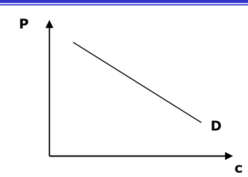
Consumption-Savings Model

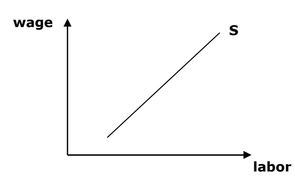
February 9, 2009

The Three Macro (Aggregate) Markets

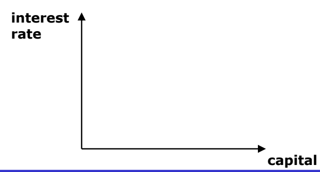
- **Goods Markets**
  - **Demand derived from C-L model**



- **Labor Markets**
  - **Supply derived from C-L model**



- **Capital/Savings/Funds/Asset Markets (aka Financial Markets)**



BASICS

- Consumption-Savings Model – provides foundation for
  - Goods-market demand function (again…but w/different interpretation)
  - Capital-market supply function
  - An application of the basic consumer theory model…
  - …we will put a macro interpretation on it

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  - Two time periods
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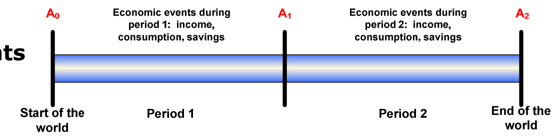
- Dynamic models are the staple of modern macroeconomic theory
- An explicit accounting of time
- Two periods are sufficient to illustrate the basic principles
  - Soon will extend to infinite number of periods (Chapter 8)

BASICS

- Timeline of events



- Notation
  - c_1: consumption in period 1
  - c_2: consumption in period 2
  - P_1: nominal price of consumption in period 1
  - P_2: nominal price of consumption in period 2
  - Y_1: nominal income in period 1 ("falls from the sky")
  - Y_2: nominal income in period 2 ("falls from the sky")

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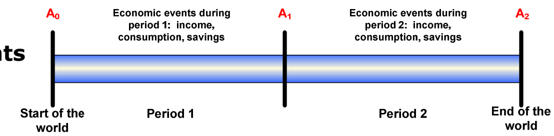

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  - A_0: nominal wealth at the beginning of period 1/end of period 0
  - A_1: nominal wealth at the beginning of period 2/end of period 1
  - A_2: nominal wealth at the beginning of period 3/end of period 2
  - i: nominal interest rate between periods
  - r: real interest rate between periods
  - π_2: net inflation rate between period 1 and period 2 $\pi_2 = \frac{P_2 - P_1}{P_1}\left(= \frac{P_2}{P_1} - 1\right)$
  - y_1: real income in period 1 (= Y_1/P_1)
  - y_2: real income in period 2 (= Y_2/P_2)

STOCKS VS. FLOWS

- Stock variables
  - Variables whose natural measurement occurs at a particular moment in time

- Flow variables
  - Variables whose natural measurement occurs over the course of a given interval of time

STOCKS VS. FLOWS

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  - Economic examples:
    - **Checking account balance**
    - **Credit card indebtedness**
    - **Mortgage loan payoff**
  - Interpret *A* in our model as net wealth (= total assets – total debts)

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Macro Fundamentals

STOCKS VS. FLOWS

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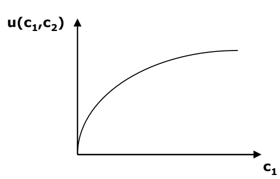
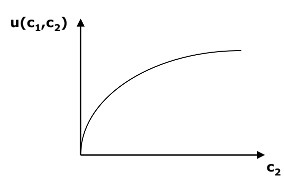
- **The two broad categories of income**
  - **Labor income**
  - **Asset income (generated by interest rate(s) on (components of) wealth)**

All income is a FLOW regardless of source

UTILITY

- **Preferences $u(c_1, c_2)$ with all the "usual properties"**
  - **Lifetime utility function**
  - **Strictly increasing in c_1**
  - **Strictly increasing in c_2**
  - **Diminishing marginal utility in c_1**
  - **Diminishing marginal utility in c_2**





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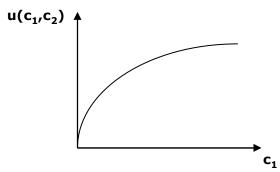
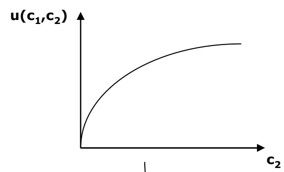
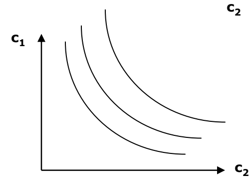




