

Economics 602

Macroeconomic Theory and Policy**Problem Set 6 Suggested Solutions**

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Fall 2009

1. **Lags in Labor Hiring.** Rather than supposing that the representative firm at the beginning of period t can decide how much labor it would like to hire for use in period t , suppose that labor used in period t must be chosen in period $t-1$. (That is, suppose n is a stock (aka state) variable.) As usual, capital for use in production in period t must be purchased in period $t-1$ because of the “time to build” surrounding capital goods. With this lag in labor hiring, construct the lifetime (in the two-period model) profit function of the firm, and show that the real interest rate now is a relevant price for labor as well as capital goods. Provide brief economic intuition. (**Hint:** Make as close an analogy with our model of firm ownership of capital as you can – in particular, think of workers in this model as being “owned” (contractually obligated to) firms.)

Solution:

With employees being contractually bound to (“owned by”) firms, the period- t nominal profits of a firm are given by

$$PR_t = P_t f(k_t, n_t) + P_t k_t + P_t w_t n_t - P_t k_{t+1} - P_t w_t n_{t+1},$$

in which labor used in production in period t , n_t , is chosen in period $t-1$ (and thus labor used in production in period $t+1$, n_{t+1} , is chosen in period t). In analogy with our model with only capital pre-determined, the employees of a firm are a valuable “asset,” with total market value $P_t w_t n_t$ -- notice that this term enters **positively** in period t profits, rather than negatively with non-pre-determined labor. What enters negatively in period t profits here is the “purchase” of period $t+1$ labor, namely the term $-P_t w_t n_{t+1}$. In the two period model, discounted nominal profits of the firm are therefore

$$PR = P_1 f(k_1, n_1) + P_1 k_1 + P_1 w_1 n_1 - P_1 k_2 - P_1 w_1 n_2 + \frac{P_2 f(k_2, n_2)}{1+i_i} + \frac{P_2 k_2}{1+i_i} + \frac{P_2 w_2 n_2}{1+i_i} - \frac{P_2 k_3}{1+i_i} - \frac{P_2 w_2 n_3}{1+i_i}$$

The usual zero-terminal-assets condition in this case means that $k_3 = 0$ **and** $n_3 = 0$ (the latter, again, because labor should be thought of as an “asset” here). Focusing attention on the choice of n_2 (since n_1 was chosen in period $t-1$), the first-order condition of the lifetime profit function with respect to n_2 is

$$-P_1 w_1 + \frac{P_2 f_n(k_2, n_2)}{1+i_1} + \frac{P_2 w_2}{1+i_1} = 0.$$

This expression can be rearranged to yield (using the exact Fisher equation)

$$(1+r_1)w_1 = f_n(k_2, n_2) + w_2.$$

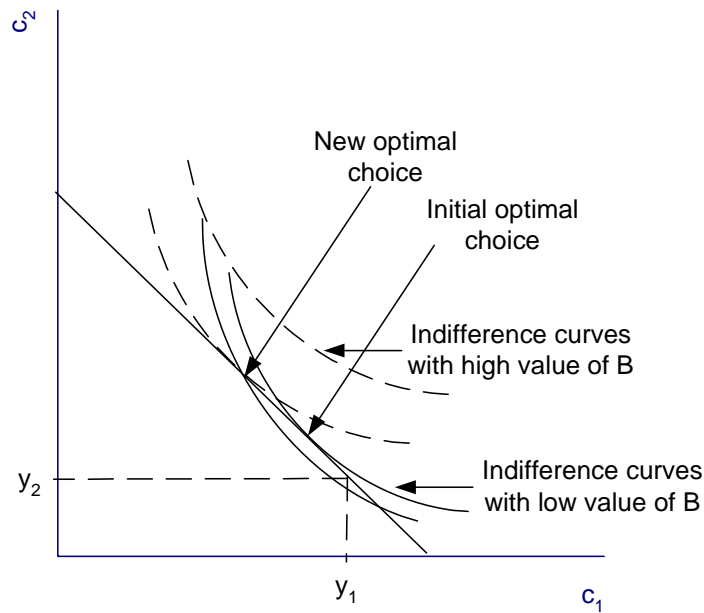
If the real wage were equal to one in each period, this condition would reduce to $r_1 = f_n(k_2, n_2)$, which would be almost identical to the condition we derived in class regarding capital demand (except of course in that case f_k is the relevant marginal product rather than f_n). The expression $r_1 = f_n(k_2, n_2)$ shows that if firms must choose labor for period 2 in period 1, the real interest rate between period 1 and period 2 is a relevant price to consider – which makes sense because there is now an interest opportunity cost associated with hiring labor (ie, “investment” in hiring).

