

CONSUMPTION-SAVINGS MODEL (CONTINUED)

FEBRUARY 16, 2009

LAGRANGE ANALYSIS: LIFETIME APPROACH

- ❑ Apply Lagrange tools to consumption-savings optimization
- ❑ Objective function: $u(c_1, c_2)$
- ❑ Constraint (assuming $A_0 = 0$): $g(c_1, c_2) = Y_1 + \frac{Y_2}{1+i} - P_1 c_1 - \frac{P_2 c_2}{1+i} = 0$

- ❑ **Step 1: Construct Lagrange function**

$$L(c_1, c_2, \lambda) = u(c_1, c_2) + \lambda \left[Y_1 + \frac{Y_2}{1+i} - P_1 c_1 - \frac{P_2 c_2}{1+i} \right]$$

- ❑ **Step 2: Compute first-order conditions with respect to c_1, c_2, λ**

- ❑ **Step 3: Solve (with focus on eliminating multiplier)**

CONSUMPTION-SAVINGS OPTIMALITY CONDITION

$$\frac{u_1(c_1^*, c_2^*)}{u_2(c_1^*, c_2^*)} = \frac{1+i}{1+\pi_2} = 1+r$$

using Fisher equation

MRS (between consumption in consecutive time periods) price ratio (across consecutive time periods)

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- Step 1: Construct Lagrange function

$$L(c_1, c_2, A_1, \lambda_1, \lambda_2) = u(c_1, c_2) + \lambda_1 [Y_1 + (1+i)A_0 - P_1c_1 - A_1] + \lambda_2 [Y_2 + (1+i)A_1 - P_2c_2 - A_2]$$

- Step 2: Compute FOCs with respect to $c_1, c_2, A_1, \lambda_1, \lambda_2$

(FOC on A_1 will be the key to asset pricing in Chapter 8...)

- Step 3: Solve (with focus on eliminating multipliers)

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using Fisher equation

MRS (between consumption in consecutive time periods)

price ratio (across consecutive time periods)

- Identical to result of lifetime formulation

TWO-PERIOD MODEL IN REAL TERMS

- Depending on application, may be useful to work with model (independent of lifetime vs. sequential approach) in nominal terms or in real terms

$$P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$$

LBC in nominal terms (assuming $A_0 = 0$)

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$$P_1 c_1 + \frac{P_2 c_2}{1+i} = Y_1 + \frac{Y_2}{1+i} \quad \text{LBC in nominal terms (assuming } A_0 = 0)$$

↓ Divide by P_1

$$c_1 + \left(\frac{P_2}{P_1(1+i)} \right) c_2 = \frac{Y_1}{P_1} + \frac{Y_2}{P_1(1+i)}$$

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↓ Multiply and divide last term on right-hand-side by P_2

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$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} \quad \text{LBC in real terms (assuming } A_0 = 0)$$

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↓ Multiply *and* divide last term on right-hand-side by P_2

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↓ Use definitions: $y_1 = Y_1/P_1$, $y_2 = Y_2/P_2$ and $1+\pi_2 = P_2/P_1$

$$c_1 + \left(\frac{1+\pi_2}{1+i} \right) c_2 = y_1 + \left(\frac{1+\pi_2}{1+i} \right) y_2$$

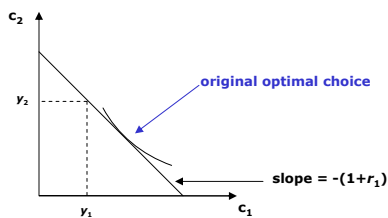
↓ Use Fisher equation: $(1+\pi_2)/(1+i) = 1/(1+r)$

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} \quad \text{LBC in real terms (assuming } A_0 = 0 \text{)}$$

Maximize $u(c_1, c_2)$ subject to the real LBC → identical consumption-savings optimality condition

MICRO-LEVEL SAVINGS

How do micro-level consumption/savings choices change as the real interest rate changes (continue assuming $A_0 = 0$ for simplicity)?

