

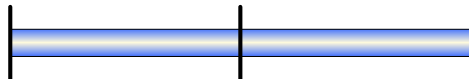
GOVERNMENT AND FISCAL POLICY IN THE CONSUMPTION-SAVINGS MODEL (CONTINUED)

FEBRUARY 23, 2009

A Government in the Two-Period Model

A DYNAMIC MODEL OF THE GOVERNMENT

- So far only consumers in our two-period world
- Introduce government in very simple form
 - Exists for both periods
 - Has spending in each period it needs to finance – can be financed via
 - Taxes
 - Issuing government debt/assets



- **Notation**
 - g_1 : real government spending in period 1
 - g_2 : real government spending in period 2
 - b_0 : government asset position at beginning of period 1/end of period 0
 - b_1 : government asset position at beginning of period 2/end of period 1
 - b_2 : government asset position at beginning of period 3/end of period 2
 - r : real interest rate between periods

GOVERNMENT BUDGET CONSTRAINT(S)

- Adopt a **lifetime** view of the budget constraint(s)
 - All analysis conducted from perspective of beginning of period 1
 - Period-1 government budget constraint $g_1 + b_1 = t_1 + (1+r)b_0$
 - Period-2 government budget constraint $g_2 + b_2 = t_2 + (1+r)b_1$

Asset position at end of period 1/beginning of period 2 the key link

Assume = 0 (fully-rational, informed, benevolent government)

- Combine into **lifetime budget constraint (LBC)**
 - Solve period-2 budget constraint for b_1 ...
 - ...and substitute into period-1 budget constraint

$$g_1 + \frac{g_2}{1+r} = t_1 + \frac{t_2}{1+r} + (1+r)b_0$$

Present discounted value (PDV) of all lifetime government expenditure Present discounted value (PDV) of all lifetime government income

IMPORTANT: Government must balance its budget over its *lifetime*, not necessarily in each period

For graphical simplicity, will often assume $b_0 = 0$ (i.e., government begins life with zero net wealth).
 Note this is a *different* assumption than $b_2 = 0$.

CONSUMER BUDGET CONSTRAINT(S)

- Introduce tax payments into consumer side of model
 - All in real terms for simplicity – can cast in nominal terms by multiplying by P
 - Period-1 budget constraint $c_1 + t_1 + a_1 - a_0 = y_1 + ra_0$
 - Period-2 budget constraint $c_2 + t_2 + a_2 - a_1 = y_2 + ra_1$
- Combine into **lifetime budget constraint (LBC)**
 - Solve period-2 budget constraint for a_1 ...
 - ...and substitute into period-1 budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 - t_1 + \frac{y_2 - t_2}{1+r} + (1+r)a_0$$

Present discounted value (PDV) of all lifetime expenditure Present discounted value (PDV) of all lifetime **disposable** income (i.e., after-tax income)

ECONOMY-WIDE RESOURCE FRONTIER

- Consumer lifetime budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 - t_1 + \frac{y_2 - t_2}{1+r} + (1+r)a_0$$

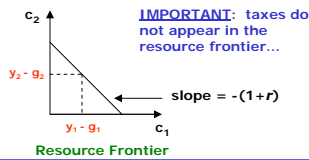
- Government lifetime budget constraint

$$g_1 + \frac{g_2}{1+r} = t_1 + \frac{t_2}{1+r} + (1+r)b_0$$

- Summing the two yields **economy-wide resource frontier**

$$c_1 + \frac{c_2}{1+r} = y_1 - g_1 + \frac{y_2 - g_2}{1+r} + (1+r)(a_0 + b_0) \quad \text{Suppose } = 0 \text{ for graphical simplicity}$$

- aka "production possibilities frontier" (PPF)
- The GDP accounting equation in two-period form



IMPORTANT: taxes do not appear in the resource frontier...

THEOREM (intermediate micro): If taxes are *lump-sum*, then consumer optimal choices can be obtained by analyzing *either* the consumer LBC *or* the economy-wide resource frontier (superimpose indifference map), and either approach will yield the same predictions.

An important theoretical result for the analysis of tax policy.

NATIONAL SAVINGS

- National savings = savings by consumers + savings by government + savings by firms

- No firms in our model (yet..), so $s_1^{firm} = 0$

- $s_1^{priv} = y_1 - t_1 - c_1$

- $s_1^{govt} = t_1 - g_1$

- $s_1^{nat} = s_1^{priv} + s_1^{govt} = y_1 - t_1 - c_1 + t_1 - g_1 = y_1 - c_1 - g_1$

EFFECTS OF TAX POLICY

- National savings = savings by consumers + savings by government + savings by firms
 - No firms in our model (yet.), so $s_1^{firm} = 0$
 - $s_1^{priv} = y_1 - t_1 - c_1$
 - $s_1^{govt} = t_1 - g_1$
 - $s_1^{nat} = s_1^{priv} + s_1^{govt} = y_1 - \cancel{t_1} - c_1 + \cancel{t_1} - g_1 = y_1 - c_1 - g_1$
- Policy Experiment: Is national savings affected by a decrease in t_1 ?
 - Suppose g_1 and g_2 do not change
 - Question 1: Effect on t_2 ?
 - t_2 must rise (examine government lifetime budget constraint)

February 23, 2009

7

EFFECTS OF TAX POLICY

- National savings = savings by consumers + savings by government + savings by firms
 - No firms in our model (yet.), so $s_1^{firm} = 0$
 - $s_1^{priv} = y_1 - t_1 - c_1$
 - $s_1^{govt} = t_1 - g_1$
 - $s_1^{nat} = s_1^{priv} + s_1^{govt} = y_1 - \cancel{t_1} - c_1 + \cancel{t_1} - g_1 = y_1 - c_1 - g_1$
- Policy Experiment: Is national savings affected by a decrease in t_1 ?
 - Suppose g_1 and g_2 do not change
 - Question 1: Effect on t_2 ?
 - t_2 must rise (examine government lifetime budget constraint)
 - Question 2: Effect of tax changes on consumers' optimal choice of period-1 consumption?

February 23, 2009

8

EFFECTS OF TAX POLICY

- National savings = savings by consumers + savings by government + savings by firms
 - No firms in our model (yet.), so $s_1^{firm} = 0$
 - $s_1^{priv} = y_1 - t_1 - c_1$
 - $s_1^{govt} = t_1 - g_1$
 - $s_1^{nat} = s_1^{priv} + s_1^{govt} = y_1 - \cancel{t_1} - c_1 + \cancel{t_1} - g_1 = y_1 - c_1 - g_1$

- Policy Experiment: Is national savings affected by a decrease in t_1 ?
 - Suppose g_1 and g_2 do not change
 - Question 1: Effect on t_2 ?
 - t_2 must rise (examine government lifetime budget constraint)
 - Question 2: Effect of tax changes on consumers' optimal choice of period-1 consumption?
 - Using intermediate micro theorem, NO EFFECT ON optimal c_1
 - Taxes are lump sum (will define/discuss next time...)
 - Economy-wide resource constraint does not depend on taxes → optimal choice of c_1 unaffected by the change in tax policy

February 23, 2009

9

EFFECTS OF TAX POLICY

- National savings = savings by consumers + savings by government + savings by firms
 - No firms in our model (yet.), so $s_1^{firm} = 0$
 - $s_1^{priv} = y_1 - t_1 - c_1$
 - $s_1^{govt} = t_1 - g_1$
 - $s_1^{nat} = s_1^{priv} + s_1^{govt} = y_1 - \cancel{t_1} - c_1 + \cancel{t_1} - g_1 = y_1 - c_1 - g_1$