However, in general of course w_1 and w_2 are not one, hence the above condition is not exactly the same as the capital demand condition. In the capital demand condition, the real price of capital goods is the same as the real price of consumption (which is one...) – note the discussion on p. 70-71 of the Lecture Notes describing that because capital goods and consumption goods are assumed to be the same goods (ie, computers can be viewed as both consumption goods and capital goods), the dollar price of each in our theoretical model is the same. The same is not true of labor – the nominal price of labor is W , which in general is different from P .

2. **Preference Shocks in the Consumption-Savings Model.** In the two-period consumption-savings model (in which the representative consumer has no control over his real labor income y_1 and y_2), suppose the representative consumer’s utility function is $u(c_1, Bc_2)$, where, as usual, c_1 denotes consumption in period 1, c_2 denotes consumption in period 2, and B is a preference parameter.

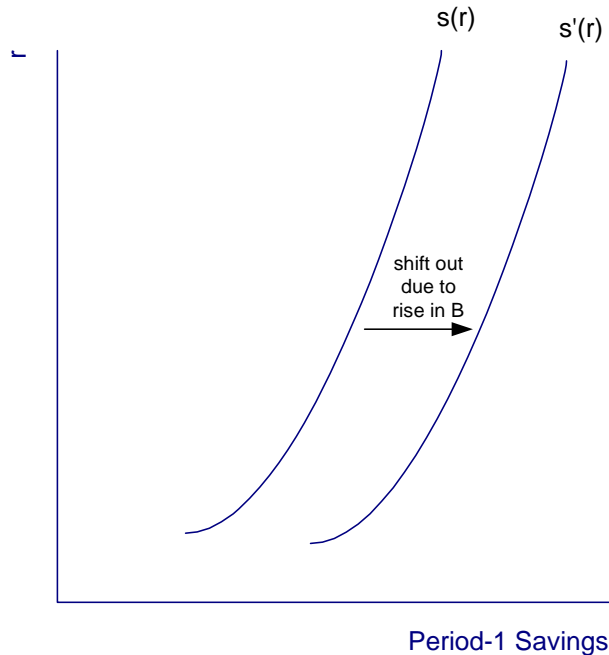
- a. Use an indifference-curve/budget-constraint diagram to illustrate the effect of an increase in B on the consumer’s optimal choice of period-1 consumption.

Solution: An increase in B means each unit of period-2 consumption delivers more utility to the consumer. Thus, in utility terms, period-2 consumption has now become more valuable relative to period-1 consumption, implying that in order to stay on a given indifference curve the consumer now needs to give up fewer units of c_2 in order to get one more unit of c_1 . In a diagram with c_2 on the vertical axis and c_1 on the horizontal axis, this is represented by a flattening of the indifference map. Because the LBC is unaffected, the flattening of the indifference map means that the new optimal choice features smaller period-1 consumption and hence larger period-2 consumption, as shown in the accompanying diagram. As drawn, consumption in period 1 is smaller than real income in period 1, but that is irrelevant.



- b. Illustrate the effect of an increase in B on the private savings function. Provide economic interpretation for the result you find.

Solution: We can deduce the effect on private savings in period 1 using the diagram in part a above. The real interest rate has not changed (in other words, the slope of the LBC has not changed), yet the representative consumer's savings in period 1 has increased. This follows directly from the observation that income y_1 is constant while consumption in period 1 falls. This result would be true for any choice of the real interest rate (in other words, no matter the slope of the LBC), hence the private savings function shifts outwards, as shown below.



- c. In the months preceding the U.S. invasion of Iraq, data shows that consumers decreased their consumption and increased their savings. Is an increase in B and the effects you analyzed in parts a and b above consistent with the idea that consumption fell and savings increased because of a looming war? If so, explain why; if not, explain why not.

Solution: Yes, these effects are consistent with developments in consumption and savings behavior in the U.S. leading up to the invasion of Iraq. An interpretation we can give using the model here is that consumers believed future macroeconomic conditions would be better than current (i.e., just before the war) macroeconomic conditions, hence a fall in consumption in the present (period 1) accompanied by a (expected) rise in consumption in the future (period 2). With B pre-multiplying consumption in the utility function (in the case here, period-2 consumption), the term B can be interpreted as a measure of “consumer confidence”: a rise in B signals that consumers are shifting their preferences towards consumption (in that period). So here, we might interpret events as consumers being more confident about the future than the present, hence they postpone some consumption until the future..