  - Plotted as indifference curves



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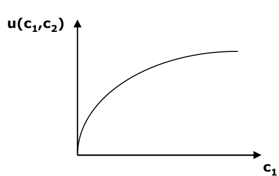

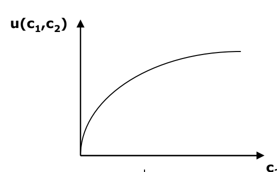

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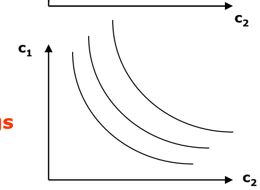

  - Utility side of consumption-savings model no different than Chapter 1 model

BUDGET CONSTRAINT(S)

- **Suppose again *Y* "falls from the sky"**
  - **Y_1 in period 1, Y_2 in period 2**
- **Need two budget constraints to describe economic opportunities and possibilities**
  - **One for each period**

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  - **Period-1 budget constraint**

$$\underbrace{P_1c_1 + A_1}_{} = \underbrace{Y_1 + (1+i)A_0}_{}$$

Total expenditure in period 1: period-1 consumption + wealth to *carry into period* 2

Total income in period 1: period-1 *Y* + income from wealth *carried into period* 1 (inclusive of interest)

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  - **Period-2 budget constraint**

$$P_2c_2 + A_2 = Y_2 + (1+i)A_1$$

Total expenditure in period 2: period-2 consumption + wealth to *carry into period* 3

Total income in period 2: period-2 Y + income from wealth *carried into period* 2 (inclusive of interest)

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can rewrite as

$$P_1c_1 + A_1 - A_0 = Y_1 + iA_0$$

Savings during period 1 (a flow) — Asset income during period 1 (a flow)

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Total expenditure in period 1: period-1 consumption + wealth to *carry into period* 2

Total income in period 1: period-1 *Y* + income from wealth *carried into period* 1 (inclusive of interest)

Savings during period 1 (a flow): $A_1 - A_0$

Asset income during period 1 (a flow): iA_0

  - Period-2 budget constraint

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Total expenditure in period 2: period-2 consumption + wealth to *carry into period* 3

Total income in period 2: period-2 *Y* + income from wealth *carried into period* 2 (inclusive of interest)

Savings during period 2 (a flow): $A_2 - A_1$

Asset income during period 2 (a flow): iA_1

DEFINITION: A consumer's savings during a given period is the *change* in his wealth during that period

BUDGET CONSTRAINT(S)

- Adopt a lifetime view of the budget constraint(s)
  - All analysis conducted from perspective of beginning of period 1
  - Period-1 budget constraint $P_1c_1 + A_1 = Y_1 + (1+i)A_0$
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Asset position at end of period 1/beginning of period 2 the key link - will think further about this soon…

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Model Structure

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$$\underbrace{P_1c_1 + \frac{P_2c_2}{1+i}}_{\text{Present discounted value (PDV) of all lifetime expenditure}} = \underbrace{Y_1 + \frac{Y_2}{1+i} + (1+i)A_0}_{\text{Present discounted value (PDV) of all lifetime income}}$$

Budget Constraint(s)

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Present discounted value (PDV) of all lifetime expenditure

Present discounted value (PDV) of all lifetime income

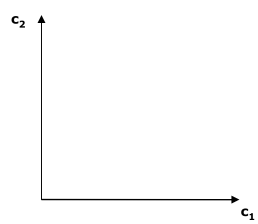
For graphical simplicity, will often assume A_0 = 0 (i.e., consumer begins life with zero net wealth).
Note this is a *different* assumption than A_2 = 0.

Lifetime Budget Constraint

Graphically

$$P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$$

Solve for c_2



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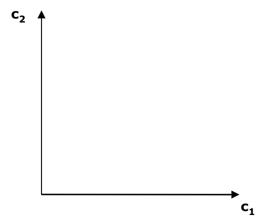

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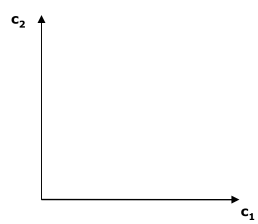

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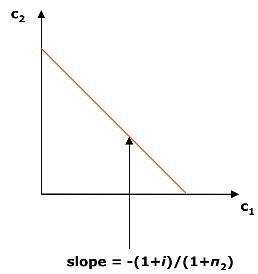

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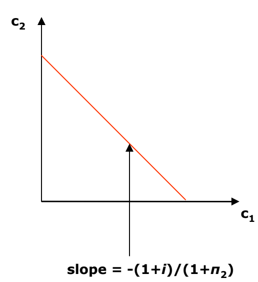