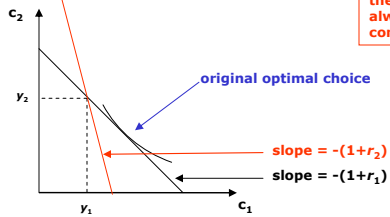


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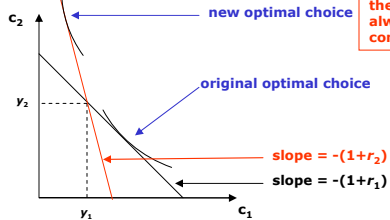


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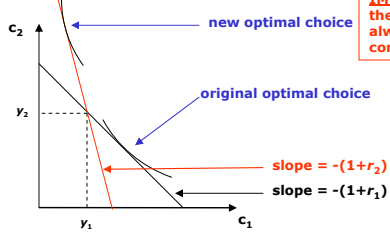
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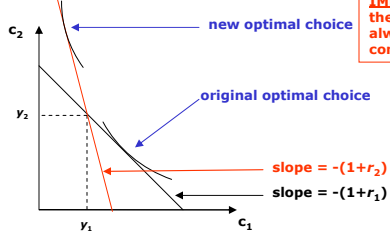
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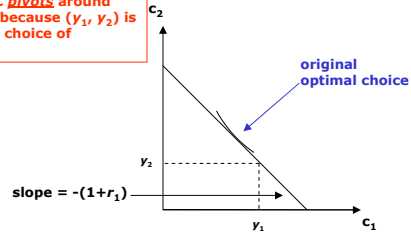
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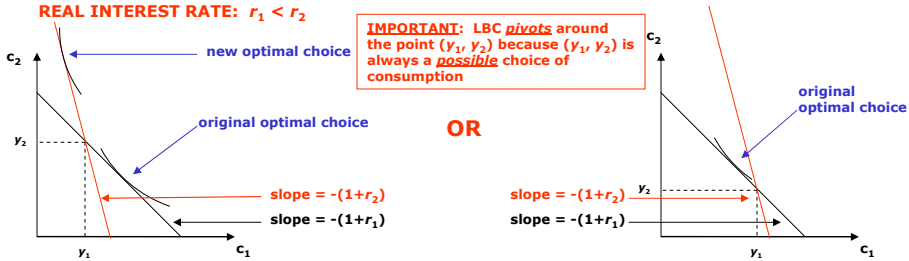
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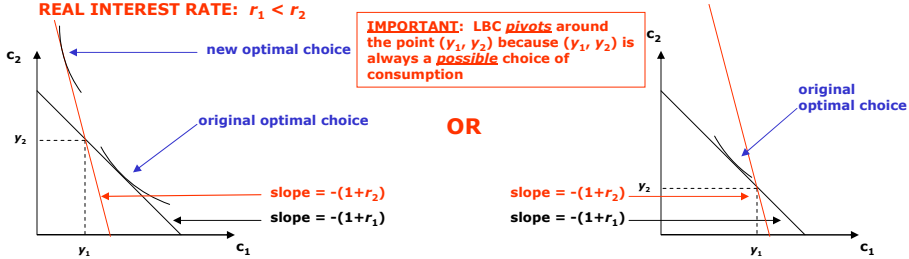
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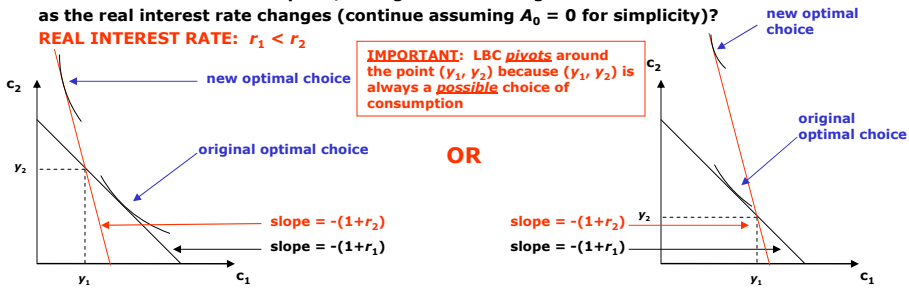
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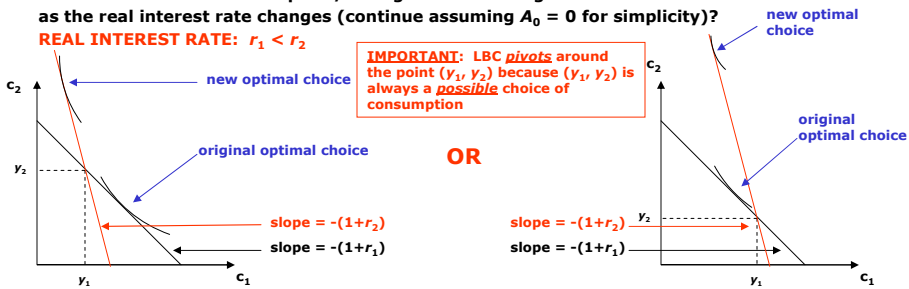
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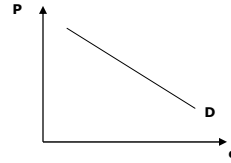
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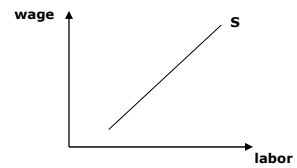
\rightarrow implying that when r rises, period-1 (i.e., "short-run") savings of all types of consumers rises...

