Labor Search Models: Basic DSGE Implementation

October 30, 2008

Firm Vacancy-Posting Problem

- **Dynamic firm profit-maximization problem**

$$\max_{v_t, n_{t+1}^f} \left[\sum_{t=0}^{\infty} \Xi_{t|0} \left(y_t - w_t n_t^f h_t - g(v_t) \right) \right]$$

Discount factor between time 0 and *t* because ***dynamic*** firm problem; in equilibrium, = household stochastic discount factor

Number of vacancies to post (how many "job advertisements")

Desired target ***future*** firm employment

Total output – sold in perfectly-competitive goods market

Total wage bill depends on both extensive and intensive employment

Total cost of posting ***v*** vacancies

- **Subject to (perceived) law of motion for firm's employment stock**

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  - Subject to (perceived) law of motion for firm's employment stock

- Baseline model
  - Shut down intensive margin: $h_t = 1$
  - Linear posting costs: $g(v) = \gamma v$
  - Firm production function: $y_t = z_t * n_t$
  - Wage-setting (process) taken as given when posting vacancies

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$$\text{s.t. } n^f_{t+1} = (1-\rho^x) n^f_t + v_t k^f(\theta_t)$$

Perceived law of motion for evolution of employment stock

Number of existing jobs that end: ρ^x exogenous separation rate, but can also endogenize

Each vacancy has probability $k^f(\theta)$ of attracting a prospective employee: depends on a ***market*** variable, θ, so taken as given

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FOCs with respect to v_t, n_{t+1}

$$-\gamma + \mu_t k^f(\theta_t) = 0$$

$$-\mu_t + E_t \left\{ \Xi_{t+1|t} \left(z_{t+1} - w_{t+1} + (1-\rho^x)\mu_{t+1} \right) \right\} = 0$$

Combine

Firm Vacancy-Posting Problem

- Vacancy posting condition (aka job creation condition)

$$\gamma = k^f(\theta_t) E_t \left\{ \Xi_{t+1|t} \left(z_{t+1} - w_{t+1} + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})} \right) \right\}$$

Cost of posting a vacancy

Expected benefit of posting a vacancy
= (probability of attracting a worker) x (expected future benefit of an additional worker)

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γ / k^f is capital value of an existing employee – because one *less* worker firm has to find in the future

EMPLOYEES ARE ASSETS

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  - **Intertemporal dimension makes discount factor potentially important**
    - **i.e., makes general equilibrium effects potentially important**

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- Vacancy-posting is a type of investment decision
  - Intertemporal dimension makes discount factor potentially important
    - i.e., makes general equilibrium effects potentially important
- Two prices affect posting decision (aside from intertemporal price)
  - (Future) wage
  - Matching probability (can often interpret probabilities as prices) which depends on the market variable θ

HOUSEHOLD PROBLEM

- Dynamic household utility-maximization problem
  - A measure [0, 1] of households (a standard assumption)
  - A measure [0, 1] of atomistic individuals live in each household
  - Thus representative household has a continuum of "family members"

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$$\text{s.t. } c_t + a_t = n_t w_t h_t + (1-n_t)b + R_t a_{t-1}$$

An (arbitrary) asset to make pricing interest rates explicit

Wage (-setting process) taken as given by household

Measure n_t of family members earn labor income (because they work) (and recall we've normalized h = 1)

Measure 1-n_t of family members receive unemployment benefits and/or engaged in home production

DSGE Labor Search Model

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Thus individual family members are risk-neutral with respect to their labor-market realization

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  - Each family member either works or is looking for work

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Wage Bargaining

- (Generalized) Nash Bargaining

$$\max_{w_t} \underbrace{\left(W(w_t)-U(w_t)\right)}^{\eta}\underbrace{\left(J(w_t)-V(w_t)\right)}^{1-\eta}$$

Bargaining over how to divide the surplus

Net payoff to an individual/household of agreeing to wage *w* and beginning production

Net payoff to a firm of agreeing to wage *w* and beginning production

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- Asset values
  - ***W***: value to (representative) household of having one additional member employed
  - ***U***: value to (representative) household of having one additional member unemployed and searching for work
  - ***J***: value to (representative) firm of having one additional employee
  - ***V***: value to (representative) firm of having a job that goes unfilled
    - Free entry in vacancy-posting → ***V*** = 0
- Will define ***W*** and ***U*** in terms of household primitives
  - i.e., based on household value equation

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- The Nash surplus-sharing rule

$$\eta\left(W'(w_t) - U'(w_t)\right) J(w_t) = (1-\eta)(-J'(w_t))\left(W(w_t) - U(w_t)\right)$$

(FOC with respect to w_t)

DSGE Labor Search Model

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    - (Most) labor search models
    - (Most) money search models
    - Political bargaining games (Albanesi 2007 ***JME***)

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- Must specify value equations ***W***(.), ***U***(.), ***J***(.)

