Department of Applied Economics Johns Hopkins University

Economics 602

Macroeconomic Theory and Policy

Problem Set 2 Suggested Solutions

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1. **Interaction of Consumption Tax and Wage Tax.** A basic idea of President Bush's economic advisers throughout his administration was to try to move the U.S. further away from a system of investment taxes (which we will discuss later in the course) and more towards a system of consumption taxes**.** A nationwide consumption tax would essentially be a national sales tax. Here, you will modify our basic consumption-leisure model to include both a proportional wage tax (which we will now denote by t_n, where, as before, $0 \le t_n < 1$) as well as a proportional consumption tax (which we will denote by t_c, where $0 \le t_c < 1$). A proportional consumption tax means that for every dollar on the price tags of items the consumer buys, the consumer must pay $(1+t_c)$ dollars. Throughout the following, suppose that economic policy has no effect on wages or prices (that is, the nominal wage W and the price of consumption P are constant throughout).

   a. Construct the budget constraint in this modified version of the consumption-leisure model. Briefly explain economically how this budget constraint differs from that in the standard consumption-leisure model we have studied in class.

Solution: The representative agent's net income from working is now given by $Y = (1-t_n)\cdot W \cdot n$, where t_n is the labor tax rate and the other notation is the same as in Chapter 2. He spends all of this income on consumption, which now costs $P\cdot(1+t_c)$ dollars per unit (inclusive of the consumption tax). Using the fact that $n = 168 - l$ in the weekly model, equating the representative agent's labor income with his expenditures on consumption gives us

$$P\cdot(1+t_c)\cdot c = (1-t_n)\cdot W\cdot(168-l).$$

If we multiply out the right-hand-side of this expression and then move the term involving the labor tax rate to the left-hand-side we obtain

$$P\cdot(1+t_c)\cdot c + (1-t_n)\cdot W\cdot l = 168\cdot(1-t_n)\cdot W .$$

Then, solving this last expression for c, we arrive at

$$c = \frac{168 \cdot (1-t_n) \cdot W}{(1+t_c) \cdot P} - \frac{(1-t_n) \cdot W}{(1+t_c) \cdot P} l .$$

This last expression can now readily be graphed with consumption on the vertical axis and leisure on the horizontal axis. As in the standard model, the horizontal intercept is $l = 168$. However, the slope is now

$$-\frac{(1-t_n) \cdot W}{(1+t_c) \cdot P} .$$

Clearly, however, if we set the consumption tax rate to zero, we recover the budget constraint in our standard consumption-leisure model – indeed, the model we studied in Chapter 2 is simply a special case of the model here. The reason the budget constraint differs here from the standard model is simple: the consumption tax is yet another tax for the consumer to take account of when making his choices about consumption and leisure. No matter the model under consideration, the budget constraint always describes all the relevant tradeoffs between two alternative use of resources, and the relevant tradeoffs involve all taxes.

b. Suppose currently the federal wage tax rate is 20 percent ($t_n = 0.20$) while the federal consumption tax rate is 0 percent ($t_c = 0$), and that the Bush economic team is considering proposing lowering the wage tax rate to 15 percent. However, they wish to leave the representative agent's optimal choice of consumption and leisure unaffected. Can they simultaneously increase the consumption tax rate from its current zero percent to achieve this goal? If so, compute the new associated consumption tax rate, and explain the economic intuition. If not, explain mathematically as well as economically why not.

Solution: From the analysis in part a above, we see that the slope of the budget constraint depends on the **relative tax** $(1-t_n)/(1+t_c)$ (in addition to the term W/P, but you are told to assume that W and P remain constant). Under the current tax policy of a 20 percent wage tax and zero consumption tax, the relative tax is $(1-0.20)/(1+0) = 0.80$. So the slope of the representative agent's budget constraint is currently $-0.80W/P$, on which he makes some optimal choice of consumption and leisure.