- Policy Experiment: Is national savings affected by a decrease in t_1 ?
 - Suppose g_1 and g_2 do not change
 - Question 1: Effect on t_2 ?
 - t_2 must rise (examine government lifetime budget constraint)
 - Question 2: Effect of tax changes on consumers' optimal choice of period-1 consumption?
 - Using intermediate micro theorem, NO EFFECT ON optimal c_1
 - Taxes are lump sum (will define/discuss next time...)
 - Economy-wide resource constraint does not depend on taxes → optimal choice of c_1 unaffected by the change in tax policy
 - Question 3: Effect of tax changes on period-1 national savings?
 - NONE – because neither g_1 nor c_1 changed

February 23, 2009

10

EFFECTS OF TAX POLICY

- National savings = savings by consumers + savings by government + savings by firms

 - No firms in our model (yet.), so $s_1^{firm} = 0$
 - $s_1^{priv} = y_1 - t_1 - c_1$
 - $s_1^{govt} = t_1 - g_1$
 - $s_1^{nat} = s_1^{priv} + s_1^{govt} = y_1 - t_1 - c_1 + t_1 - g_1 = y_1 - c_1 - g_1$
- Analyzing effects of changes in tax policy on optimal consumption choices is the key
- Policy Experiment: Is national savings affected by a decrease in t_1 ?

 - Suppose g_1 and g_2 do not change
 - Question 1: Effect on t_2 ?
 - t_2 must rise (examine government lifetime budget constraint)
 - Question 2: Effect of tax changes on consumers' optimal choice of period-1 consumption?
 - Using intermediate micro theorem, NO EFFECT ON optimal c_1
 - Taxes are lump sum (will define/discuss next time...)
 - Economy-wide resource constraint does not depend on taxes → optimal choice of c_1 unaffected by the change in tax policy
 - Question 3: Effect of tax changes on period-1 national savings?
 - NONE – because neither g_1 nor c_1 changed

RICARDIAN EQUIVALENCE

- **Ricardian Equivalence Theorem:** For a given time-path of government spending, neither consumption nor national savings is affected by the precise timing of lump-sum taxes
- A benchmark result/concept in the theory of macroeconomic policy

RICARDIAN EQUIVALENCE

- ❑ **Ricardian Equivalence Theorem:** For a given time-path of government spending, neither consumption nor national savings is affected by the precise timing of lump-sum taxes
- ❑ A benchmark result/concept in the theory of macroeconomic policy
- ❑ **Intuition:** Rational consumers understand that a tax cut today means a tax increase in the future (because total government liabilities are unchanged)
 - ❑ Thus entire tax cut is saved by consumers in order to pay higher taxes in the future
 - ❑ Private savings and government savings move in exactly offsetting ways

RICARDIAN EQUIVALENCE

- ❑ **Ricardian Equivalence Theorem:** For a given time-path of government spending, neither consumption nor national savings is affected by the precise timing of lump-sum taxes
- ❑ A benchmark result/concept in the theory of macroeconomic policy
- ❑ **Intuition:** Rational consumers understand that a tax cut today means a tax increase in the future (because total government liabilities are unchanged)
 - ❑ Thus entire tax cut is saved by consumers in order to pay higher taxes in the future
 - ❑ Private savings and government savings move in exactly offsetting ways
- ❑ Ricardian Equivalence is to tax theory what perfect competition is to standard economic theory
 - ❑ Idea relies crucially on lump-sum taxes...
 - ❑ ...even though in reality lump-sum taxes do not exist...

NATURE OF TAXATION

- ❑ **Lump-Sum Tax**
 - ❑ A tax whose total incidence (i.e., total amount paid) does not depend in any way on any economic decisions/choices an individual makes
 - ❑ Real-world examples:

NATURE OF TAXATION

- ❑ **Lump-Sum Tax**
 - ❑ A tax whose total incidence (i.e., total amount paid) does not depend in any way on any economic decisions/choices an individual makes
 - ❑ Real-world examples: ?...

NATURE OF TAXATION

- ❑ **Lump-Sum Tax**
 - ❑ A tax whose total incidence (i.e., total amount paid) does not depend in any way on any economic decisions/choices an individual makes
 - ❑ Real-world examples: ?...

- ❑ Taxes in our two-period framework so far
 - ❑ **Lump-sum!** Total amounts t_1 and t_2 paid by consumer are independent of any of their decisions/choices

$$\text{Period-1 budget constraint} \\ c_1 + t_1 + a_1 - a_0 = y_1 + ra_0$$

$$\text{Period-2 budget constraint} \\ c_2 + t_2 + a_2 - a_1 = y_2 + ra_1$$

NATURE OF TAXATION

- ❑ **Lump-Sum Tax**
 - ❑ A tax whose total incidence (i.e., total amount paid) does not depend in any way on any economic decisions/choices an individual makes
 - ❑ Real-world examples: ?...

- ❑ Taxes in our two-period framework so far
 - ❑ **Lump-sum!** Total amounts t_1 and t_2 paid by consumer are independent of any of their decisions/choices

$$\text{Period-1 budget constraint} \\ c_1 + t_1 + a_1 - a_0 = y_1 + ra_0$$

$$\text{Period-2 budget constraint} \\ c_2 + t_2 + a_2 - a_1 = y_2 + ra_1$$

- ❑ **Proportional (aka distortionary) Tax**
 - ❑ A tax whose total incidence depends on economic decisions/choices an individual makes

NATURE OF TAXATION

- ❑ **Lump-Sum Tax**
 - ❑ A tax whose total incidence (i.e., total amount paid) does not depend in any way on any economic decisions/choices an individual makes
 - ❑ Real-world examples: ?...

- ❑ Taxes in our two-period framework so far
 - ❑ **Lump-sum!** Total amounts t_1 and t_2 paid by consumer are independent of any of their decisions/choices

Period-1 budget constraint

$$c_1 + t_1 + a_1 - a_0 = y_1 + ra_0$$

Period-2 budget constraint

$$c_2 + t_2 + a_2 - a_1 = y_2 + ra_1$$

- ❑ **Proportional (aka distortionary) Tax**
 - ❑ A tax whose total incidence depends on economic decisions/choices an individual makes
 - ❑ In simple two-period framework: consumers only make consumption choices c_1 and c_2

r is consumption tax rate (aka sales tax rate)

Period-1 budget constraint
 $(1 + \tau_1)c_1 + a_1 - a_0 = y_1 + ra_0$

Period-2 budget constraint
 $(1 + \tau_2)c_2 + a_2 - a_1 = y_2 + ra_1$

PROPORTIONAL TAXATION

r is consumption tax rate (aka sales tax rate)

Period-1 budget constraint
 $(1 + \tau_1)c_1 + a_1 - a_0 = y_1 + ra_0$

Period-2 budget constraint
 $(1 + \tau_2)c_2 + a_2 - a_1 = y_2 + ra_1$

- ❑ **Combine into consumer LBC**

$$(1 + \tau_1)c_1 + \frac{(1 + \tau_2)c_2}{1 + r} = y_1 + \frac{y_2}{1 + r} + (1 + r)a_0$$

PROPORTIONAL TAXATION

r is consumption tax rate (aka sales tax rate)

