}} \right)$$

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Impose steady state i.e., $c_{t+1} = c_t = c$, $i_{t+1} = i_t = i$, $n_t = n$, and $\mu_t = \mu$

$$\frac{1+\mu}{1+\pi} = \frac{c}{c} \cdot \left(\frac{1+i}{i} \right) \left(\frac{i}{1+i} \right)$$

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$\mu = \pi$ **IN LONG RUN, RATE OF MONEY GROWTH = RATE OF INFLATION**

MONETARISM

$$\mu = \pi$$

IN LONG RUN, RATE OF MONEY
GROWTH = RATE OF INFLATION

- ❑ In steady state, inflation determined solely by how quickly central bank (Fed) expands (or shrinks) the nominal money supply
- ❑ This relationship the basis for the **monetarist** school of thought
 - ❑ Milton Friedman's famous dictum: "Inflation is always and everywhere a monetary phenomenon"
 - ❑ Policy translation: "A central bank should not worry about/try to control anything other than how quickly the money supply in the economy is growing. Keeping money growth under control will keep inflation under control."

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 - ❑ Largest policy influence in U.K., short-lived policy influence in U.S.
 - ❑ Largely died out as basis for serious policy advice by mid-1980's

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 - ❑ Largest policy influence in U.K., short-lived policy influence in U.S.
 - ❑ Largely died out as basis for serious policy advice by mid-1980's
- ❑ Nevertheless still viewed as fundamental "law" of macroeconomics
 - ❑ A concern today: Fed's "easy monetary policy" (read: Fed has increased money supply very rapidly) will spawn a burst of inflation

MONETARY POLICY

- ❑ In short-run, do changes in monetary policy have effects on consumption and real GDP?
 - ❑ Keynesian/New Keynesian view: **yes** because prices are sticky
 - ❑ RBC view: **no** because prices are not sticky
- ❑ In long-run, changes in money growth rate
 - ❑ **Only have effects on inflation**
 - ❑ Have no effects on consumption and real GDP

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- ❑ **Next: interactions between monetary policy and fiscal policy**

FISCAL AND MONETARY INTERACTIONS

APRIL 13, 2009

Introduction

MONETARY POLICY AND FISCAL POLICY

- ❑ **Chapter 7: studied fiscal policy in isolation from monetary policy**
 - ❑ Illustrated some core issues of fiscal policy (i.e., lifetime budget constraint of government, Ricardian Equivalence)

- ❑ **Chapter 14: studied monetary policy in isolation from fiscal policy**
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 - ❑ Both occur simultaneously
 - ❑ The conduct of fiscal policy can place **restrictions** on what monetary policy can do, and vice-versa

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- ❑ Chapter 15: Domestic-economy interactions of fiscal and monetary policy – **focus on dynamic unfolding of events**

MONETARY POLICY AND FISCAL POLICY

- ❑ Representative consumer will be “in the background,” not the focus, of our analyses in Chapter 15
 - ❑ No explicit utility maximization problems, etc.
 - ❑ But we know where optimal choices of c_t and M_t/P_t etc. come from...
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- ❑ An infinite-period framework

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- ❑ An infinite-period model
- ❑ Two distinct “sides” of the government
 - ❑ **Fiscal authority – i.e., Congress**
 - ❑ Controls government spending g_t
 - ❑ Collects taxes T_t (will assume only lump-sum taxes throughout)
 - ❑ Issues (sells) new bonds (for various financing needs)
 - ❑ **Monetary authority (aka central bank) – i.e., Fed**
 - ❑ Controls money supply of economy...
 - ❑ ...by engaging in open-market operations

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 - **Receives “profits” from central bank (because it legally charters C.B.)**
 - **Monetary authority (aka central bank) – i.e., Fed**
 - Controls money supply of economy...
 - ...by engaging in open-market operations
 - **Turns over any “profits” it earns to fiscal authority**

The crucial linkage between fiscal and monetary policy

FISCAL AUTHORITY

- Fiscal authority's actions described by the period- t budget constraint

$$P_t g_t + B_{t-1}^f = T_t + P_t^b B_t^f + RCB_t$$

Total expenditure in period t Total income in period t

- **RCB_t : receipts (profits) turned over from the central bank to the fiscal authority in period t**