The larger is $(1+i)/(1+\pi 2)$, the steeper is the budget line

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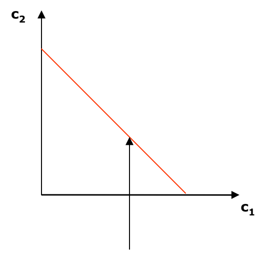


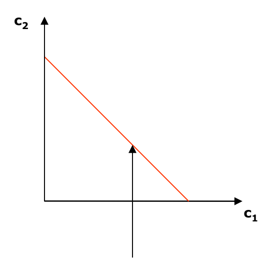
slope = $-(1+i)/(1+\pi_2)$

The larger is $(1+i)/(1+\pi_2)$, the steeper is the budget line

IMPORTANT: Changes in nominal interest rates (Fed) and/or inflation affect the slope of the LBC

Consumer Optimization

- **Consumer's decision problem:** maximize lifetime utility subject to lifetime budget constraint – bring together both cost side and benefit side
  - Choose c_1 and c_2 subject to $P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$
  - Plot budget line



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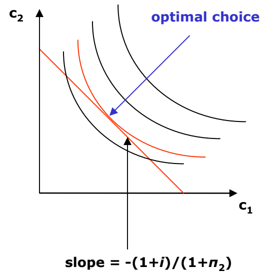

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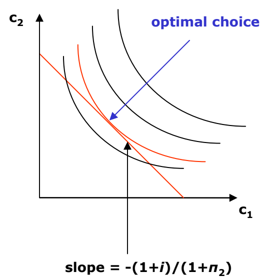

- **At the optimal choice**

CONSUMPTION-SAVINGS OPTIMALITY CONDITION

- A key building block of modern macro models

$$\underbrace{\frac{u_1(c_1^*, c_2^*)}{u_2(c_1^*, c_2^*)}}_{\text{MRS (between consumption in consecutive time periods)}} = \underbrace{\frac{1+i}{1+\pi_2}}_{\text{price ratio (across consecutive time periods)}}$$

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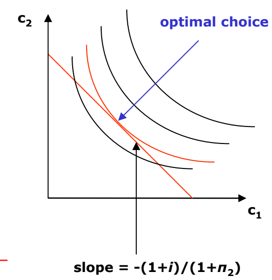

- **At the optimal choice**

CONSUMPTION-SAVINGS OPTIMALITY CONDITION

- A key building block of modern macro models

$$\underbrace{\frac{u_1(c_1^*, c_2^*)}{u_2(c_1^*, c_2^*)}}_{\text{MRS (between consumption in consecutive time periods)}} = \underbrace{\frac{1+i}{1+\pi_2}}_{\text{price ratio (across consecutive time periods)}}$$

Derive consumption-leisure optimality condition using Lagrange analysis

LAGRANGE ANALYSIS

- **Apply Lagrange tools to consumption-savings optimization**
- **Objective function: $u(c_1,c_2)$**
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The Mathematics of the Consumption-Savings Model

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CONSUMPTION-SAVINGS OPTIMALITY CONDITION

MRS (between consumption in consecutive time periods)

price ratio (across consecutive time periods)

Savings and Asset Positions

- **Definition: A consumer's savings during a given time period is the *change* in his wealth during that time period**

- **Assets/wealth (whether positive or negative) are a means for "transferring income over time"**

Macro Fundamentals

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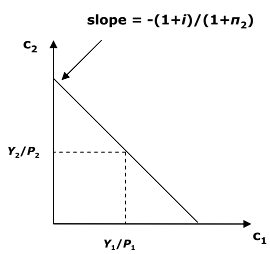

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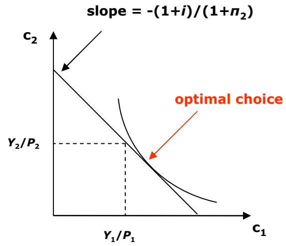

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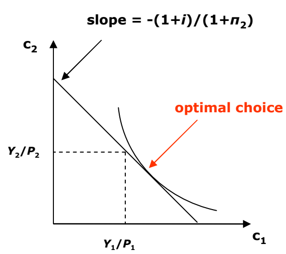

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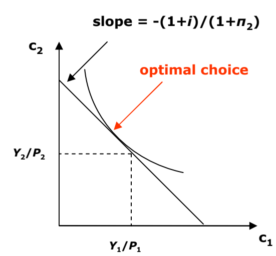

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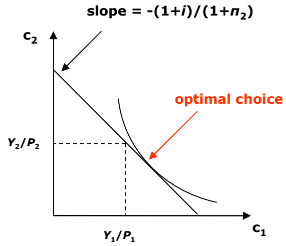