THE THREE MACRO (AGGREGATE) MARKETS

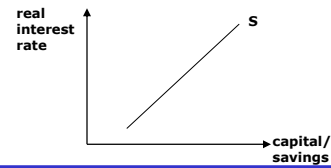
- ❑ **Goods Markets**
 - ❑ Demand derived from C-L model



- ❑ **Labor Markets**
 - ❑ Supply derived from C-L model



- ❑ **Capital/Savings/Funds/Asset Markets (aka Financial Markets)**
 - ❑ Supply derived from C-S model



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ASSESSING THE EFFECTS OF THE "CREDIT CRUNCH"

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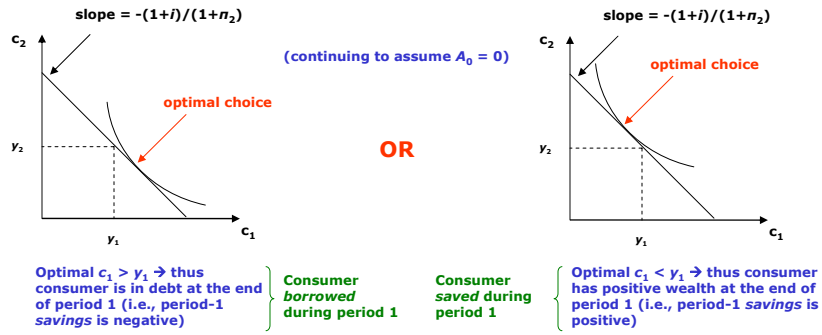
ASSESSING THE EFFECTS OF THE CREDIT CRUNCH

- **"Credit crunch" – financial sector has restricted the quantity of loans it is willing to extend to consumers in the "short run"**
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 - **Interpret "short run" to be period 1 (i.e., 2008-2009)**