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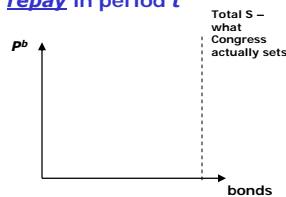
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OPEN MARKET FOR BONDS

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- Fiscal authority's period $t+1$ budget constraint

$$\underbrace{P_{t+1} g_{t+1} + B_t^T}_{\text{Total expenditure in period } t+1} = \underbrace{T_{t+1} + P_{t+1}^b B_{t+1}^T}_{\text{Total income in period } t+1} + RCB_{t+1}$$

Total expenditure in period $t+1$ Total income in period $t+1$

- And so on for $t+2, t+3$, etc.

MONETARY AUTHORITY

- Monetary authority's actions described by the period- t budget constraint

$$\underbrace{P_t^b B_t^M + RCB_t}_{\text{Total expenditure in period } t} = \underbrace{B_{t-1}^M + M_t - M_{t-1}}_{\text{Total income in period } t}$$

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- **M_t - M_{t-1}**: the change in the money supply engineered by the central bank during the course of period t
 - M_t and M_{t-1} individually are stock variables, but $M_t - M_{t-1}$ is a flow variable

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- And so on for period $t+2$, $t+3$, etc.

CONSOLIDATED GOVERNMENT BUDGET

- Can view the two sides of the government as one consolidated entity

$$P_t^b B_t^M + RCB_t = B_{t-1}^M + M_t - M_{t-1}$$

$$P_t g_t + B_{t-1}^T = T_t + P_t^b B_t^T + RCB_t$$

Combine by eliminating RCB_t

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Combine by eliminating RCB_t

$$P_t g_t + B_{t-1}^T - B_{t-1}^M = T_t + P_t^b (B_t^T - B_t^M) + M_t - M_{t-1}$$

- B^T is the total quantity of fiscally-issued bonds...
- ...of which B^M is purchased by the central bank on the open market
- → $B^T - B^M$ is total quantity of fiscally-issued bonds held by the private sector (i.e., consumers, mutual funds, domestic investors, foreign investors, etc.) – i.e., the quantity available on the open market

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- Define $B = B^T - B^M$ as bonds held by the private sector
 - What really matters for policy actions and interactions
 - → Consolidated flow government budget constraint

$$P_t g_t + B_{t-1} = T_t + P_t^b B_t + M_t - M_{t-1}$$

CONSOLIDATED GOVERNMENT BUDGET

- Consolidated flow government budget constraint highlights the limits that fiscal policy places on monetary policy and vice-versa

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- All analysis from the perspective of the very beginning of period t

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- All analysis from the perspective of the very beginning of period t
- Fiscal policy in period t is a particular setting for all three of its instruments (g_t, T_t, B_t)
 - Fiscal policy has three instruments (aka policy tools)
- Monetary policy in period t is a particular setting for all one of its instruments (M_t)
 - Monetary policy has one instrument (aka policy tool)
- (Problem Set 2: A **policy** is defined by **unique** settings for **each** available instrument)

ACTIVE VS. PASSIVE POLICY

- **Definition:** A policy authority is **active** if ***every instrument*** at its disposal can be completely freely chosen, without any heed for the consolidated government budget constraint
 - Active authority does not engage in policy in such a way as to make sure the consolidated government budget balances

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- At beginning of period t , B_{t-1} and M_{t-1} are fixed (assume no default)
- Fiscal authority sets g_t , B_t , and T_t } Question: How will "consistency" between them be guaranteed?
- Monetary authority sets M_t }

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$$P_t g_t + B_{t-1} = T_t + P_t^b B_t + M_t - M_{t-1}$$

i.e., how will **equality** of the consolidated budget be achieved?

- At beginning of period t , B_{t-1} and M_{t-1} are fixed (assume no default)
- Fiscal authority sets g_t , B_t , and T_t } Question: How will "**consistency**" between them be guaranteed?
- Monetary authority sets M_t } Two possibilities...