Period-1 budget constraint
 $(1 + \tau_1)c_1 + a_1 - a_0 = y_1 + ra_0$

Period-2 budget constraint
 $(1 + \tau_2)c_2 + a_2 - a_1 = y_2 + ra_1$

- **Combine into consumer LBC**

$$(1 + \tau_1)c_1 + \frac{(1 + \tau_2)c_2}{1 + r} = y_1 + \frac{y_2}{1 + r} + (1 + r)a_0$$

- **Slope is** $-\left(\frac{1 + \tau_1}{1 + \tau_2}\right)(1 + r)$

- **Non-lump-sum taxes: optimal consumption choices must be determined using consumer LBC, not economy's resource frontier (i.e., intermediate micro theorem does not apply)**

- **Changes in tax *rates* do affect optimal consumption choices because they *change slope of consumer LBC***

February 23, 2009

21

PROPORTIONAL TAXATION

r is consumption tax rate (aka sales tax rate)

Period-1 budget constraint
 $(1 + \tau_1)c_1 + a_1 - a_0 = y_1 + ra_0$

Period-2 budget constraint
 $(1 + \tau_2)c_2 + a_2 - a_1 = y_2 + ra_1$

- **Combine into consumer LBC**

$$(1 + \tau_1)c_1 + \frac{(1 + \tau_2)c_2}{1 + r} = y_1 + \frac{y_2}{1 + r} + (1 + r)a_0$$

- **Slope is** $-\left(\frac{1 + \tau_1}{1 + \tau_2}\right)(1 + r)$

- **Non-lump-sum taxes: optimal consumption choices must be determined using consumer LBC, not economy's resource frontier (i.e., intermediate micro theorem does not apply)**

- **Changes in tax *rates* do affect optimal consumption choices because they *change slope of consumer LBC***

- **Ricardian Equivalence Theorem does not apply**

- **Changes in tax rates *do* affect consumption and thus national savings**

February 23, 2009

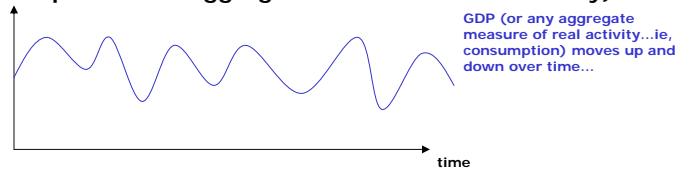
22

RICARDIAN EQUIVALENCE?

- ❑ So why the fascination with Ricardian Equivalence?
- ❑ A benchmark result/concept in the theory of macroeconomic policy
 - ❑ Effects of actual policy proposals can be compared to the Ricardian Equivalence benchmark

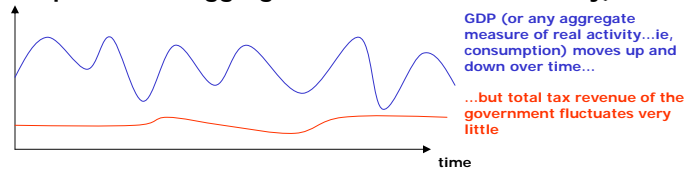
RICARDIAN EQUIVALENCE?

- ❑ So why the fascination with Ricardian Equivalence?
- ❑ A benchmark result/concept in the theory of macroeconomic policy
 - ❑ Effects of actual policy proposals can be compared to the Ricardian Equivalence benchmark
- ❑ At aggregate level, total tax collections sometime "seem" lump-sum (i.e., independent of aggregate macroeconomic activity)



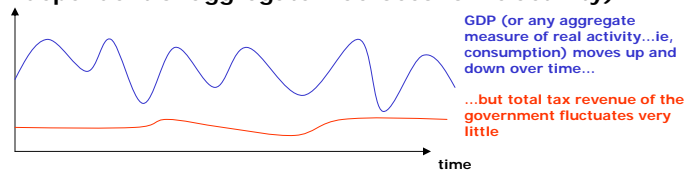
RICARDIAN EQUIVALENCE?

- ❑ So why the fascination with Ricardian Equivalence?
- ❑ A benchmark result/concept in the theory of macroeconomic policy
 - ❑ Effects of actual policy proposals can be compared to the Ricardian Equivalence benchmark
- ❑ At aggregate level, total tax collections sometime “seem” lump-sum (i.e., independent of aggregate macroeconomic activity)



RICARDIAN EQUIVALENCE?

- ❑ So why the fascination with Ricardian Equivalence?
- ❑ A benchmark result/concept in the theory of macroeconomic policy
 - ❑ Effects of actual policy proposals can be compared to the Ricardian Equivalence benchmark
- ❑ At aggregate level, total tax collections sometime “seem” lump-sum (i.e., independent of aggregate macroeconomic activity)



- ❑ Ricardian Equivalence
 - ❑ Is a theoretical benchmark
 - ❑ Is an empirical benchmark
- } Ricardian Equivalence is about the (lack of) effects of *changes in tax policy*, holding total government liabilities fixed. If g_1 and/or g_2 change, Ric. Equiv. does *not* apply.

INFINITE-PERIOD ECONOMY

FEBRUARY 23, 2009

Introduction

BASICS

- ❑ Modern workhorse macroeconomic models feature an **infinite** number of periods
 - ❑ A more realistic (?) view of time
- ❑ Especially useful for thinking about asset accumulation and asset pricing
 - ❑ The intersection of modern macro theory and modern finance theory

BASICS

- ❑ Modern workhorse macroeconomic models feature an **infinite** number of periods
 - ❑ A more realistic (?) view of time
- ❑ Especially useful for thinking about asset accumulation and asset pricing
 - ❑ The intersection of modern macro theory and modern finance theory
- ❑ Here, assume just one **real** asset
 - ❑ Call it a “stock” – i.e., a share in the S&P 500
 - ❑ (In Chapter 14, two nominal assets: bonds and money)

BASICS

- ❑ Modern workhorse macroeconomic models feature an **infinite** number of periods
 - ❑ A more realistic (?) view of time
- ❑ Especially useful for thinking about asset accumulation and asset pricing
 - ❑ The intersection of modern macro theory and modern finance theory
- ❑ Here, assume just one **real** asset
 - ❑ Call it a “stock” – i.e., a share in the S&P 500
 - ❑ (In Chapter 14, two nominal assets: bonds and money)
- ❑ Index time periods by arbitrary indexes t , $t+1$, $t+2$, etc.
 - ❑ **Important: all of our analysis will be conducted from the perspective of the very beginning of period t ...**
 - ❑ **...so an “infinite future” (period $t+1$, period, $t+2$, period $t+3$, ...) for which to save**

BASICS

□ Timeline of events



□ Notation

- c_t : consumption in period t
- P_t : nominal price of consumption in period t
- Y_t : nominal income in period t ("falls from the sky")
- a_{t-1} : real wealth (stock) holdings at beginning of period t /end of period $t-1$

February 23, 2009

 a_{t-1} Economic events during
period t - income,
consumption, savings a_t Economic
period t
consumpPeriod t

Peri

BASICS

□ Timeline of events



□ Notation

- c_t : consumption in period t
- P_t : nominal price of consumption in period t
- Y_t : nominal income in period t ("falls from the sky")
- a_{t-1} : real wealth (stock) holdings at beginning of period t /end of period $t-1$
- S_t : nominal price of a unit of stock in period t
- D_t : nominal dividend paid in period t by each unit of stock held at the start of t