Now the government wants to lower the labor tax rate to $t_n = 0.15$ but wants to leave the representative agent's optimal choice of consumption and leisure unchanged. This means that whatever the government does, it must make sure that the slope of his budget constraint does not change – which means that the relative tax must remain 0.80. We can then solve for the new consumption tax rate that yields this relative tax: $(1-0.15)/(1+t_c) = 0.80$ means that the government must set a consumption tax rate of $t_c = 0.0625$. The economic reasoning is that the relative tax has two free variables in it, the labor tax and the consumption tax. There are an infinite number of combinations that yield any particular value of the relative tax. Think of the following simple example: if you have two numbers x and y, and you are asked to come up with a combination of the two variables such that $x/y = 0.80$, there are obviously an infinite number of combinations that work.

c. A **tax policy** is defined as a particular combination of tax rates. For example a labor tax rate of 20 percent combined with a consumption tax rate of zero percent is one particular tax policy. A labor tax rate of five percent combined with a consumption tax rate of 10 percent is a different tax policy. Based on what you found in parts a and b above, address the following statement: a government can use many different tax policies to induce the same level of consumption by individuals.

Solution: The statement is true, and it follows from the discussion given in part b above. If the government believes that W and P are unaffected by its tax policies (which is not true – we will address this issue soon), then it has two tax rates it can alter to achieve its goals, but it is only the relative tax that affects the representative agent's budget constraint.

d. Consider again the Bush proposal to lower the wage tax rate from 20 percent to 15 percent. This time, however, policy discussion is focused on trying to boost overall consumption. Is it possible for this goal to be achieved if the consumption tax rate is raised from its current zero percent?

Solution: We saw in the standard consumption-leisure model that as the budget line became steeper, consumption increases. This is still true in this version of the consumption-leisure model. The current tax policy has $t_n = 0.20$ and $t_c = 0$ so that the relative tax is $(1-0.20)/(1+0) = 0.80$. Any new tax policy which features a larger value of $(1-t_n)/(1+t_c)$ (and hence a steeper budget constraint) will thus achieve the desired goal of higher overall consumption. With a labor tax rate of $t_n = 0.15$, we thus need

$$\frac{(1-0.15)}{(1+t_c)} > 0.80 .$$

Solving this inequality for t_c, we have that

$$t_c < 0.0625$$

achieves the desired goal. So **any** tax policy with $t_n = 0.15$ and $t_c < 0.0625$ achieves the desired policy role. So the conclusion is: yes, the consumption tax rate can be raised and the desired goal still be achieved.

e. Using a Lagrangian, derive the consumer's consumption-leisure optimality condition (for an arbitrary utility function) as a function of the real wage and the consumption and labor tax rates.

Solution: The Lagrangian is

$$L(c,l,\lambda) = u(c,l) + \lambda\left[(1-t_n)W(168-l) - P(1+t_c)c\right]$$

The FOCs with respect to consumption and leisure are (we'll ignore the one with respect to the multiplier because in order to generate the consumption-leisure optimality condition, we actually don't need it):

$$u_c(c,l) - \lambda P(1+t_c) = 0$$

$$u_l(c,l) - \lambda W(1-t_n) = 0$$

To generate the consumption-leisure optimality condition, we must combine these two expressions by eliminating λ between them. Doing so, and expressing one side of the resulting expressing as the MRS between consumption and leisure, we have

$$\frac{u_l(c,l)}{u_c(c,l)} = \frac{(1-t_n)W}{(1+t_c)P}.$$

The left-hand side is the representative consumer's MRS between consumption and leisure, and the right-hand-side is the real wage rate (W/P) adjusted by **both** the labor and consumption taxes.

2. **Non-Backward-Bending Labor Supply Curve.** Consider an economy populated by 100 individuals who have identical preferences over consumption and leisure. In this economy, the aggregate labor supply curve is upward-sloping. For simplicity, suppose throughout this question that the labor tax rate is zero.

