NOMINAL RIGIDITIES IN A DSGE MODEL: BASIC CALVO-YUN MODEL

OCTOBER 9, 2008

DIFFERENTIATED-GOODS FIRMS

- Optimal-pricing condition

$$E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{t+s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\left[\underbrace{\frac{\varepsilon-1}{\varepsilon}\frac{P_{it}}{P_s}}_{\text{Real marginal revenue}}-mc_s\right]\right\}=0$$

- With sticky prices, optimal P_i balances current and future marginal revenues against current and future marginal costs until the next (expected) price re-optimization

- Differentiated firm *i*'s (and hence the aggregate) markup will be time-varying
  - As "initial marginal revenues" > "initial marginal costs" to balance against "later marginal revenues" < "later marginal costs"
  - See King and Wolman (1999)

  (As inflation erodes the *relative* price of firm *i*)

- Conduct full non-linear analysis (around distorted steady state)
  - Later: typical New Keynesian analysis (around efficient steady state)

Optimal-Pricing Condition

- **Optimal-pricing condition**

$$E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{t+s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\left[\frac{\varepsilon-1}{\varepsilon}\frac{P_{it}}{P_s}-mc_s\right]\right\}=0$$

- **Define**

$$P_t x_t^1 = E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{t+s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\frac{\varepsilon-1}{\varepsilon}\frac{P_{it}}{P_s}\right\}$$

PDV of nominal marginal revenues until next price change

$$P_t x_t^2 = E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{t+s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s mc_s\right\}$$

PDV of nominal marginal costs until next price change

- **Optimal-pricing condition:** $x_t^1 = x_t^2$
  - **Emphasizes that optimal *P_i* balances current and future *mr* against current and future *mc***

- **Write x_t^1, x_t^2 recursively (following SGU (2005 *NBER Macro Annual*))**

DSGE Calvo-Yun Model

Optimal-Pricing Condition

- **Some notation and definitions**

P_{it}	Nominal price of good *i* in period *t*
$p_{it} \equiv \frac{P_{it}}{P_t}$	Relative price of good *i* in period *t*
$P_{it+1} = P_{it}$	Evolution of nominal price if no price change

Optimal-Pricing Condition

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$$\downarrow$$

$$p_{it+1} = \frac{P_{it+1}}{P_{t+1}} = \frac{P_{it}}{P_{t+1}} = \frac{P_{it}}{P_t}\frac{P_t}{P_{t+1}} = \frac{p_{it}}{\pi_{t+1}}$$

As long as no nominal price change, a firm's relative price erodes at the rate of inflation

($\pi_{t+1} = P_{t+1}/P_t$)

Optimal-Pricing Condition

- **Write x_t^1 recursively**

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$\downarrow$ Divide by P_t; write out first two terms

$$x_t^1 = \frac{P_t}{P_t}\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon}y_t\frac{\varepsilon-1}{\varepsilon} + \alpha E_t\left\{\Xi_{t+1|t}\frac{P_{t+1}}{P_t}\left(\frac{P_{it}}{P_{t+1}}\right)^{1-\varepsilon}y_{t+1}\frac{\varepsilon-1}{\varepsilon}\right\} + E_t\left\{\sum_{s=t+2}^{\infty}\alpha^{s-t}\Xi_{t+s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s mc_s\right\}$$

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↓ $P_{it+1} = P_{it}$ while no price change opportunity

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↓ Use $p_{it+1} = p_{it}/\pi_{t+1}$

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DSGE Calvo-Yun Model

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Multiply and divide by $p_{it+1}^{1-\varepsilon}$

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Have generated a recursive term

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Express recursively

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1}^{\varepsilon} \left(\frac{p_{it}}{p_{it+1}} \right)^{1-\varepsilon} x_{t+1}^1 \right\}$$

x^1 expressed recursively

OPTIMAL-PRICING CONDITION

- Both x_t^1, x_t^2 recursively

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$$x_t^2 = p_{it}{}^{-\varepsilon} y_t mc_t + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1}^{1+\varepsilon} \left(\frac{p_{it}}{p_{it+1}} \right)^{-\varepsilon} x_{t+1}^2 \right\}$$ *x^2 expressed recursively*