The "defining
features" of
stock

February 23, 2009

32

BASICS

Timeline of events



Notation

- c_t : consumption in period t
 - P_t : nominal price of consumption in period t
 - Y_t : nominal income in period t ("falls from the sky")
 - a_{t-1} : real wealth (stock) holdings at beginning of period t /end of period $t-1$
 - S_t : nominal price of a unit of stock in period t
 - D_t : nominal dividend paid in period t by each unit of stock held at the start of t
 - π_{t+1} : net inflation rate between period t and period $t+1$
- $$\pi_{t+1} = \frac{P_{t+1} - P_t}{P_t} \left(= \frac{P_{t+1}}{P_t} - 1 \right)$$

- y_t : real income in period t ($= Y_t/P_t$)

 a_{t-1}

Economic events during
period t : income,
consumption, savings

 a_t

Economic
period t
consump

February 23, 2009

Period t

Peri

BASICS

Timeline of events



Notation

- c_{t+1} : consumption in period $t+1$
- P_{t+1} : nominal price of consumption in period $t+1$
- Y_{t+1} : nominal income in period $t+1$ ("falls from the sky")
- a_t : real wealth (stock) holdings at beginning of period $t+1$ /end of period t
- S_{t+1} : nominal price of a unit of stock in period $t+1$
- D_{t+1} : nominal dividend paid in period t by each unit of stock held at the start of $t+1$
- π_{t+2} : net inflation rate between period $t+1$ and period $t+2$

$$\pi_{t+2} = \frac{P_{t+2} - P_{t+1}}{P_{t+1}} \left(= \frac{P_{t+2}}{P_{t+1}} - 1 \right)$$

- y_{t+1} : real income in period $t+1$ ($= Y_{t+1}/P_{t+1}$)

February 23, 2009

34

BASICS

- Timeline of events



- Notation

- And so on for period $t+2$, $t+3$, etc...

February 23, 2009

a_{t-1}

Economic events during
period t - income,
consumption, savings

a_t

Economic
period t
consump

Period t

Peri

SUBJECTIVE DISCOUNT FACTOR

- Infinite number of periods a more serious view of time
- **Impatience** potentially an issue when taking a serious view of time

February 23, 2009

36

SUBJECTIVE DISCOUNT FACTOR

- ❑ Infinite number of periods a more serious view of time
- ❑ **Impatience** potentially an issue when taking a serious view of time
- ❑ Individuals (i.e., consumers) are impatient
 - ❑ All else equal, would rather have outcome X today than identical outcome X at some future date
 - ❑ An introspective statement about the world
 - ❑ An empirical statement about the world

SUBJECTIVE DISCOUNT FACTOR

- ❑ Infinite number of periods a more serious view of time
- ❑ **Impatience** potentially an issue when taking a serious view of time
- ❑ Individuals (i.e., consumers) are impatient
 - ❑ All else equal, would rather have outcome X today than identical outcome X at some future date
 - ❑ An introspective statement about the world
 - ❑ An empirical statement about the world
- ❑ Subjective discount factor
 - ❑ A simple model of consumer impatience
 - ❑ **β (a number between zero and one) measures impatience**
 - ❑ The lower is β , the less does individual value future utility

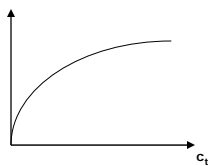
SUBJECTIVE DISCOUNT FACTOR

- ❑ Infinite number of periods a more serious view of time
- ❑ **Impatience** potentially an issue when taking a serious view of time
- ❑ Individuals (i.e., consumers) are impatient
 - ❑ All else equal, would rather have outcome X today than identical outcome X at some future date
 - ❑ An introspective statement about the world
 - ❑ An empirical statement about the world
- ❑ Subjective discount factor
 - ❑ A simple model of consumer impatience
 - ❑ **β (a number between zero and one) measures impatience**
 - ❑ The lower is β , the less does individual value future utility
 - ❑ Simple assumption about how "impatience" builds up over time
 - ❑ Multiplicatively: i.e., discount one period ahead by β , discount two periods ahead by β^2 , discount three periods ahead by β^3 , etc.
 - ❑ Do individuals' impatience really build up over time in this way?...limited empirical evidence so really don't know...

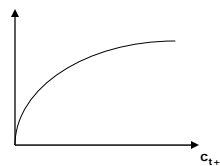
UTILITY

- ❑ Preferences $v(c_t, c_{t+1}, c_{t+2}, \dots)$ with all the "usual properties"
 - ❑ **Lifetime utility function**
 - ❑ Strictly increasing in $c_t, c_{t+1}, c_{t+2}, c_{t+3}, \dots$
 - ❑ Diminishing marginal utility in $c_t, c_{t+1}, c_{t+2}, c_{t+3}, \dots$

$v(c_t, c_{t+1}, c_{t+2}, c_{t+3}, \dots)$



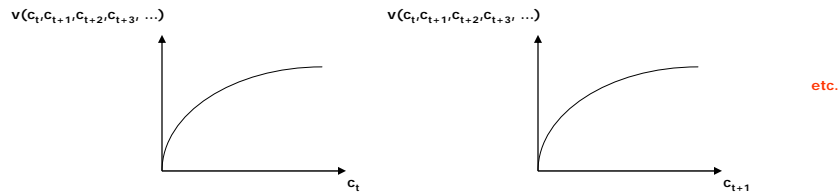
$v(c_t, c_{t+1}, c_{t+2}, c_{t+3}, \dots)$



etc.

UTILITY

- Preferences $v(c_t, c_{t+1}, c_{t+2}, \dots)$ with all the “usual properties”
 - **Lifetime utility function**
 - Strictly increasing in $c_t, c_{t+1}, c_{t+2}, c_{t+3}, \dots$
 - Diminishing marginal utility in $c_t, c_{t+1}, c_{t+2}, c_{t+3}, \dots$

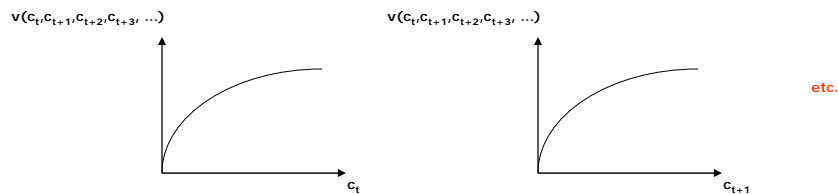


- Lifetime utility function additively-separable across time (a simplifying assumption), starting at time t

$$v(c_t, c_{t+1}, c_{t+2}, c_{t+3}, \dots) = u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \dots$$

UTILITY

- Preferences $v(c_t, c_{t+1}, c_{t+2}, \dots)$ with all the “usual properties”
 - **Lifetime utility function**
 - Strictly increasing in $c_t, c_{t+1}, c_{t+2}, c_{t+3}, \dots$
 - Diminishing marginal utility in $c_t, c_{t+1}, c_{t+2}, c_{t+3}, \dots$



- Lifetime utility function additively-separable across time (a simplifying assumption), starting at time t

$$v(c_t, c_{t+1}, c_{t+2}, c_{t+3}, \dots) = u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \dots$$

- **Utility side of infinite-period model no different than Chapter 1 model – except no longer possible to represent graphically...**

BUDGET CONSTRAINT(S)

- ❑ Suppose again Y "falls from the sky"
 - ❑ Y_t in period t , Y_{t+1} in period $t+1$, Y_{t+2} in period $t+2$, etc.
- ❑ Need **infinite** budget constraints to describe economic opportunities and possibilities
 - ❑ One for each period