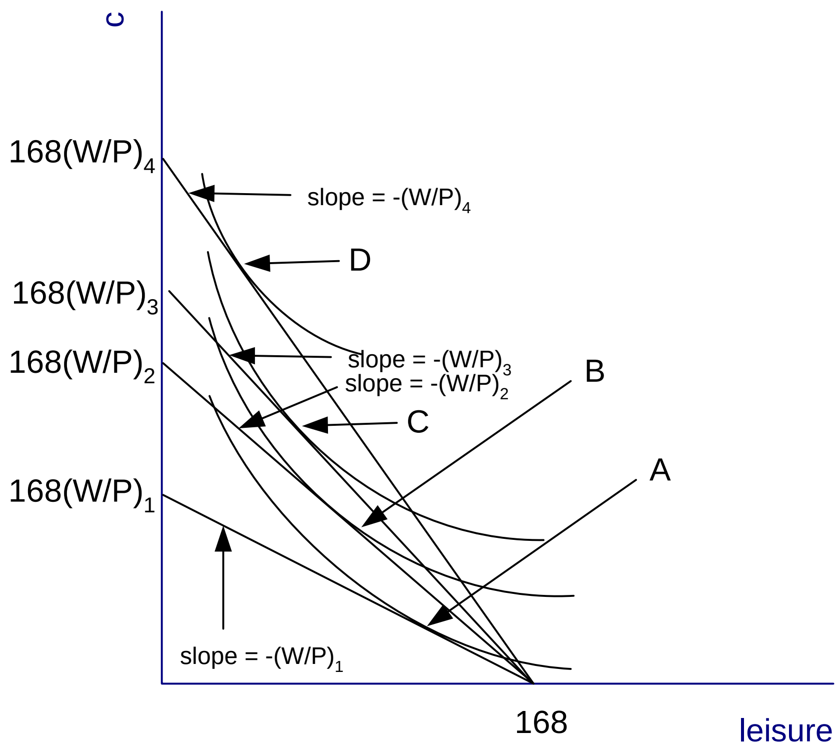
   a. For such a labor supply curve, how does the substitution effect compare with the income effect?

   Solution: The upward-sloping region of any individual's labor supply curve arises because the substitution effect of a higher real wage dominates the income effect of a higher real wage. (See the discussion in Chapter 2.) Thus, if the individual's labor supply curve is always upward-sloping, then it must be that for this individual the substitution effect always outweighs the income effect. With 100 identical individuals in the economy, the aggregate labor supply curve is simple the sum of each individual's labor supply curve, and thus inherits the properties of the individuals' labor supply curves.

   Extended Note: the labor supply curve cannot literally be **always** upward-sloping. The upper-limit on the labor axis is of course (for the weekly model) 168 hours. Once that upper limit is reached (i.e., a person is doing nothing but working), any further rise in the real wage cannot increase hours worked – hence the labor supply curve becomes vertical. But this latter effect should probably strike you as uninteresting because then the individual does not enjoy any leisure at all. Indeed, if we have a "usual" indifference map over consumption and leisure, we will never have that an indifference curve is tangent to the budget line on either axis, a necessary implication of an optimal choice that has zero leisure (try drawing this to convince yourself).

b. Using indifference curves and budget constraints, show how such a labor supply curve arises.

Solution: We must have that any rise in the real wage leads to a higher optimal choice of consumption **and** a lower optimal choice of leisure (with of course a natural zero lower bound on leisure – see the Extended Note above), irrespective of the current real wage.



In the above diagram, as the real wage rises from $(W/P)_1$ to $(W/P)_2$ to $(W/P)_3$ to $(W/P)_4$, the optimal choice moves from point A to B to C to D, respectively. Clearly, as the real wage rises, the quantity of leisure demanded (and hence the quantity of labor supplied) rises, consistent with a labor-supply curve that does not bend backwards.

3. **A Backward-Bending Aggregate Labor Supply Curve?** Despite our use of the backward-bending labor supply curve as arising from the representative agent's preferences, there is controversy in macroeconomics about whether this is a good representation. Specifically, even though a backward-bending labor supply curve may be a good description of a given individual's decisions, it does **not** immediately