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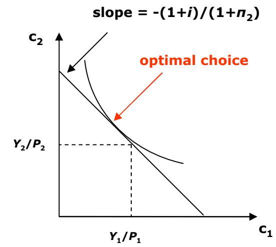

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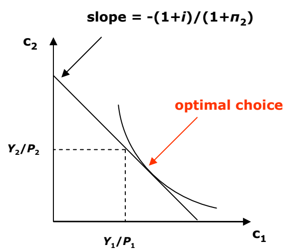
Assessing the Credit Crunch



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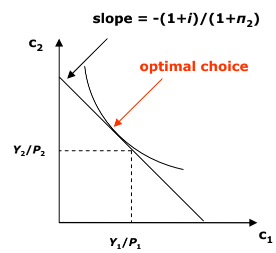

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Use this framework to analyze the channel by which financial market problems have been affecting consumption activity

Fisher Equation

- Nominal interest rate – measured in dollars
- Real interest rate – measured in goods

- Fisher equation: a link between the nominal interest rate, inflation rate, and real interest rate
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More useful for our theoretical model

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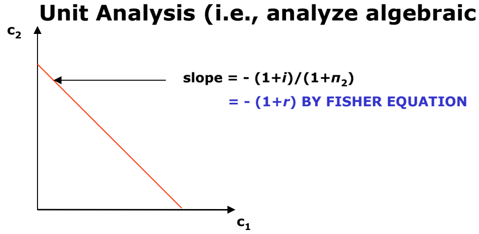
$$r=i-\pi$$

A useful rule of thumb

Real Interest Rate

- *r* a key variable for macroeconomic analysis

- Unit Analysis (i.e., analyze algebraic units of variables)



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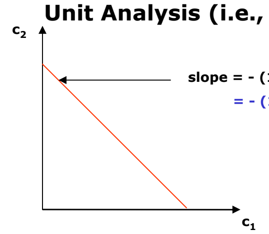

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Slope measures how much c_2 must be given up in order to obtain one more unit of c_1 ("rise over run") when borrowing or lending at market prices

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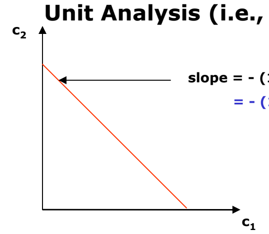

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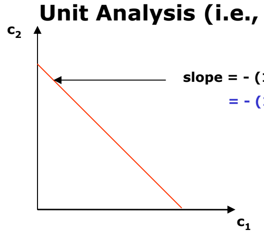

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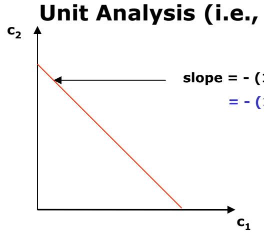

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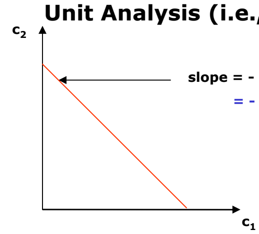

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- **Currently: nominal *i* (short-term) ≈ 0%, π ≈ 0% (CPI measure)**
  - **Real interest rate ≈ 0 right now…**

CONSUMPTION-SAVINGS OPTIMALITY CONDITION

- **Emphasizing *i* and π** $$\frac{u_1(c_1^*,c_2^*)}{u_2(c_1^*,c_2^*)}=\frac{1+i}{1+\pi}$$

Fisher equation

- **Emphasizing *r*** $$\frac{u_1(c_1^*,c_2^*)}{u_2(c_1^*,c_2^*)}=1+r$$

- **Can also analyze two-period model sequentially, rather than from a lifetime perspective**

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Sequential formulation highlights the role of net wealth (A_1) between period 1 and period 2**
  - **Accords better with the explicit timing of economic events than the lifetime approach…**
  - **…but yields the same result**
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Consumption-Savings Model: Sequential Formulation

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  - **See Math Refresher, Chapter -1**

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- Could pursue sequential approach in either nominal or real terms

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

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using Fisher equation

- **Identical to result of lifetime formulation**

Two-Period Model in Real Terms

- **Depending on application, may be useful to work with model (independent of lifetime vs. sequential approach) in nominal terms or in real terms**

$$P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$$

LBC in nominal terms (assuming $A_0 = 0$)

From Nominal to Real

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Use Fisher equation: $(1+\pi_2)/(1+i) = 1/(1+r)$

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Use Fisher equation: $(1+\pi_2)/(1+i) = 1/(1+r)$

Maximize $u(c_1,c_2)$ subject to the real LBC → identical consumption-savings optimality condition (details in recitations)

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$ LBC in real terms (assuming $A_0 = 0$)