BUDGET CONSTRAINT(S)

- ❑ Suppose again Y "falls from the sky"
 - ❑ Y_t in period t , Y_{t+1} in period $t+1$, Y_{t+2} in period $t+2$, etc.
- ❑ Need **infinite** budget constraints to describe economic opportunities and possibilities
 - ❑ One for each period
 - ❑ **Period- t budget constraint**

$$P_t c_t + S_t a_t = Y_t + S_t a_{t-1} + D_t a_{t-1}$$

Total expenditure in period t :
 period- t consumption + wealth
 to carry into period $t+1$

Total income in period t : period- t Y
 + income from stock-holdings
 carried into period t (has value S_t
 and pays dividend D_t)

BUDGET CONSTRAINT(S)

- Suppose again **Y** "falls from the sky"
 - Y_t in period t , Y_{t+1} in period $t+1$, Y_{t+2} in period $t+2$, etc.
- Need **infinite** budget constraints to describe economic opportunities and possibilities
 - One for each period
 - Period- t budget constraint

$$P_t c_t + S_t a_t = Y_t + S_t a_{t-1} + D_t a_{t-1}$$

Total expenditure in period t :
period- t consumption + wealth
to carry into period $t+1$

Total income in period t : period- t Y
+ income from stock-holdings
carried into period t (has value S_t
and pays dividend D_t)

- Period $t+1$ budget constraint

$$P_{t+1} c_{t+1} + S_{t+1} a_{t+1} = Y_{t+1} + S_{t+1} a_t + D_{t+1} a_t$$

Total expenditure in period $t+1$:
period- $t+1$ consumption +
wealth to carry into period $t+2$

Total income in period $t+1$: period-
 $t+1$ Y + income from stock-
holdings carried into period $t+1$
(has value S_{t+1} and pays dividend
 D_{t+1})

BUDGET CONSTRAINT(S)

- Suppose again **Y** "falls from the sky"
 - Y_t in period t , Y_{t+1} in period $t+1$, Y_{t+2} in period $t+2$, etc.
- Need **infinite** budget constraints to describe economic opportunities and possibilities
 - One for each period
 - Period- t budget constraint

$$P_t c_t + S_t a_t = Y_t + S_t a_{t-1} + D_t a_{t-1}$$

← can rewrite as

$$P_t c_t + S_t (a_t - a_{t-1}) = Y_t + D_t a_{t-1}$$

Savings during period t (a flow) Dividend income during period t (a flow)
↓

Total expenditure in period t :
period- t consumption + wealth
to carry into period $t+1$

Total income in period t : period- t Y
+ income from stock-holdings
carried into period t (has value S_t
and pays dividend D_t)

- Period $t+1$ budget constraint

$$P_{t+1} c_{t+1} + S_{t+1} a_{t+1} = Y_{t+1} + S_{t+1} a_t + D_{t+1} a_t$$

← can rewrite as

$$P_{t+1} c_{t+1} + S_{t+1} (a_{t+1} - a_t) = Y_{t+1} + D_{t+1} a_t$$

Savings during period $t+1$ (a flow) Dividend income during period $t+1$ (a flow)
↓

Total expenditure in period $t+1$:
period- $t+1$ consumption +
wealth to carry into period $t+2$

Total income in period $t+1$: period-
 $t+1$ Y + income from stock-
holdings carried into period $t+1$
(has value S_{t+1} and pays dividend
 D_{t+1})

BUDGET CONSTRAINT(S)

- Suppose again Y "falls from the sky"
 - Y_t in period t , Y_{t+1} in period $t+1$, Y_{t+2} in period $t+2$, etc.

- Need **infinite** budget constraints to describe economic opportunities and possibilities

- One for each period
- Period- t budget constraint

$$P_t c_t + S_t a_t = Y_t + S_t a_{t-1} + D_t a_{t-1}$$

← can rewrite as

$$P_t c_t + S_t (a_t - a_{t-1}) = Y_t + D_t a_{t-1}$$

Savings during period t (a flow)

Dividend income during period t (a flow)

Total expenditure in period t : period- t consumption + wealth to carry into period $t+1$

Total income in period t : period- t Y + income from stock-holdings carried into period t (has value S_t and pays dividend D_t)

Savings during period $t+1$ (a flow)

Dividend income during period $t+1$ (a flow)

- Period $t+1$ budget constraint

$$P_{t+1} c_{t+1} + S_{t+1} a_{t+1} = Y_{t+1} + S_{t+1} a_t + D_{t+1} a_t$$

← can rewrite as

$$P_{t+1} c_{t+1} + S_{t+1} (a_{t+1} - a_t) = Y_{t+1} + D_{t+1} a_t$$

Total expenditure in period $t+1$: period- $t+1$ consumption + wealth to carry into period $t+2$

Total income in period $t+1$: period- $t+1$ Y + income from stock-holdings carried into period $t+1$ (has value S_{t+1} and pays dividend D_{t+1})

And identical-looking budget constraints for $t+2$, $t+3$, $t+4$, etc...

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- Sequential formulation highlights the role of stock holdings (a_t) between period t and period $t+1$
 - Accords better with the explicit timing of economic events than the lifetime approach...
 - ...but yields the same result
 - Advantage: allows us to think about interaction between asset prices and macroeconomic events (intersection of finance theory and macro theory)

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- ❑ Sequential formulation highlights the role of stock holdings (a_t) between period t and period $t+1$
 - ❑ Accords better with the explicit timing of economic events than the lifetime approach...
 - ❑ ...but yields the same result
 - ❑ Advantage: allows us to think about interaction between asset prices and macroeconomic events (intersection of finance theory and macro theory)
- ❑ Apply Lagrange tools to consumption-savings optimization
- ❑ Objective function: $v(c_t, c_{t+1}, c_{t+2}, \dots)$
- ❑ Constraints:
 - ❑ Period- t budget constraint: $Y_t + S_t a_{t-1} + D_t a_{t-1} - P_t c_t - S_t a_t = 0$
 - ❑ Period- $t+1$ budget constraint: $Y_{t+1} + S_{t+1} a_t + D_{t+1} a_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1} = 0$
 - ❑ Period- $t+2$ budget constraint: $Y_{t+2} + S_{t+2} a_{t+1} + D_{t+2} a_{t+1} - P_{t+2} c_{t+2} - S_{t+2} a_{t+2} = 0$
 - ❑ etc...

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- ❑ Sequential formulation highlights the role of stock holdings (a_t) between period t and period $t+1$
 - ❑ Accords better with the explicit timing of economic events than the lifetime approach...
 - ❑ ...but yields the same result
 - ❑ Advantage: allows us to think about interaction between asset prices and macroeconomic events (intersection of finance theory and macro theory)
 - ❑ Apply Lagrange tools to consumption-savings optimization
 - ❑ Objective function: $v(c_t, c_{t+1}, c_{t+2}, \dots)$
 - ❑ Constraints:
 - ❑ Period- t budget constraint: $Y_t + S_t a_{t-1} + D_t a_{t-1} - P_t c_t - S_t a_t = 0$
 - ❑ Period- $t+1$ budget constraint: $Y_{t+1} + S_{t+1} a_t + D_{t+1} a_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1} = 0$
 - ❑ Period- $t+2$ budget constraint: $Y_{t+2} + S_{t+2} a_{t+1} + D_{t+2} a_{t+1} - P_{t+2} c_{t+2} - S_{t+2} a_{t+2} = 0$
 - ❑ etc...
- INFINITE constraints {
- ❑ Sequential Lagrange formulation requires infinite multipliers

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Step 1:** Construct Lagrange function (starting from t)

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Step 1:** Construct Lagrange function (starting from t)

$$u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \dots$$

First the lifetime utility function....