Value Equations

- Individual/household value equations (constructed from household problem)

$$W(w_t)=w_t+\beta E_t\left\{(1-\rho^x)W(w_{t+1})+\rho^x U(w_{t+1})\right\}$$

Value to household of having the marginal individual employed

Contemporaneous return is wage

Expected future return takes into account transition probabilities

$$U(w_t)=b+\beta E_t\left\{k^h(\theta_t)W(w_{t+1})+(1-k^h(\theta_t))U(w_{t+1})\right\}$$

Value to household of having the marginal individual unemployed and searching

Contemporaneous return is unemployment benefit/home production

Expected future return takes into account transition probabilities

VALUE EQUATIONS

- **Individual / household value equations (constructed from household problem)**

Each searching individual has probability $k^h(\theta)$ of finding a job opening: depends on a *market* variable, θ, so taken as given

$$W(w_t) = \underbrace{w_t}_{\text{Contemporaneous return is wage}} + \beta E_t \underbrace{\left\{(1-\rho^x)W(w_{t+1}) + \rho^x U(w_{t+1})\right\}}_{\text{Expected future return takes into account transition probabilities}}$$

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Value to household of having the marginal individual unemployed and searching

- **Firm value equation**

$$J(w_t) = \underbrace{z_t - w_t}_{\text{Contemporaneous return is marginal output net of wage payment}} + E_t \underbrace{\left\{\Xi_{t+1|t}(1-\rho^x)J(w_{t+1})\right\}}_{\text{Expected future return takes into account transition probabilities}}$$

Value to firm of the marginal employee

WAGE BARGAINING

- **The Nash surplus-sharing rule**

$$\eta\left(W'(w_t)-U'(w_t)\right)J(w_t)=(1-\eta)(-J'(w_t))\left(W(w_t)-U(w_t)\right)$$

(FOC with respect to w_t)

↓ **Insert marginal values**

$$\eta J(w_t)=(1-\eta)\left(W(w_t)-U(w_t)\right)$$

Firm's surplus *J* a constant fraction of household's surplus *W – U*

NOTE: NOT a general property of Nash bargaining; here due to the linearity of *W, U,* and *J* with respect to wage

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↓ **Using definitions of *W, U,* and *J,* and some algebra**

$$w_t=\eta\left[z_t+\gamma\theta_t\right]+(1-\eta)b$$

Bargained wage a convex combination of gains from consummating the match and the gains from walking away from the match

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Gains from production depend on market outcomes through θ

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Labor Market Matching

- **Aggregate matching function displays CRS**

$$m(u_t, v_t)$$

$u_t = 1 - n_t$ is measure of individuals searching for work

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- For any given individual vacancy or individual (partial equilibrium), matching probabilities depend only on ***v* / *u***

$$\frac{m(u_t,v_t)}{v_t} = m\left(\frac{u_t}{v_t},1\right) = m\left(\theta_t^{-1},1\right) \equiv k^f(\theta_t)$$

Probability a given vacancy / job posting attracts a worker

$$\frac{m(u_t,v_t)}{u_t} = m\left(1,\frac{v_t}{u_t}\right) = m\left(1,\theta_t\right) \equiv k^h(\theta_t)$$

Probability a given individual finds a job opening

$$\theta_t \equiv \frac{v_t}{u_t}$$

Market tightness: measures relative number of traders on opposite sides of market

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Probability a given individual finds a job opening

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- Market tightness an allocational signal
  - Because matching probabilities depend on it
  - e.g., the higher (lower) is ***v* / *u*,** the easier (harder) it is for a given individual to find a job opening

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In matching models, θ is the key driving force of efficiency and therefore optimal policy prescriptions (Hosios 1990 *ReStud* the key reference)