- d. Using a Lagrangian and assuming the utility function is $u(c_1, B \cdot c_2) = \ln(c_1) + \ln(B \cdot c_2)$, show how the representative consumer’s MRS (and hence optimal choices of consumption over time) depends on B .

Solution: Setting up the Lagrangian in the two-period model as always, we have

$$\ln(c_1) + \ln(B \cdot c_2) + \lambda \left[y_1 + \frac{y_2}{1+r_1} - c_1 - \frac{c_2}{1+r_1} \right],$$

in which for simplicity we have assumed the initial assets equal zero because it does not at all affect the consumption-savings optimality condition (verify this yourself). The FOCs on c_1 and c_2 are, respectively,

$$\frac{1}{c_1} - \lambda = 0$$

$$\frac{B}{B \cdot c_2} - \frac{\lambda}{1+r_1} = 0$$

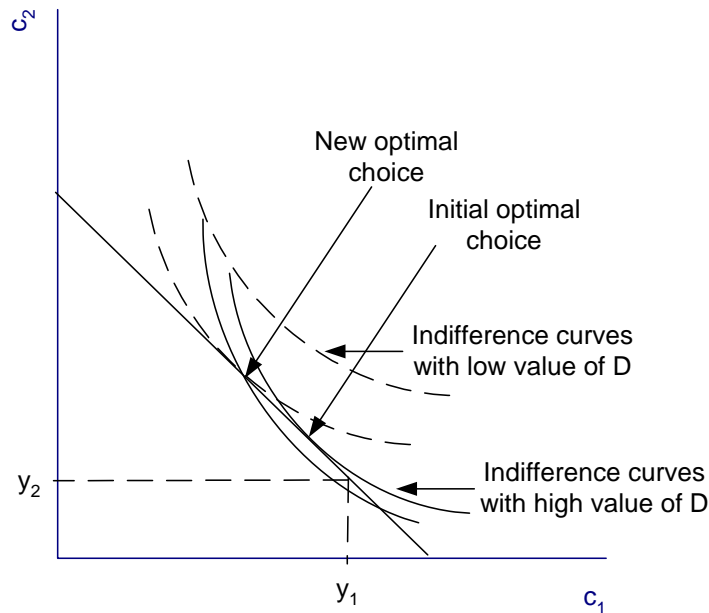
In the FOC on c_2 , note that the B term ends up canceling out (because, recall, the derivative of an expression such as $\ln(2x)$ is $2/(2x) = 1/x$). Combining these two FOCs as usual then yields that at the optimal choice,

$$\frac{1/c_1}{1/c_2} = 1 + r_1,$$

the left-hand-side of which is the intertemporal MRS, as always. Note that it is **independent** of the preference shifter B , which turns out to be a special feature of the log utility function.

- e. How would your analysis in parts a and b change if the consumer's utility function were $u(Dc_1, c_2)$ (instead of $u(c_1, Bc_2)$) and you were told that the value D decreased? (D is simply some other measure of preference shocks.)

Solution: Here, we return to a general utility specification, not necessarily log. With the utility function written as $u(Dc_1, c_2)$ and a decrease in D , the analysis above is completely unchanged. The fall in D makes consumption in period 1 less valuable in utility terms relative to period-2 consumption, which means that in order to obtain one more unit of period-2 consumption while remaining on the same indifference curve the consumer must give up more units of period-1 consumption than he had to before the fall in D . But in a diagram with c_2 on the vertical axis and c_1 on the horizontal axis, this simply means that the indifference curves become flatter, just as in part a.



This exercise cautions you to think about the underlying economics – specifically, how the consumer’s marginal rate of substitution (refer to Chapter 1) is affected – when analyzing preference shocks. We cannot make a blanket statement such as “the indifference map flattens when the measure of the preference shock increases” because it depends on exactly how we introduce the preference shock into our theoretical model. Here in part d we introduced the preference shock by attaching it to period-1 consumption, whereas earlier we introduced the preference shock by attaching it to period-2 consumption.