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ACTIVE FISCAL/PASSIVE MONETARY POLICY

- Suppose fiscal authority sets all of its policy instruments (all **three** of them) without heed for the consolidated flow GBC

- Monetary authority must **react** by setting M_t to ensure the consolidated GBC holds

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 - **Monetary authority is passive**

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 - Fiscal authority is active
- Monetary authority must **react** by setting M_t to ensure the consolidated GBC holds
 - Monetary authority is passive
- Game-theoretic undertones
 - Fiscal authority is the “dominant” policy-maker
 - Fiscal authority is the “leader”
 - Monetary authority is the “lagging” policy-maker
 - Monetary authority is the “follower”
- Policy pressure (by fiscal authority on monetary authority) is implicit and (largely...) through market forces
 - U.S. Fed is independent of Congress
 - But in developing countries?...

ACTIVE MONETARY/PASSIVE FISCAL POLICY

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ACTIVE VS. PASSIVE POLICY

- ❑ Which regime describes the U.S.?
 - ❑ Matter of a lot of debate
 - ❑ Maybe there's “regime switching” – i.e., each authority “takes turns” being the follower and the leader
 - ❑ Through the rise and fall of political power?
 - ❑ Through the ascendancy of strong central bankers?
- ❑ Game theory a compelling way to study monetary-fiscal interactions (more advanced course)
- ❑ Core issue: there are **limits** or **restrictions** that each policy-setting authority places on the actions of the others

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- ❑ A more nuanced view: the period- t choices of one policy authority **may restrict the choices of the other policy authority in period t and/or period $t+1$ and/or period $t+2$ and/or period $t+3$, ...**

NEXT

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FISCAL AND MONETARY INTERACTIONS: AN INTERTEMPORAL VIEW

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ACTIVE VS. PASSIVE POLICY

- ❑ Core issue: there are **limits** or **restrictions** that each policy-setting authority places on the actions of the other
- ❑ Analysis so far: the period- t choices of one policy authority **restrict the choices of the other policy authority *in period t***

$$P_t g_t + B_{t-1} = T_t + P_t^b B_t + M_t - M_{t-1} \quad \text{Period-}t \text{ consolidated GBC}$$

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- ❑ A more nuanced view: the period- t choices of one policy authority may **restrict the choices of the other policy authority in period t and/or period $t+1$ and/or period $t+2$ and/or period $t+3$, ...**
- ❑ Emphasizes that the limits may not be realized immediately, but can occur later (in the economy's/government's lifetime)
- ❑ Requires deriving/analyzing the **lifetime (aka intertemporal) consolidated government budget constraint**

DERIVING THE LIFETIME CONSOLIDATED GBC

$$P_t g_t + B_{t-1} = T_t + P_t^b B_t + M_t - M_{t-1} \quad \text{Period-}t \text{ consolidated GBC}$$

↓ Divide by P_t to put in real terms

$$g_t + \frac{B_{t-1}}{P_t} = \frac{T_t}{P_t} + \frac{P_t^b B_t}{P_t} + \frac{M_t - M_{t-1}}{P_t}$$

SEIGNORAGE REVENUE

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- **Definition:** **seignorage revenue** is the real quantity of resources the government raises for itself through the act of money creation
 - Abbreviate sr_t
- **Printing money is a source of income for the government!**

SEIGNORAGE REVENUE

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- **Definition:** **seignorage revenue** is the real quantity of resources the government raises for itself through the act of money creation
 - Abbreviate sr_t
- **Printing money is a source of income for the government!**
 - Unimportant in the U.S. (less than 1% of government revenue) and other developed countries
 - **But can be important in developing countries (because of poorly-developed tax collection systems and corruption)**
 - (Plays an important role in how fiscal-monetary interactions affect exchange rate systems (Chapter 16))

DERIVING THE LIFETIME CONSOLIDATED GBC

$$P_t g_t + B_{t-1} = T_t + P_t^b B_t + M_t - M_{t-1} \quad \text{Period-}t \text{ consolidated GBC}$$

↓ Divide by P_t to put in real terms

$$g_t + \frac{B_{t-1}}{P_t} = \frac{T_t}{P_t} + \frac{P_t^b B_t}{P_t} + sr_t$$

DERIVING THE LIFETIME CONSOLIDATED GBC

$$P_t g_t + B_{t-1} = T_t + P_t^b B_t + M_t - M_{t-1} \quad \text{Period-}t \text{ consolidated GBC}$$