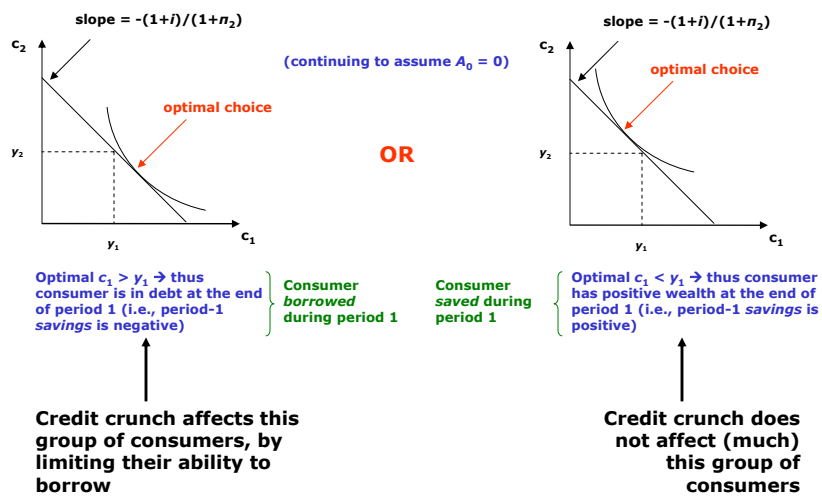
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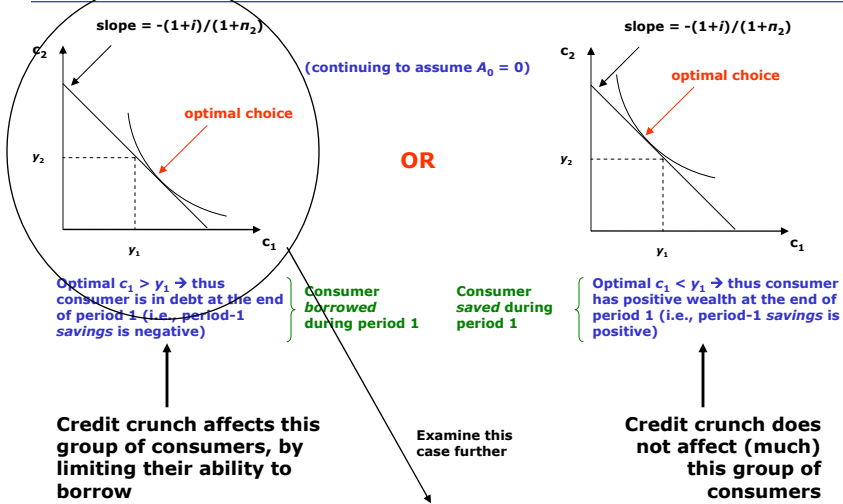
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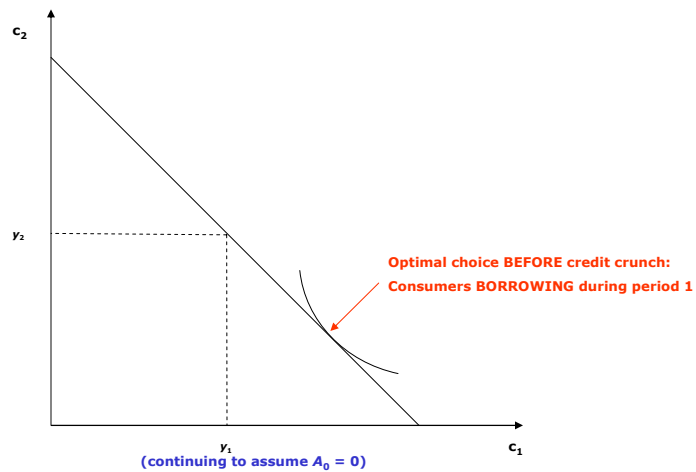
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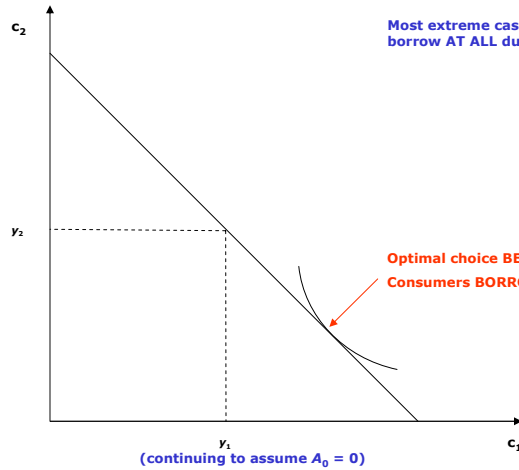
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"Credit crunch" in period 1 limits the ability of consumers to borrow during period 1.

Most extreme case: consumers cannot borrow AT ALL during period 1.