3. **Intertemporal Consumption-Leisure Model – A Numerical Look.** Consider the intertemporal consumption-savings model. Suppose the lifetime utility function is given by $v(B_1c_1, l_1, B_2c_2, l_2) = u(B_1c_1, l_1) + u(B_2c_2, l_2)$, which is a slight modification of the utility function presented in Chapter 5. The modification is that preference shifters B_1 and B_2 enter the lifetime utility function, with B_1 the preference shifter in period one and B_2 the preference shifter in period two. In each of the two periods the function u takes the form

$$u(B_t c_t, l_t) = 2\sqrt{B_t c_t} + 2\sqrt{l_t}.$$

Note the t subscripts -- $t = 1, 2$ depending on which period we are considering. Labor tax rates, real wages, the real interest rate between period one and period two, and the preference realizations are given by: $t_1 = 0.15$, $t_2 = 0.2$, $w_1 = 0.2$, $w_2 = 0.25$, $r = 0.15$, $B_1 = 1$, $B_2 = 1.2$. Finally, the initial assets of the consumer are zero.

Solution: Note that you needed to compute the marginal utility functions. For the given lifetime utility function, the marginal utility functions are, for $t = 1, 2$:

$$v_{c_t} = \frac{\sqrt{B_t}}{\sqrt{c_t}}; v_{l_t} = \frac{1}{\sqrt{l_t}}$$

- a. Construct the marginal rate of substitution functions between consumption and leisure in each of period one and period two (**Hint:** these expressions will be functions of consumption and leisure – you are not being asked to solve for any numerical values yet). How does the preference shifter affect this intratemporal margin?

Solution: As by now is routine, the consumption-leisure marginal rate of substitution function is $MRS_{c,l_t} = v_{l_t} / v_{c_t}$. With the given functions, the marginal rate of substitution function in period t , where t is either 1 or 2, is thus

$$MRS_{c,l_t}(c_t, l_t) = \frac{\sqrt{c_t}}{\sqrt{B_t} \sqrt{l_t}}.$$

Again, note that this function is the MRS function for period $t = 1, 2$. From this function it is clear that a rise in B_t lowers this MRS, meaning a rise in B_t flattens the indifference map over consumption and leisure within a given period.

- b. Construct the marginal rate of substitution function between period-one consumption and period-two consumption. (**Hint:** Again, you are not being asked to solve for any numerical values yet.) How do the preference shifters affect this intertemporal margin?

Solution: Again as by now should be routine, the intertemporal MRS function is given by $MRS_{c_1 c_2} = v_{c_1} / v_{c_2}$. Note the subscripts: v_{c_1} denotes the marginal utility function with respect to period-one consumption, and v_{c_2} denotes the marginal utility function with respect to period-two consumption. Using the given v_c function, we have

$$MRS_{c_1 c_2}(c_1, c_2) = \frac{\sqrt{B_1}}{\sqrt{B_2}} \cdot \frac{\sqrt{c_2}}{\sqrt{c_1}}.$$

The ratio of B values across the two periods affects the slope of the indifference map between period-one and period-two consumption. The larger is the ratio B_1 / B_2 , the steeper is the indifference map across consumption in the two periods – the interpretation of this is that the larger is B_1 relative to B_2 , the more “confident” (recall our interpretation of B from class) consumers are about the present (period one) than they are about the future (period two), hence the more period-two consumption they are willing to give up for a given increase in period-one consumption (which is our usual interpretation of the slope of an indifference curve with c_1 plotted on the horizontal axis and c_2 plotted on the vertical axis).

- c. Using the expressions you developed in parts a and b along with the lifetime budget constraint (expressed in **real** terms...) and the given numerical values, solve numerically for the optimal choices of consumption in each of the two periods and of leisure in the two periods. (**Hint:** You need to set up and solve the appropriate Lagrangian.) (**Note:** the computations here are messy and the final answers do not necessarily work out “nicely.” **To preserve some numerical accuracy, carry out your computations to at least four decimal places.**)

Solution: The LBC in real terms is

$$c_1 + \frac{c_2}{1+r} = (1-t_1)w_1(168-l_1) + \frac{(1-t_2)w_2(168-l_2)}{1+r}. \quad (0.1)$$

This expression follows readily from expression (34) on p. 60 of the Lecture Notes (it’s probably a good idea to derive this from expression (34) if you don’t see it immediately), with zero initial assets imposed. This LBC involves the four unknowns, c_1 , c_2 , l_1 , and l_2 , which are the variables you are asked to solve for. We need three other expressions involving these variables – these three are the two consumption-leisure optimality conditions (one for each of period one and period two) and the one consumption-savings optimality condition. **By now you should know how these optimality conditions can be obtained by formulating the appropriate Lagrangian** – for ease of exposition the Lagrangian is omitted here. Suffice it to say it is simply the above consumption-leisure and consumption-savings optimality conditions that emerge from the Lagrangian. The consumption-leisure optimality conditions for period one and period two and the consumption-savings optimality condition are, respectively,

$$MRS_{c_1 l_1} = \frac{\sqrt{c_1}}{\sqrt{B_1} \sqrt{l_1}} = (1-t_1)w_1, \quad (0.2)$$

$$MRS_{c_2 l_2} = \frac{\sqrt{c_2}}{\sqrt{B_2} \sqrt{l_2}} = (1-t_2)w_2, \quad (0.3)$$

$$MRS_{c_1 c_2} = \frac{\sqrt{B_1}}{\sqrt{B_2}} \frac{\sqrt{c_2}}{\sqrt{c_1}} = 1+r. \quad (0.4)$$