DSGE Calvo-Yun Model

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- Optimal-pricing condition expressed compactly

$$x_t^1 = x_t^2$$

- Now can use usual numerical solution methods

AGGREGATE PRICE LEVEL

- Aggregate price index follows from Dixit-Stiglitz aggregation

non-adjusters ↓ adjusters ↓

$$P_t^{1-\varepsilon} = \int_0^1 P_{it}^{1-\varepsilon} di = \int_0^\alpha P_{it-1}^{1-\varepsilon} di + \int_\alpha^1 P_{it}^{*1-\varepsilon} di$$

Obtained by substituting demand functions into D-S aggregator

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$$= \alpha P_{t-1}^{1-\varepsilon} + (1-\alpha) P_t^{*1-\varepsilon}$$

KEY: Because adjusters were *randomly* selected, average (aggregate) price of non-adjusters is same as previous period's average (aggregate) price

Fraction 1 - α re-set price optimally (and symmetrically)

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EQUILIBRIUM EVOLUTION OF AGGREGATE INFLATION – depends on relative price set by firms currently adjusting nominal price

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Together form the "aggregate supply" block of New Keynesian sticky-price model

$$x_t^1 = x_t^2$$

Optimal-pricing condition

PRICE DISPERSION

- Calvo model implies dispersion of relative prices
  - As does Taylor model (see Chari, Kehoe, McGrattan (2000 *Econometrica* for an example))…
  - …but not Rotemberg model (quadratic cost of nominal price adjustment)

DSGE Calvo-Yun Model

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  - Because it implies quantity dispersion across intermediate producers…
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The basic driving force of optimal policy in any NK model

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↓ Symmetric choices of k/n ratio across all firms i…

$$y_t \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di = z_t f\left(\frac{k_t}{n_t}, 1\right) \int_0^1 n_{it} di$$

PRICE DISPERSION

- For firm i, $y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} y_t = z_t f(k_{it}, n_{it})$
- Integrating over i

$$y_t \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di = z_t \int_0^1 f(k_{it}, n_{it}) di$$

↓ Symmetric choices of k/n ratio across all firms i…

$$y_t \underbrace{\int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di}_{\equiv s_t} = z_t f\left(\frac{k_t}{n_t}, 1\right) \int_0^1 n_{it} di$$

A measure of dispersion: relative price dispersion leads to dispersion of factor usage across differentiated firms, hence dispersion of quantity across differentiated firms

↓ Express s_t recursively

PRICE DISPERSION

$$s_t = \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di = \int_\alpha^1 \left(\frac{P_{it}^*}{P_t}\right)^{-\varepsilon} di + \int_0^\alpha \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di$$

↓ Re-setters all choose same price

$$= (1-\alpha)\left(\frac{P_t^*}{P_t}\right)^{-\varepsilon} + \int_0^\alpha \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di$$

DSGE Calvo-Yun Model

PRICE DISPERSION

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↓ Multiply by $(P_{t-1}/P_{t-1})^{-\varepsilon}$

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Because of Calvo *random* adjustment and *all* adjusters choose *same* price

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NOTE:

$\alpha = 0$: $s_t = 1$ (no dispersion)

$\alpha > 0$: $s_t > 1$ (dispersion)

Resource Constraint

- Summarized by *three* conditions

$$y_t = c_t + k_{t+1} - (1-\delta)k_t + g_t$$ "Usual" resource constraint

And using factor market clearing conditions here

$k_t = \int_0^1 k_{it} di, \; n_t = \int_0^1 n_{it} di$

$$y_t = \frac{z_t f(k_t, n_t)}{s_t}$$ Some output is a pure deadweight loss (note s_t < 1 cannot occur)

$$s_t = (1-\alpha) p_t^{*-\varepsilon} + \alpha \pi_t^{\varepsilon} s_{t-1}$$ Law of motion for deadweight loss

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- Summarized by *three* conditions

= 0 in Yun model

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- Law of motion for s_t represented using laws of motion for both P_{t-1} and P^*_{t-1}
  - See equations (25) and (26)

OTHER MODEL DETAILS

- Cash / credit to motivate money demand
  - i.e., Lucas and Stokey (1983), Chari, Christiano, and Kehoe (1991)