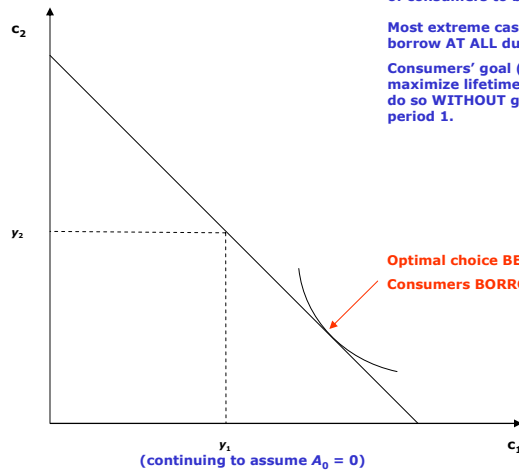


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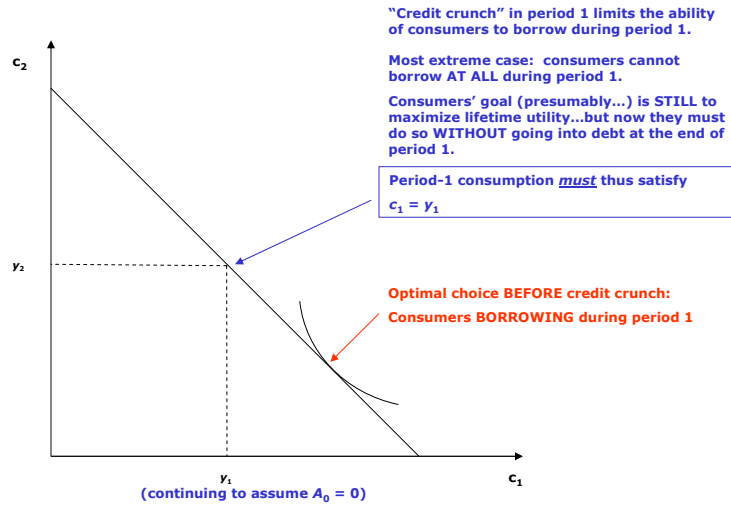
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Consumers' goal (presumably...) is STILL to maximize lifetime utility...but now they must do so WITHOUT going into debt at the end of period 1.



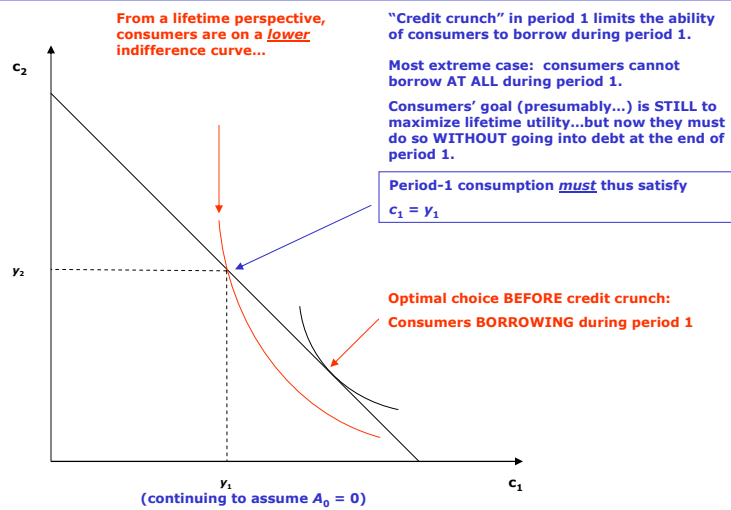
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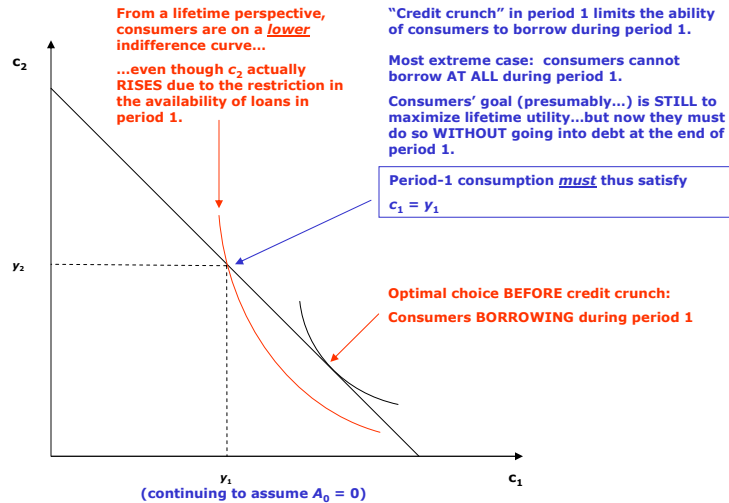
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- ❑ Can analyze macroeconomic consequences of shrinkage of credit availability using two-period model
 - ❑ Interpret "short run" to be period 1 (i.e., 2008-2009)
- ❑ Consequences
 - ❑ A large fraction of consumers (though not all) unable to borrow to pay for their desired period-1 consumption → their period-1 (i.e., "short run") consumption falls
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- OVERALL (i.e., lifetime utility of consumers FALLS (lower indiff. curve))