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Step 1: Construct Lagrange function (starting from t)**

$$u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \dots$$

First the lifetime utility function...

$$+ \lambda_t [Y_t + (S_t + D_t)a_{t-1} - P_t c_t - S_t a_t]$$

...then the period t constraint...

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Step 1: Construct Lagrange function (starting from t)**

$$u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \dots$$

First the lifetime utility function...

$$+ \lambda_t [Y_t + (S_t + D_t)a_{t-1} - P_t c_t - S_t a_t]$$

...then the period t constraint...

$$+ \beta \lambda_{t+1} [Y_{t+1} + (S_{t+1} + D_{t+1})a_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1}]$$

...then the period $t+1$ constraint...

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

□ **Step 1: Construct Lagrange function (starting from t)**

$$\begin{aligned}
 &u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \dots && \text{First the lifetime utility function...} \\
 &+ \lambda_t [Y_t + (S_t + D_t)a_{t-1} - P_t c_t - S_t a_t] && \dots \text{then the period } t \text{ constraint...} \\
 &+ \beta \lambda_{t+1} [Y_{t+1} + (S_{t+1} + D_{t+1})a_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1}] && \dots \text{then the period } t+1 \text{ constraint...} \\
 &+ \beta^2 \lambda_{t+2} [Y_{t+2} + (S_{t+2} + D_{t+2})a_{t+1} - P_{t+2} c_{t+2} - S_{t+2} a_{t+2}] && \dots \text{then the period } t+2 \text{ constraint...}
 \end{aligned}$$

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

□ **Step 1: Construct Lagrange function (starting from t)**

$$\begin{aligned}
 &u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \dots && \text{First the lifetime utility function...} \\
 &+ \lambda_t [Y_t + (S_t + D_t)a_{t-1} - P_t c_t - S_t a_t] && \dots \text{then the period } t \text{ constraint...} \\
 &+ \beta \lambda_{t+1} [Y_{t+1} + (S_{t+1} + D_{t+1})a_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1}] && \dots \text{then the period } t+1 \text{ constraint...} \\
 &+ \beta^2 \lambda_{t+2} [Y_{t+2} + (S_{t+2} + D_{t+2})a_{t+1} - P_{t+2} c_{t+2} - S_{t+2} a_{t+2}] && \dots \text{then the period } t+2 \text{ constraint...} \\
 &+ \beta^3 \lambda_{t+3} [Y_{t+3} + (S_{t+3} + D_{t+3})a_{t+2} - P_{t+3} c_{t+3} - S_{t+3} a_{t+3}] && \dots \text{then the period } t+3 \text{ constraint...}
 \end{aligned}$$

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

□ **Step 1: Construct Lagrange function (starting from t)**

$$\begin{aligned}
 & u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \dots && \text{First the lifetime utility function...} \\
 & + \lambda_t [Y_t + (S_t + D_t)a_{t-1} - P_t c_t - S_t a_t] && \dots \text{then the period } t \text{ constraint...} \\
 & + \beta \lambda_{t+1} [Y_{t+1} + (S_{t+1} + D_{t+1})a_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1}] && \dots \text{then the period } t+1 \text{ constraint...} \\
 & + \beta^2 \lambda_{t+2} [Y_{t+2} + (S_{t+2} + D_{t+2})a_{t+1} - P_{t+2} c_{t+2} - S_{t+2} a_{t+2}] && \dots \text{then the period } t+2 \text{ constraint...} \\
 & + \beta^3 \lambda_{t+3} [Y_{t+3} + (S_{t+3} + D_{t+3})a_{t+2} - P_{t+3} c_{t+3} - S_{t+3} a_{t+3}] && \dots \text{then the period } t+3 \text{ constraint...} \\
 & + \dots && \text{Infinite number of terms}
 \end{aligned}$$

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

□ **Step 1: Construct Lagrange function (starting from t)**

IMPORTANT:
Discount factor β
multiplies both
future utility and
future budget
constraints

Everything (utility
and income) about
the future is
discounted

$$\begin{aligned}
 & u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \dots && \text{First the lifetime utility function...} \\
 & + \lambda_t [Y_t + (S_t + D_t)a_{t-1} - P_t c_t - S_t a_t] && \dots \text{then the period } t \text{ constraint...} \\
 & + \beta \lambda_{t+1} [Y_{t+1} + (S_{t+1} + D_{t+1})a_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1}] && \dots \text{then the period } t+1 \text{ constraint...} \\
 & + \beta^2 \lambda_{t+2} [Y_{t+2} + (S_{t+2} + D_{t+2})a_{t+1} - P_{t+2} c_{t+2} - S_{t+2} a_{t+2}] && \dots \text{then the period } t+2 \text{ constraint...} \\
 & + \beta^3 \lambda_{t+3} [Y_{t+3} + (S_{t+3} + D_{t+3})a_{t+2} - P_{t+3} c_{t+3} - S_{t+3} a_{t+3}] && \dots \text{then the period } t+3 \text{ constraint...} \\
 & + \dots && \text{Infinite number of terms}
 \end{aligned}$$

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

□ **Step 1: Construct Lagrange function (starting from t)**

IMPORTANT:
Discount factor β
multiplies both
future utility and
future budget
constraints

Everything (utility
and income) about
the future is
discounted

$$\begin{aligned}
 & u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \dots \\
 & + \lambda_t [Y_t + (S_t + D_t)a_{t-1} - P_t c_t - S_t a_t] \\
 & + \beta \lambda_{t+1} [Y_{t+1} + (S_{t+1} + D_{t+1})a_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1}] \\
 & + \beta^2 \lambda_{t+2} [Y_{t+2} + (S_{t+2} + D_{t+2})a_{t+1} - P_{t+2} c_{t+2} - S_{t+2} a_{t+2}] \\
 & + \beta^3 \lambda_{t+3} [Y_{t+3} + (S_{t+3} + D_{t+3})a_{t+2} - P_{t+3} c_{t+3} - S_{t+3} a_{t+3}] \\
 & + \dots
 \end{aligned}$$

First the lifetime utility function....

...then the period t constraint...

...then the period t+1 constraint...

...then the period t+2 constraint...

...then the period t+3 constraint...

Infinite number of terms

□ **Step 2: Compute FOCs with respect to $c_t, a_t, c_{t+1}, a_{t+1}, c_{t+2}, \dots$**

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

□ **Step 1: Construct Lagrange function (starting from t)**

IMPORTANT:
Discount factor β
multiplies both
future utility and
future budget
constraints

Everything (utility
and income) about
the future is
discounted

$$\begin{aligned}
 & u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \dots \\
 & + \lambda_t [Y_t + (S_t + D_t)a_{t-1} - P_t c_t - S_t a_t] \\
 & + \beta \lambda_{t+1} [Y_{t+1} + (S_{t+1} + D_{t+1})a_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1}] \\
 & + \beta^2 \lambda_{t+2} [Y_{t+2} + (S_{t+2} + D_{t+2})a_{t+1} - P_{t+2} c_{t+2} - S_{t+2} a_{t+2}] \\
 & + \beta^3 \lambda_{t+3} [Y_{t+3} + (S_{t+3} + D_{t+3})a_{t+2} - P_{t+3} c_{t+3} - S_{t+3} a_{t+3}] \\
 & + \dots
 \end{aligned}$$

First the lifetime utility function....