Could have been cashless…

- (Habit persistence (i.e., time-non-separability) in leisure)
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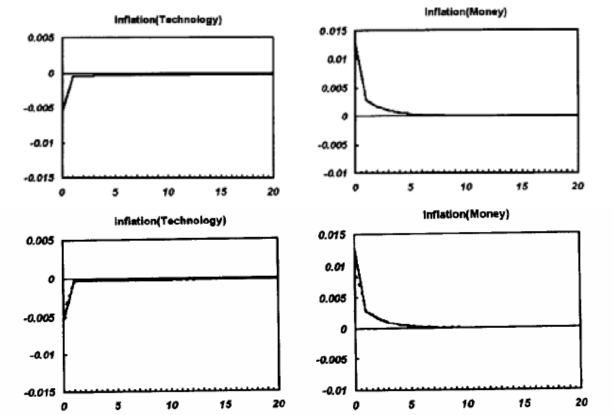
- Exogenous AR(1) TFP process
- Indexation of prices to average (i.e., steady-state) inflation
  - For firms not re-setting price, $P_{it} = \pi P_{it-1}$ (will see again in Christiano, Eichenbaum, and Evans (2005 *JPE*))
- Approximated and simulated using "usual" methods
  - Using King, Plosser, Rebelo (1988) linear approximation method
    - (One…) predecessor to SGU algorithm

Nominal Effects of Sticky Prices

- Effects on inflation not very different compared to flex-price case



Flexible Prices

Sticky Prices

Yun 1996 Model

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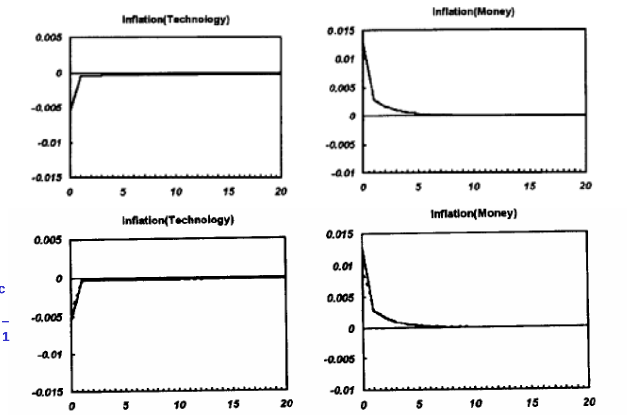


Flexible Prices

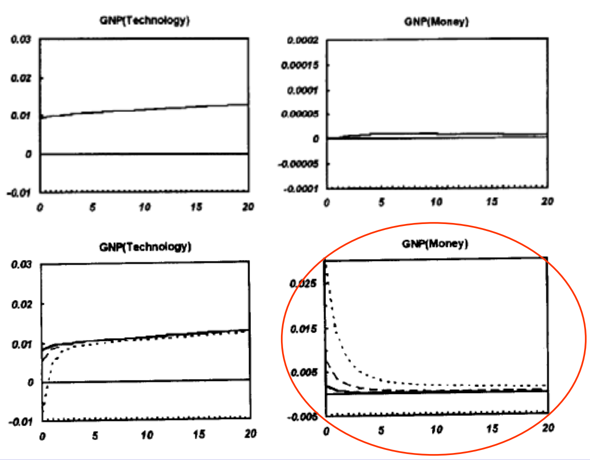
Sticky Prices

Lack of inflation persistence in basic Calvo-Yun model (now) well-known – see Steinsson (2001 *JME*)

Yun 1996 Model

Real Effects of Sticky Prices

- Effects on GDP much bigger the stickier are prices



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MARKUP DYNAMICS

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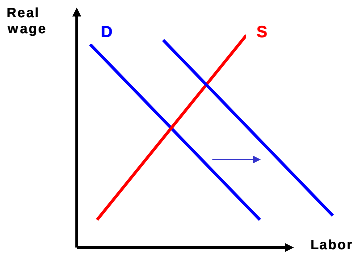

$$\frac{w_t}{z_t f_n(k_t, n_t)} = \underbrace{\frac{1}{\mu_t}}_{= mc_t}$$

Sticky-price (or sticky-wage) model delivers endogenously-countercyclical markup

DSGE Sticky-Price Models

- Nominal rigidities embedded in DSGE model
  - Monetary shifts → quantitatively "big" effects on output
  - (Re-)articulates "old" Keynesian ideas
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- A Phillips Curve?
- Optimal policy?