By now you should know the interpretation of these optimality conditions: they simply represent the tangency between a relevant budget constraint and a relevant indifference curve. Equations (0.1), (0.2), (0.3), and (0.4) are now four equations in the four unknowns c_1 , c_2 , l_1 and l_2 , so we can solve with some algebraic effort.

Let’s decide to express the unknowns c_2 , l_1 , and l_2 all in terms of c_1 . Once we have done this, we can substitute into the LBC and solve for c_1 . From (0.4), we get that

$$c_2 = \frac{B_2}{B_1}(1+r)^2 c_1; \quad (0.5)$$

from (0.2), we get that

$$l_1 = \left(\frac{1}{(1-t_1)w_1} \right)^2 \frac{1}{B_1} c_1; \quad (0.6)$$

and from (0.3) we similarly get that

$$l_2 = \left(\frac{1}{(1-t_2)w_2} \right)^2 \frac{1}{B_2} c_2. \quad (0.7)$$

In (0.7), we need to substitute out c_2 using (0.5) (because, recall, we are trying to express the unknowns in terms of c_1), giving us

$$l_2 = \left(\frac{1+r}{(1-t_2)w_2} \right)^2 \frac{1}{B_1} c_1. \quad (0.8)$$

Now, substitute into the LBC using (0.5), (0.6), and (0.8). Doing so and collecting all the resulting terms involving c_1 on the left-hand-side (you should perform these algebraic steps yourself....) gives us

$$c_1 \cdot \left[1 + \frac{B_2}{B_1}(1+r) + \frac{1}{(1-t_1)w_1 B_1} + \frac{1+r}{(1-t_2)w_2 B_1} \right] = 168(1-t_1)w_1 + \frac{168(1-t_2)w_2}{1+r}, \quad (0.9)$$

in which the only unknown, as desired, is c_1 . Inserting all of the given numerical values, we finally find that $c_1^* = 4.1233$. Then using (0.5), (0.6), and (0.8) we find $c_2^* = 6.5437$, $l_1^* = 142.6754$, and $l_2^* = 136.3272$. The individual thus works $168 - 142.6754 = 25.3246$ hours per week in the first period and $168 - 136.3272 = 31.6728$ hours per week in the second period.

- d. Based on your answer in part c, how much (in real terms) does the consumer save in period one? What is the asset position that the consumer begins period two with?

Solution: Recall that real private savings (inclusive of taxes is) income minus tax payments minus consumption. Given the solution above, total real income in period one is $(168 - l_1)w_1 = 5.0649$, of which the amount paid in taxes is $t_1(168 - l_1)w_1 = 0.7597$. Disposable income (gross income less taxes) in period one is thus $5.0649 - 0.7597 = 4.3052$. Subtracting period-one consumption, we have that real savings in period one is $4.3052 - 4.1233 = 0.1819$. Because the consumer began period one with zero assets, at the end of period one his real asset position is thus 0.1819. (Then, with positive assets to begin period two, the individual is able to consume more than his income in period two – perform this calculation to verify this for yourself.)

- e. Suppose B_2 were instead higher, at 1.6. How are your solutions in parts c and d affected? Provide brief interpretation in terms of “consumer confidence.”

Solution: Examining the solution (0.9), we see that B_2 enters the solution in only one place. It is easy to conclude from (0.9) that a higher value of B_2 will lead to a lower value of optimal period-one consumption. Specifically, $c_1^* = 3.9923$, which then implies $c_2^* = 8.4476$, $l_1^* = 138.1405$, and $l_2 = 131.9941$.

With B_2 higher relative to B_1 (and with the particular way B enters the utility function, specifically, multiplying c), the consumer is more confident about the economic state in the future (period two) than in the present (period one). He thus works and consumes less in period one, and works and consumes more in period two due to the rise in B_2 . Savings in period one rises to $(1 - t_2)w_2(168 - l_2) - c_1 = 1.0839$, consistent with the increased desire to postpone consumption until period two.