↓ Divide by P_t to put in real terms

$$g_t + \frac{B_{t-1}}{P_t} = \frac{T_t}{P_t} + \frac{P_t^b B_t}{P_t} + sr_t$$

↓ Define $b_t = B_t/P_t$, $t_t = T_t/P_t$ and rearrange terms

$$\frac{B_{t-1}}{P_t} = sr_t + (t_t - g_t + P_t^b b_t) \quad \text{Period-}t \text{ consolidated GBC}$$

Revenue generated by monetary authority actions Revenue generated by fiscal authority actions

DERIVING THE LIFETIME CONSOLIDATED GBC

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Revenue generated by monetary authority actions Revenue generated by fiscal authority actions

$$\frac{B_t}{P_{t+1}} = sr_{t+1} + (t_{t+1} - g_{t+1} + P_{t+1}^b b_{t+1}) \quad \text{Period-}t+1 \text{ consolidated GBC}$$

(same thing, just update subscripts)

DERIVING THE LIFETIME CONSOLIDATED GBC

$$P_t g_t + B_{t-1} = T_t + P_t^b B_t + M_t - M_{t-1} \quad \text{Period-}t \text{ consolidated GBC}$$

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↓ Combine

DERIVING THE LIFETIME CONSOLIDATED GBC

$$P_t g_t + B_{t-1} = T_t + P_t^b B_t + M_t - M_{t-1} \quad \text{Period-}t \text{ consolidated GBC}$$

↓ Divide by P_t to put in real terms

$$g_t + \frac{B_{t-1}}{P_t} = \frac{T_t}{P_t} + \frac{P_t^b B_t}{P_t} + sr_t$$

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Revenue generated by monetary authority actions Revenue generated by fiscal authority actions

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↓ Combine

↓ Use Fisher equation

DERIVING THE LIFETIME CONSOLIDATED GBC

$$P_t g_t + B_{t-1} = T_t + P_t^b B_t + M_t - M_{t-1} \quad \text{Period-}t \text{ consolidated GBC}$$

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$$g_t + \frac{B_{t-1}}{P_t} = \frac{T_t}{P_t} + \frac{P_t^b B_t}{P_t} + sr_t$$

↓ Define $b_t = B_t/P_t$, $t_t = T_t/P_t$ and rearrange terms

$$\frac{B_{t-1}}{P_t} = sr_t + (t_t - g_t + P_t^b b_t) \quad \text{Period-}t \text{ consolidated GBC}$$

Revenue generated by monetary authority actions Revenue generated by fiscal authority actions

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Complete mathematical derivation in Chapter 15

↓ Combine
↓ Use Fisher equation
↓ Substitute in $t+2$ and $t+3$ and $t+4$, etc.... budget constraints
↓ Use Fisher equation
↓ Combine

LIFETIME CONSOLIDATED GBC

$$\frac{B_{t-1}}{P_t} = \sum_{s=0}^{\infty} \frac{t_{t+s} - g_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})} + \sum_{s=0}^{\infty} \frac{sr_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})} \quad \text{Lifetime consolidated GBC}$$

REAL value of government debt that must be repaid at start of period t

- The period- t real value of government debt that must be repaid can be repaid by...

LIFETIME CONSOLIDATED GBC

$$\frac{B_{t-1}}{P_t} = \underbrace{\sum_{s=0}^{\infty} \frac{t_{t+s} - g_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all fiscal surpluses starting in period } t} + \underbrace{\sum_{s=0}^{\infty} \frac{sr_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all seignorage revenues starting in period } t}$$

REAL value of government debt that must be repaid at start of period t

Present-discounted value of all fiscal surpluses starting in period t

Lifetime consolidated GBC

- The period- t real value of government debt that must be repaid can be repaid by...
- ...either period- t **and/or later** fiscal surpluses ($t - g$ is fiscal surplus...recall terminology from Chapter 7)...