...then the period t constraint...

...then the period t+1 constraint...

...then the period t+2 constraint...

...then the period t+3 constraint...

Infinite number of terms

□ **Step 2: Compute FOCs with respect to $c_t, a_t, c_{t+1}, a_{t+1}, c_{t+2}, \dots$**

with respect to c_t :

with respect to a_t :

with respect to c_{t+1} :

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

□ **Step 1: Construct Lagrange function (starting from t)**

IMPORTANT:
Discount factor β
multiplies both
future utility and
future budget
constraints
Everything (utility
and income) about
the future is
discounted

$$\begin{aligned}
 & u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \dots && \text{First the lifetime utility function...} \\
 & + \lambda_t [Y_t + (S_t + D_t)a_{t-1} - P_t c_t - S_t a_t] && \dots \text{then the period } t \text{ constraint...} \\
 & + \beta \lambda_{t+1} [Y_{t+1} + (S_{t+1} + D_{t+1})a_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1}] && \dots \text{then the period } t+1 \text{ constraint...} \\
 & + \beta^2 \lambda_{t+2} [Y_{t+2} + (S_{t+2} + D_{t+2})a_{t+1} - P_{t+2} c_{t+2} - S_{t+2} a_{t+2}] && \dots \text{then the period } t+2 \text{ constraint...} \\
 & + \beta^3 \lambda_{t+3} [Y_{t+3} + (S_{t+3} + D_{t+3})a_{t+2} - P_{t+3} c_{t+3} - S_{t+3} a_{t+3}] && \dots \text{then the period } t+3 \text{ constraint...} \\
 & + \dots && \text{Infinite number of terms}
 \end{aligned}$$

□ **Step 2: Compute FOCs with respect to $c_t, a_t, c_{t+1}, a_{t+1}, c_{t+2}, \dots$**

Identical except for time subscripts

with respect to c_t :	$u'(c_t) - \lambda_t P_t = 0$	Equation 1
with respect to a_t :	$-\lambda_t S_t + \beta \lambda_{t+1} (S_{t+1} + D_{t+1}) = 0$	Equation 2
with respect to c_{t+1} :	$\beta u'(c_{t+1}) - \beta \lambda_{t+1} P_{t+1} = 0$	Equation 3

THE BASICS OF ASSET PRICING

$$u'(c_t) - \lambda_t P_t = 0 \quad \text{Equation 1}$$

$$-\lambda_t S_t + \beta \lambda_{t+1} (S_{t+1} + D_{t+1}) = 0 \quad \text{Equation 2}$$

$$u'(c_{t+1}) - \lambda_{t+1} P_{t+1} = 0 \quad \text{Equation 3}$$

□ **Equation 2 \rightarrow** $S_t = \left(\frac{\beta \lambda_{t+1}}{\lambda_t} \right) (S_{t+1} + D_{t+1})$ BASIC ASSET-PRICING EQUATION

THE BASICS OF ASSET PRICING

$$u'(c_t) - \lambda_t P_t = 0 \quad \text{Equation 1}$$

$$-\lambda_t S_t + \beta \lambda_{t+1} (S_{t+1} + D_{t+1}) = 0 \quad \text{Equation 2}$$

$$u'(c_{t+1}) - \lambda_{t+1} P_{t+1} = 0 \quad \text{Equation 3}$$

□ Equation 2 →
$$S_t = \left(\frac{\beta \lambda_{t+1}}{\lambda_t} \right) (S_{t+1} + D_{t+1})$$
 BASIC ASSET-PRICING EQUATION

Period- t stock price = Pricing kernel × Future return

Two components:
 1. Future price of stock
 2. Future dividend payment

THE BASICS OF ASSET PRICING

$$u'(c_t) - \lambda_t P_t = 0 \quad \text{Equation 1}$$

$$-\lambda_t S_t + \beta \lambda_{t+1} (S_{t+1} + D_{t+1}) = 0 \quad \text{Equation 2}$$

$$u'(c_{t+1}) - \lambda_{t+1} P_{t+1} = 0 \quad \text{Equation 3}$$

□ Equation 2 →
$$S_t = \left(\frac{\beta \lambda_{t+1}}{\lambda_t} \right) (S_{t+1} + D_{t+1})$$
 BASIC ASSET-PRICING EQUATION

Period- t stock price = Pricing kernel × Future return

Two components:
 1. Future price of stock
 2. Future dividend payment

- Much of finance theory concerned with pricing kernel
 - Theoretical properties
 - Empirical models of kernels
- Pricing kernel where macro theory and finance theory intersect

THE BASICS OF ASSET PRICING

$$u'(c_t) - \lambda_t P_t = 0 \quad \text{Equation 1}$$

$$-\lambda_t S_t + \beta \lambda_{t+1} (S_{t+1} + D_{t+1}) = 0 \quad \text{Equation 2}$$

$$u'(c_{t+1}) - \lambda_{t+1} P_{t+1} = 0 \quad \text{Equation 3}$$

□ Equation 2 →
$$S_t = \left(\frac{\beta \lambda_{t+1}}{\lambda_t} \right) (S_{t+1} + D_{t+1})$$
 BASIC ASSET-PRICING EQUATION

Period- t stock price = Pricing kernel × Future return

Two components:
1. Future price of stock
2. Future dividend payment

- Much of finance theory concerned with pricing kernel
 - Theoretical properties
 - Empirical models of kernels
- Pricing kernel where macro theory and finance theory intersect
 - Solve equations 1 and 3 for λ_t and λ_{t+1}
 - Insert in asset-pricing equation

MACROECONOMIC EVENTS AFFECT ASSET PRICES

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)} \right) (S_{t+1} + D_{t+1}) \left(\frac{P_t}{P_{t+1}} \right)$$

- Consumption across time (c_t and c_{t+1}) affects stock prices
 - Fluctuations over time in aggregate consumption impact S_t

MACROECONOMIC EVENTS AFFECT ASSET PRICES

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)} \right) (S_{t+1} + D_{t+1}) \left(\frac{P_t}{P_{t+1}} \right)$$

↓ Using definition of inflation: $1 + \pi_{t+1} = P_{t+1} / P_t$

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)} \right) (S_{t+1} + D_{t+1}) \left(\frac{1}{1 + \pi_{t+1}} \right)$$

- Consumption across time (c_t and c_{t+1}) affects stock prices
 - Fluctuations over time in aggregate consumption impact S_t
- Inflation affects stock prices
 - Fluctuations over time in inflation impact S_t

MACROECONOMIC EVENTS AFFECT ASSET PRICES

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)} \right) (S_{t+1} + D_{t+1}) \left(\frac{P_t}{P_{t+1}} \right)$$

↓ Using definition of inflation: $1 + \pi_{t+1} = P_{t+1} / P_t$

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)} \right) (S_{t+1} + D_{t+1}) \left(\frac{1}{1 + \pi_{t+1}} \right)$$

- Consumption across time (c_t and c_{t+1}) affects stock prices
 - Fluctuations over time in aggregate consumption impact S_t
- Inflation affects stock prices
 - Fluctuations over time in inflation impact S_t
- **Wall Street Journal page A1, Feb. 21, 2008:**
 "Stocks [prices] fell on the [higher-than-expected] inflation reading."
 Interpretation: Increase in π_t (which typically would also be associated with increase in π_{t+1}) led to fall in S_t .