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Labor-Market Equilibrium

- Aggregate law of motion of employment

$$N_{t+1} = (1-\rho^x)N_t + m(u_t, v_t)$$

- Flow equilibrium conditions (an accounting identity…)

$$m(u_t, v_t) = u_t k^h(\theta_t) = v_t k^f(\theta_t)$$

- Vacancy-posting (aka job-creation) condition

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- **Basic labor-theory literature: impose ss on these and analyze, do comparative statics, etc. (exogenous real interest rate)**
  - **Pissarides Chapter 1, RSW 2005 *JEL***

GENERAL EQUILIBRIUM

- **Aggregate law of motion for employment**
- **Vacancy-posting (aka job-creation) condition**
- **Wage determination**

} The labor market equilibrium (*partial* equilibrium from the perspective of the entire environment)

- **Consumption-savings optimality condition (endogenizes real interest rate)**

$$1 = R_t E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \right\}$$

- **Aggregate resource constraint**

$$c_t + g_t + \gamma v_t = z_t N_t h_t + (1-N_t)b$$

Vacancy posting costs and "outside option" are real uses of resources

- **Exogenous LOMs for TFP and any other driving processes…**

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- **Consumption-savings optimality condition (endogenizes real interest rate)**

$$1 = R_t E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \right\}$$

- **Aggregate resource constraint**

Often interpreted as the output of a home production sector – only the unemployed produce in the home sector

$$c_t + g_t + \gamma v_t = z_t N_t h_t + (1 - N_t) b$$

Vacancy posting costs and "outside option" are real uses of resources

- **Exogenous LOMs for TFP and any other driving processes…**

STEADY STATE OF LABOR MARKET

- **Imposing deterministic steady state on labor-market equilibrium conditions**

(1) $$1 - u = (1 - \rho^x)(1 - u) + m(u, v)$$ (using $N = 1 - u$)

(2) $$\gamma = \beta k^f(\theta) \left(z - w + \frac{(1 - \rho^x)\gamma}{k^f(\theta)} \right)$$

(3) $$w = \eta \left[z + \gamma\theta \right] + (1 - \eta) b$$

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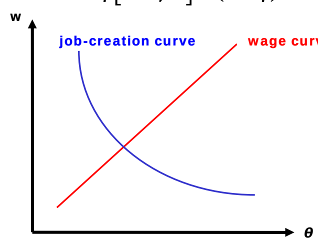
Pissarides Figure 1.1



"Labor supply curve" and "labor demand curve" replaced by "wage curve" and "job-creation curve"

The relevant "quantity" variable θ – but can also think of θ as a "price" because it governs matching probabilities…

STEADY STATE OF LABOR MARKET

- **Imposing deterministic steady state on labor-market equilibrium conditions**

(1)
$$u = \frac{m(u,v)+\rho^x}{\rho^x}$$

For a given (w,θ), v and u negatively related (given CRS matching function)

(2)
$$\gamma = \beta k^f\left(\frac{v}{u}\right)\left(z - w + \frac{(1-\rho^x)\gamma}{k^f\left(\frac{v}{u}\right)}\right)$$

For a given (w,θ), v and u positively related (given CRS matching function)

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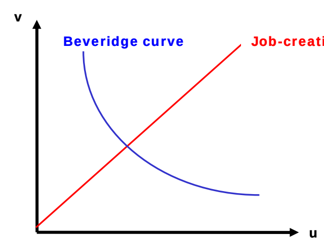
Pissarides Figure 1.2



BEVERIDGE CURVE: Empirical relationship in both long run and short run (i.e., cyclical)

Steady State of Labor Market

- **Labor-market equilibrium is (*w, u, θ*) satisfying (1), (2), (3)**

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  - **A rise in *b*…**
    - **…raises *w***
    - **…lowers *θ***
    - **…lowers *v* and raises *u***

Higher value (ue benefit) of unemployment requires a higher wage to induce individuals to work, which reduces firm incentives to create jobs

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  - **A fall in β (or a rise in ρ^x)…**
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    - **…lowers θ**
    - **…raises u**
    - **…ambiguous effect on v**

    Higher real rate and / or faster job separations (i.e., "faster depreciation of employment stock") makes posting jobs (FOR FIXED u) less attractive for firms (both erode firm profits)

Long-Run Analysis

STEADY STATE OF LABOR MARKET

- **Labor-market equilibrium is (w, u, θ) satisfying (1), (2), (3)**
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- **See Pissarides Chapter 1 and RSW (2005 *JEL*) for more**
- **Next: dynamic stochastic partial equilibrium (Shimer 2005, Hall 2005, and Hagedorn and Manovskii 2008)**