LIFETIME CONSOLIDATED GBC

$$\frac{B_{t-1}}{P_t} = \underbrace{\sum_{s=0}^{\infty} \frac{t_{t+s} - g_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all fiscal surpluses starting in period } t} + \underbrace{\sum_{s=0}^{\infty} \frac{sr_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all seignorage revenues starting in period } t}$$

REAL value of government debt that must be repaid at start of period t

Present-discounted value of all fiscal surpluses starting in period t

Present-discounted value of all seignorage revenues starting in period t

Lifetime consolidated GBC

- The period- t real value of government debt that must be repaid can be repaid by...
- ...either period- t **and/or later** fiscal surpluses ($t - g$ is fiscal surplus...recall terminology from Chapter 7)...
- ...or period- t **and/or later** seignorage revenues...
- ...or both

LIFETIME CONSOLIDATED GBC

$$\frac{B_{t-1}}{P_t} = \underbrace{\sum_{s=0}^{\infty} \frac{t_{t+s} - g_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all fiscal surpluses starting in period } t} + \underbrace{\sum_{s=0}^{\infty} \frac{sr_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all seignorage revenues starting in period } t}$$

Lifetime consolidated GBC

REAL value of government debt that must be repaid at start of period t

- The period- t real value of government debt that must be repaid can be repaid by...
 - ...either period- t **and/or later** fiscal surpluses ($t - g$ is fiscal surplus...recall terminology from Chapter 7)...
 - ...or period- t **and/or later** seignorage revenues...
 - ...or both
- Both **fiscal surpluses** and **seignorage revenue** are used in the consolidated government budget to repay government debt
 - Question: "Which" authority is "most responsible" for repaying debt?
 - Question: "When" is this authority responsible for repaying it?

Controlled by fiscal authority

Controlled by central bank

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RICARDIAN VS. NON-RICARDIAN POLICY

- When thinking about dynamic (i.e., over many periods) interactions between fiscal and monetary policy, usually the most relevant case is when fiscal authority is active (i.e., the "leader")

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RICARDIAN VS. NON-RICARDIAN POLICY

- When thinking about dynamic (i.e., over many periods) interactions between fiscal and monetary policy, usually the most relevant case is when fiscal authority is active (i.e., the “leader”)
- **Definition:** A **Ricardian** fiscal policy is in place if the fiscal authority sets its planned **sequence** of tax and spending policy to ensure that the lifetime consolidated GBC is satisfied
- **Definition:** A **non-Ricardian** fiscal policy is in place if the fiscal authority sets its planned **sequence** of tax and spending policy without regard for whether the lifetime consolidated GBC is satisfied

RICARDIAN VS. NON-RICARDIAN POLICY

- When thinking about dynamic (i.e., over many periods) interactions between fiscal and monetary policy, the most relevant case is when fiscal authority is active (i.e., the “leader”)
- **Definition:** A **Ricardian** fiscal policy is in place if the fiscal authority sets its planned **sequence** of tax and spending policy to ensure that the lifetime consolidated GBC is satisfied
- **Definition:** A **non-Ricardian** fiscal policy is in place if the fiscal authority sets its planned **sequence** of tax and spending policy without regard for whether the lifetime consolidated GBC is satisfied
- What matters is
 - The fiscal authority's entire plan for $t_{t'}, t_{t+1'}, t_{t+2'}, t_{t+3'}, t_{t+4'}, \dots$
 - The fiscal authority's entire plan for $g_{t'}, g_{t+1'}, g_{t+2'}, g_{t+3'}, g_{t+4'}, \dots$
 - Whether **and** when the monetary authority “reacts” to what the fiscal authority chooses

RICARDIAN CHANGES IN FISCAL POLICY

$$\frac{B_{t-1}}{P_t} = \underbrace{\sum_{s=0}^{\infty} \frac{t_{t+s} - g_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all fiscal surpluses starting in period } t} + \underbrace{\sum_{s=0}^{\infty} \frac{sr_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all seignorage revenues starting in period } t}$$

Lifetime consolidated GBC

REAL value of government debt that must be repaid at start of period t

Present-discounted value of all fiscal surpluses starting in period t

Present-discounted value of all seignorage revenues starting in period t

□ **Start of analysis:**

- Fiscal authority has in place entire planned **sequence** for t and g
- Monetary authority has planned **sequence** for sr (i.e., money creation)