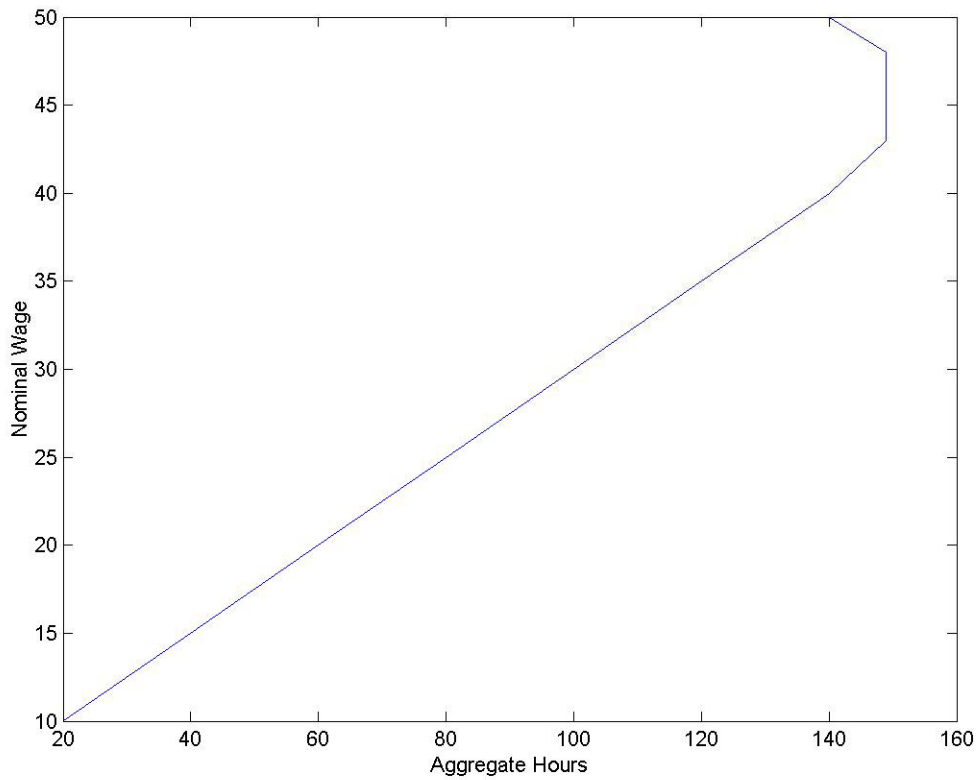
follow that the representative agent's preferences should also feature a backward-bending labor supply curve. In this exercise you will uncover for yourself this problem. For simplicity, assume that the labor tax rate is $t = 0$ throughout all that follows.

a. Suppose the economy is made up of five individuals, person A, person B, person C, person D, and person E, each of whom has the labor supply schedule given below. Using the indicated wage rates, graph each individual's labor supply curve **as well as** the aggregate labor supply curve.

Solution: In the table below, the aggregate (total) number of hours worked by all persons in the economy at each wage rate is now shown (this was not given to you).

Nominal Wage, W	Person A	Person B	Person C	Person D	Person E	Aggregate
$10	20 hours	0 hours	0 hours	0 hours	0 hours	20 hours
$15	25	15	0	0	0	40
$20	30	22	8	0	0	60
$25	33	27	15	5	0	80
$30	35	30	20	15	0	100
$35	37	32	25	20	6	120
$40	36	31	27	25	21	140
$45	35	30	26	28	30	149
$50	33	29	24	25	29	140

The aggregate labor supply curve simply plots the values in the last column in the table above against the wage rate (with, recall, the labor tax rate held constant at $t_n = 0$ throughout for simplicity), as shown below. Clearly, most of the aggregate labor supply curve is upward-sloping, with only the very top portion backward-bending. For brevity, the individuals' labor supply curves are omitted – they are of course simply each individual's hours worked plotted against the wage, and it should be clear even from the table that each individual in the economy has a backward-bending labor supply curve.



Now suppose that in this economy, the "usual" range of the nominal wage is between $10 and $45.

b. Restricting attention to this range, is the aggregate labor supply curve backward-bending?

Solution: If the usual range of the nominal wage is $10-$45 in the economy, then clearly no (see the Figure above), the aggregate labor supply is **not** backward-bending.

c. At a theoretical level, if we want to use the representative-agent paradigm and restrict attention to this usual range of the wage, does a backward-bending labor supply curve make sense?

Solution: The point of the representative-agent framework is to represent theoretically the "average" person in the economy in all aspects of his economic life (in so far as such theoretical modeling is possible…), including of course his labor supply decisions. The "average" person in the economy does not earn the highest wages in the economy.

d. Explain qualitatively the relationship you find between the individuals' labor supply curves and the aggregate labor supply curve over the range \$10 – \$45. Especially address the "backward-bending" nature of the curves.

Solution: Over the range \$10 - \$45, the labor supply curves of person A, person B, and person C are backward-bending, while the labor supply curves of person D and person E are not (notice that the labor supply curves of person D and person E do not bend backwards until the range \$45 - \$50). The aggregate labor supply curve is always upward-sloping in this range of the wage. The fundamental issue here is that people are different from each other in such a way that the average person, over the range \$10-\$45, "looks like" only 2 of the 5 people in this economy (person D and person E). We could easily construct another example in which the representative agent's labor supply "looked like" well less than 40% of the population over some "usual" range of income. This illustrates that microeconomic phenomena (in this case the backward-bending labor supply curve) when summed together do not necessarily give qualitatively the same phenomena at the macroeconomic level – a cautionary note in using the representative-agent approach to macroeconomics.