GOVERNMENT AND FISCAL POLICY IN THE CONSUMPTION-SAVINGS MODEL

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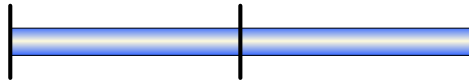
A Government in the Two-Period Model

A DYNAMIC MODEL OF THE GOVERNMENT

- So far only consumers in our two-period world
- Introduce government in very simple form
 - Exists for both periods
 - Has spending in each period it needs to finance – can be financed via
 - Taxes
 - Issuing government debt/assets

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- **Notation**
 - g_1 : real government spending in period 1
 - g_2 : real government spending in period 2
 - b_0 : government asset position at beginning of period 1/end of period 0
 - b_1 : government asset position at beginning of period 2/end of period 1
 - b_2 : government asset position at beginning of period 3/end of period 2
 - r : real interest rate between periods

A DYNAMIC MODEL OF THE GOVERNMENT

- Economic activities/actions described by period budget constraints
 - Period-1 government budget constraint

$$g_1 + b_1 = (1+r)b_0 + t_1$$

Total expenditure in period 1:
period-1 spending + wealth to carry into period 2

Total income in period 1:
period-1 tax collections + income from wealth carried into period 1 (inclusive of interest)

Period 1

government activities during period 1: government spending and tax collection

b_1

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← can rewrite as →

$$g_1 + b_1 - b_0 = t_1 + rb_0$$

Savings during period 1 (a flow) Asset income during period 1 (a flow)

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$$g_2 + b_2 = (1+r)b_1 + t_2$$

← can rewrite as →

$$g_2 + b_2 - b_1 = t_2 + rb_1$$

Total expenditure in period 2:
period-2 spending + wealth to
carry into period 3

Total income in period 2:
period-2 tax collections +
income from wealth carried into
period 2 (inclusive of interest)

Savings during period 2 (a flow) Asset income during period 2 (a flow)

□ Definition: A government's savings during a given period is the **change in its wealth during that period**

- "Fiscal surplus" if government savings is positive
- "Fiscal deficit" if government savings is negative

Surplus/deficit is a flow measure

GOVERNMENT BUDGET CONSTRAINT(S)

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- All analysis conducted from perspective of beginning of period 1

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For graphical simplicity, will often assume $b_0 = 0$ (i.e., government begins life with zero net wealth).
 Note this is a *different* assumption than $b_2 = 0$.

CONSUMER BUDGET CONSTRAINT(S)

- **Introduce tax payments into consumer side of model**
 - **All in real terms for simplicity – can cast in nominal terms by multiplying by P**
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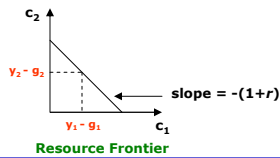
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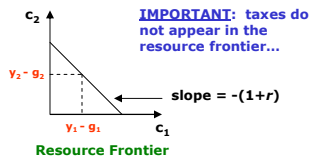
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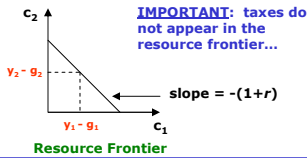
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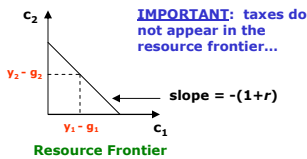
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An important theoretical result for the analysis of tax policy.

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Analyzing effects of changes in tax policy on optimal consumption choices is the key

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- ❑ Ricardian Equivalence is to tax theory what perfect competition is to standard economic theory
 - ❑ Idea relies crucially on lump-sum taxes...
 - ❑ ...even though in reality lump-sum taxes do not exist...