RICARDIAN CHANGES IN FISCAL POLICY

$$\frac{B_{t-1}}{P_t} = \underbrace{\sum_{s=0}^{\infty} \frac{t_{t+s} - g_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all fiscal surpluses starting in period } t} + \underbrace{\sum_{s=0}^{\infty} \frac{sr_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all seignorage revenues starting in period } t}$$

Lifetime consolidated GBC

REAL value of government debt that must be repaid at start of period t

Present-discounted value of all fiscal surpluses starting in period t

Present-discounted value of all seignorage revenues starting in period t

□ **Start of analysis:**

- Fiscal authority has in place entire planned **sequence** for t and g
- Monetary authority has planned **sequence** for sr (i.e., money creation)

□ **Fiscal authority then changes the precise timing of t collection but does so in a Ricardian way**

- i.e., makes sure it changes t collection so as to satisfy the consolidated GBC

RICARDIAN CHANGES IN FISCAL POLICY

$$\frac{B_{t-1}}{P_t} = \underbrace{\sum_{s=0}^{\infty} \frac{t_{t+s} - g_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all fiscal surpluses starting in period } t} + \underbrace{\sum_{s=0}^{\infty} \frac{sr_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all seignorage revenues starting in period } t}$$

Lifetime consolidated GBC

REAL value of government debt that must be repaid at start of period t

- **Start of analysis:**
 - Fiscal authority has in place entire planned **sequence** for t and g
 - Monetary authority has planned **sequence** for sr (i.e., money creation)
- Fiscal authority then changes the precise timing of t collection **but does so in a Ricardian way**
 - i.e., makes sure it changes t collection so as to satisfy the consolidated GBC
- Question: What is the impact on monetary policy (i.e., on sr collection)?
 - **NONE**, because fiscal policy is being conducted in Ricardian way
 - **(The basis for Ricardian Equivalence from Chapter 7...)**

FISCAL THEORY OF INFLATION (FTI)

$$\frac{B_{t-1}}{P_t} = \underbrace{\sum_{s=0}^{\infty} \frac{t_{t+s} - g_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all fiscal surpluses starting in period } t} + \underbrace{\sum_{s=0}^{\infty} \frac{sr_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all seignorage revenues starting in period } t}$$

Lifetime consolidated GBC

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- **Start of analysis:**
 - Fiscal authority has in place entire planned **sequence** for t and g
 - Monetary authority has planned **sequence** for sr (i.e., money creation)
- Fiscal authority then changes the precise timing of t collection **but does so in a non-Ricardian way**
- Question: What is the impact on monetary policy (i.e., on sr collection)?

FISCAL THEORY OF INFLATION (FTI)

$$\frac{B_{t-1}}{P_t} = \underbrace{\sum_{s=0}^{\infty} \frac{t_{t+s} - g_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all fiscal surpluses starting in period } t} + \underbrace{\sum_{s=0}^{\infty} \frac{sr_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all seignorage revenues starting in period } t}$$

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 - Monetary authority has planned **sequence** for sr (i.e., money creation)
- Fiscal authority then changes the precise timing of t collection **but does so in a non-Ricardian way**
- **Question: What is the impact on monetary policy (i.e., on sr collection)?**
 - If monetary authority alters its plan for sr (i.e., money creation), it has "reacted" ("passive") to ensure lifetime consolidated GBC holds

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FISCAL THEORY OF INFLATION (FTI)

$$\frac{B_{t-1}}{P_t} = \underbrace{\sum_{s=0}^{\infty} \frac{t_{t+s} - g_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all fiscal surpluses starting in period } t} + \underbrace{\sum_{s=0}^{\infty} \frac{sr_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all seignorage revenues starting in period } t}$$

Lifetime consolidated GBC

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- **Start of analysis:**
 - Fiscal authority has in place entire planned **sequence** for t and g
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- Fiscal authority then changes the precise timing of t collection **but does so in a non-Ricardian way**
- **Question: What is the impact on monetary policy (i.e., on sr collection)?**
 - If monetary authority alters its plan for sr (i.e., money creation), it has "reacted" ("passive") to ensure lifetime consolidated GBC holds
- **Money-creation leads to inflation (monetarist link of Chapter 14)**
 - → Terminology "Fiscal Theory of Inflation"