MACROECONOMIC EVENTS AFFECT ASSET PRICES

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)} \right) (S_{t+1} + D_{t+1}) \left(\frac{P_t}{P_{t+1}} \right)$$

↓ Using definition of inflation: $1 + \pi_{t+1} = P_{t+1} / P_t$

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)} \right) (S_{t+1} + D_{t+1}) \left(\frac{1}{1 + \pi_{t+1}} \right)$$

- Consumption across time (c_t and c_{t+1}) affects stock prices
 - Fluctuations over time in aggregate consumption impact S_t
- Inflation affects stock prices
 - Fluctuations over time in inflation impact S_t
- **Wall Street Journal page A1, Oct. 2, 2008:**
 "Stock prices fell on fears of short-run weakness in the economy."
 Interpretation: A decrease in c_t (induced by the "credit crunch") makes stocks **right now** (i.e., in period t) a less attractive asset – demand for it falls, hence its price (S_t) falls.

February 23, 2009

69

MACROECONOMIC EVENTS AFFECT ASSET PRICES

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)} \right) (S_{t+1} + D_{t+1}) \left(\frac{P_t}{P_{t+1}} \right)$$

↓ Using definition of inflation: $1 + \pi_{t+1} = P_{t+1} / P_t$

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)} \right) (S_{t+1} + D_{t+1}) \left(\frac{1}{1 + \pi_{t+1}} \right)$$

- Consumption across time (c_t and c_{t+1}) affects stock prices
 - Fluctuations over time in aggregate consumption impact S_t
- Inflation affects stock prices
 - Fluctuations over time in inflation impact S_t
- ANY factor (monetary policy, fiscal policy, globalization, etc.) that affects inflation and GDP in principle impacts stock/asset markets
- Direction of causality?...

February 23, 2009

70

MACROECONOMIC EVENTS AFFECT ASSET PRICES

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)} \right) (S_{t+1} + D_{t+1}) \left(\frac{P_t}{P_{t+1}} \right)$$

↓ Using definition of inflation: $1 + \pi_{t+1} = P_{t+1} / P_t$

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)} \right) (S_{t+1} + D_{t+1}) \left(\frac{1}{1 + \pi_{t+1}} \right)$$

VIEW AS A CONSUMPTION-SAVINGS OPTIMALITY CONDITION →

- ❑ Consumption across time (c_t and c_{t+1}) affects stock prices
 - ❑ Fluctuations over time in aggregate consumption impact S_t
- ❑ Inflation affects stock prices
 - ❑ Fluctuations over time in inflation impact S_t
- ❑ ANY factor (monetary policy, fiscal policy, globalization, etc.) that affects inflation and GDP in principle impacts stock/asset markets
- ❑ Direction of causality?...

CONSUMER OPTIMIZATION

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)} \right) (S_{t+1} + D_{t+1}) \left(\frac{P_t}{P_{t+1}} \right)$$

↓ Move $u'(c_t)$ and $\beta u'(c_{t+1})$ terms to left-hand-side, and S_t to right-hand-side

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \left(\frac{S_{t+1} + D_{t+1}}{S_t} \right) \left(\frac{1}{1 + \pi_{t+1}} \right)$$

CONSUMER OPTIMIZATION

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)} \right) (S_{t+1} + D_{t+1}) \left(\frac{P_t}{P_{t+1}} \right)$$

↓ Move $u'(c_t)$ and $\beta u'(c_{t+1})$ terms to left-hand-side, and S_t to right-hand-side

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \left(\frac{S_{t+1} + D_{t+1}}{S_t} \right) \left(\frac{1}{1 + \pi_{t+1}} \right)$$

i.e., ratio of marginal utilities

MRS between period t consumption and period $t+1$ consumption

Some sort of price ratio...

CONSUMER OPTIMIZATION

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)} \right) (S_{t+1} + D_{t+1}) \left(\frac{P_t}{P_{t+1}} \right)$$

↓ Move $u'(c_t)$ and $\beta u'(c_{t+1})$ terms to left-hand-side, and S_t to right-hand-side

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \left(\frac{S_{t+1} + D_{t+1}}{S_t} \right) \left(\frac{1}{1 + \pi_{t+1}} \right)$$

CONSUMPTION-SAVINGS OPTIMALITY CONDITION

i.e., ratio of marginal utilities

MRS between period t consumption and period $t+1$ consumption

Analogy with Chapters 3 & 4: must be $(1+r_t)$

Recall real interest rate is a price

CONSUMER OPTIMIZATION

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)} \right) (S_{t+1} + D_{t+1}) \left(\frac{P_t}{P_{t+1}} \right)$$

Move $u'(c_t)$ and $\beta u'(c_{t+1})$ terms to left-hand-side, and S_t to right-hand-side

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \left(\frac{S_{t+1} + D_{t+1}}{S_t} \right) \left(\frac{1}{1 + \pi_{t+1}} \right)$$

CONSUMPTION-SAVINGS OPTIMALITY CONDITION

i.e., ratio of marginal utilities

MRS between period t consumption and period $t+1$ consumption

Analogy with Chapters 3 & 4: must be $(1+r_t)$

Recall real interest rate is a price

- Recover Chapter 3 & 4 model by setting $t = 1$ and $\beta = 1$

CONSUMER OPTIMIZATION

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)} \right) (S_{t+1} + D_{t+1}) \left(\frac{P_t}{P_{t+1}} \right)$$

Move $u'(c_t)$ and $\beta u'(c_{t+1})$ terms to left-hand-side, and S_t to right-hand-side

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \left(\frac{S_{t+1} + D_{t+1}}{S_t} \right) \left(\frac{1}{1 + \pi_{t+1}} \right)$$

CONSUMPTION-SAVINGS OPTIMALITY CONDITION

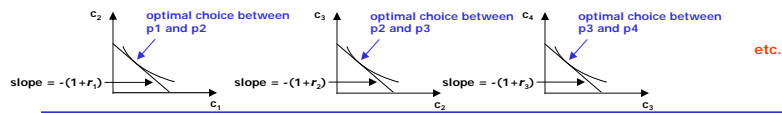
i.e., ratio of marginal utilities

MRS between period t consumption and period $t+1$ consumption

Analogy with Chapters 3 & 4: must be $(1+r_t)$

Recall real interest rate is a price

- Recover Chapter 3 & 4 model by setting $t = 1$ and $\beta = 1$
- Infinite-period model is sequence of overlapping two-period models



A LONG-RUN THEORY OF MACRO

- Consumption-savings optimality condition at the heart of modern macro models
 - Emphasize the dynamic nature of aggregate economic events
 - Foundation for understanding the periodic ups and downs (“business cycles”) of the economy
 - (Chapter 13: business cycle models)

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = 1 + r_t$$

A LONG-RUN THEORY OF MACRO

- Consumption-savings optimality condition at the heart of modern macro models
 - Emphasize the dynamic nature of aggregate economic events
 - Foundation for understanding the periodic ups and downs (“business cycles”) of the economy
 - (Chapter 13: business cycle models)

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = 1 + r_t$$

↓
 NEXT: Impose “steady state”
 and examine long-run
 relationship between interest
 rates and consumer impatience

$$\frac{1}{\beta} = 1 + r$$