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FISCAL THEORY OF THE PRICE LEVEL (FTPL)

$$\frac{B_{t-1}}{P_t} = \underbrace{\sum_{s=0}^{\infty} \frac{t_{t+s} - g_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all fiscal surpluses starting in period } t} + \underbrace{\sum_{s=0}^{\infty} \frac{sr_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all seignorage revenues starting in period } t}$$

Lifetime consolidated GBC

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- **Start of analysis:**
 - Fiscal authority has in place entire planned **sequence** for t and g
 - Monetary authority has planned **sequence** for sr (i.e., money creation)
- Fiscal authority then changes the precise timing of t collection **but does so in a non-Ricardian way**
- Question: What is the impact on monetary policy (i.e., on sr collection)?
 - If monetary authority does **not** alter its sr plan...

FISCAL THEORY OF THE PRICE LEVEL (FTPL)

$$\frac{B_{t-1}}{P_t} = \underbrace{\sum_{s=0}^{\infty} \frac{t_{t+s} - g_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all fiscal surpluses starting in period } t} + \underbrace{\sum_{s=0}^{\infty} \frac{sr_{t+s}}{\prod_{s=0}^{\infty} (1+r_{t+s})}}_{\text{Present-discounted value of all seignorage revenues starting in period } t}$$

Lifetime consolidated GBC

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 - Monetary authority has planned **sequence** for sr (i.e., money creation)
- Fiscal authority then changes the precise timing of t collection **but does so in a non-Ricardian way**
- Question: What is the impact on monetary policy (i.e., on sr collection)?
 - If monetary authority does **not** alter its sr plan...
 - ...the entire adjustment must come via a change in the period- t price level

FISCAL THEORY OF THE PRICE LEVEL (FTPL)

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Lifetime consolidated GBC

REAL value of government debt that must be repaid at start of period t

Present-discounted value of all fiscal surpluses starting in period t

Present-discounted value of all seignorage revenues starting in period t

- **Start of analysis:**
 - Fiscal authority has in place entire planned *sequence* for t and g
 - Monetary authority has planned *sequence* for sr (i.e., money creation)
- Fiscal authority then changes the precise timing of t collection **but does so in a non-Ricardian way**
- Question: What is the impact on monetary policy (i.e., on sr collection)?
 - If monetary authority does **not** alter its sr plan...
 - ...the entire adjustment must come via a change in the period- t price level
- **A one-time change in prices, not a sustained increase in prices**

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MONETARY-FISCAL INTERACTIONS

- Does Congress act in a way to ensure *lifetime* budget balance?
 - Sometimes seems yes...sometimes seems no...

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MONETARY-FISCAL INTERACTIONS

- ❑ Does Congress act in a way to ensure *lifetime* budget balance?
 - ❑ Sometimes seems yes...sometimes seems no...
- ❑ If not, then FTI and FTPL are the two relevant ways that lifetime government budget balance can be brought about
- ❑ **FTI:** Any non-Ricardian changes in fiscal policy announced in period t will lead to inflation in period t *and/or* in future periods
- ❑ **FTPL:** Any non-Ricardian changes in fiscal policy announced in period t will lead to a *one-time* (i.e., in period t) change in prices, but no further inflation in future periods

MONETARY-FISCAL INTERACTIONS

- ❑ Does Congress act in a way to ensure *lifetime* budget balance?
 - ❑ Sometimes seems yes...sometimes seems no...
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- ❑ **FTI:** Any non-Ricardian changes in fiscal policy announced in period t will lead to inflation in period t *and/or* in future periods
- ❑ **FTPL:** Any non-Ricardian changes in fiscal policy announced in period t will lead to a *one-time* (i.e., in period t) change in prices, but no further inflation in future periods
- ❑ FTI and FTPL differ in their predictions of **when** fiscal pressures have their consequences on economy-wide prices
 - ❑ Immediately: FTPL
 - ❑ In the future: FTI
- ❑ (FTPL the foundation for exchange rate analysis in Chapter 16)