

LABOR SEARCH MODELS: PARTIAL-EQUILIBRIUM DYNAMICS

NOVEMBER 6, 2008

DSGE Labor Search Model

LABOR-MARKET EQUILIBRIUM

- **Aggregate law of motion of employment**

$$N_{t+1} = (1 - \rho^x)N_t + m(u_t, v_t)$$

- **Flow equilibrium conditions (an accounting identity...)**

$$m(u_t, v_t) = u_t k^h(\theta_t) = v_t k^f(\theta_t)$$

- **Vacancy-posting (aka job-creation) condition**

$$\gamma = k^f(\theta_t) E_t \left\{ \Xi_{t+1|t} \left(z_{t+1} - w_{t+1} + \frac{(1 - \rho^x)\gamma}{k^f(\theta_{t+1})} \right) \right\}$$

- **Wage determination**

$$w_t = \eta [z_t + \gamma \theta_t] + (1 - \eta)b$$

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- **Wage determination**

$$w_t = \eta [z_t + \gamma \theta_t] + (1 - \eta)b$$

- **Shimer (2005) and Hall (2005): analyze the stochastic dynamics of the labor market equilibrium**

- **Not** general equilibrium dynamics

Does a good job explaining long-run (steady-state) phenomena

BASIC ISSUES AND RESULTS

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 - Unemployment u_t (equivalently, $N_t = 1 - u_t$)
 - Vacancies v_t
 - Labor-market tightness θ_t

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- **Main Model Shortcoming:** the wage-setting process (i.e., assumption of Nash bargaining)
 - Exogenous rise in productivity is nearly-fully absorbed by a rise in the wage → virtually no change in firms' incentives to post vacancies
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Recall $z_{t+1} - w_{t+1}$ the (future) payoff governing vacancy-posting decision

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EMPIRICAL FACTS

Basic cyclical labor-market facts

Data Sources:
CPS, JOLTS,
and
Conference
Board

TABLE 1—SUMMARY STATISTICS, QUARTERLY U.S. DATA, 1951–2003

	u	v	du	f	s	p	
Standard deviation	0.190	0.202	0.382	0.118	0.075	0.020	
Quarterly autocorrelation	0.936	0.940	0.941	0.908	0.733	0.878	
Correlation matrix	u	1	-0.894	-0.971	-0.949	0.709	-0.408
	v	—	1	0.975	0.897	-0.684	0.364
	du	—	—	1	0.948	-0.715	0.396
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Data displays a cyclical Beveridge Curve

Labor-market tightness θ

Worker matching rate $k^b(\theta)$

Extremely high correlation consistent with basic labor-matching model (in which k^b depends on *only* θ)

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Estimated matching function elasticity $m(u_i, v_i) = u_i^\alpha v_i^{1-\alpha} : \alpha = 0.72$

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Annotations:
 - Data displays a cyclical Beveridge Curve (arrow pointing to -0.894)
 - Labor-market tightness θ (arrow pointing to 0.202)
 - Worker matching rate $k^b(\theta)$ (arrow pointing to 0.897)
 - Extremely high correlation consistent with basic labor-matching model (in which k^b depends on only θ) (arrow pointing to 0.948)

Estimated matching function elasticity $m(u_t, v_t) = u_t^\alpha v_t^{1-\alpha} : \alpha = 0.72$

Question: How well can stochastic dynamic (partial-equilibrium) labor-search model match key labor-market business cycle facts?

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MODEL DETAILS

Exogenous processes

- Labor productivity, z
- Separation rate, ρ^x
- (Markov processes, continuous time \rightarrow can re-cast as AR(1)'s in discrete time)

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MODEL DETAILS

- **Exogenous processes**
 - Labor productivity, z
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- **Calibration**
 - Mean productivity $z = 1$ (normalization)
 - Implies real wage $\approx z$ (but $< z$ because of posting costs)
 - Worker Nash bargaining power $\eta = 0.72$ ($= a$)
 - Satisfies Hosios (1990 *ReStud*) condition for search efficiency
 - Mean quarterly separation rate $\rho^x = 0.1$
 - Unemployment benefit $b = 0.4$
 - Replacement rate about 40 percent of labor income
 - But also measures flow value of leisure/home production
 - A critical parameter (Hagedorn and Manovskii (2008))
 - (Vacancy-posting cost γ and matching-function constant highly model-specific)

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 - (Vacancy-posting cost γ and matching-function constant highly model-specific)
- Accounting profit $z - w$ each period

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MODEL SIMULATION RESULTS

☐ Productivity shocks alone

Model displays a cyclical Beveridge Curve

TABLE 3—LABOR PRODUCTIVITY SHOCKS

	u	v	v/u	f	p
Standard deviation	0.009 (0.001)	0.027 (0.004)	0.035 (0.005)	0.010 (0.001)	0.020 (0.003)
Quarterly autocorrelation	0.949 (0.018)	0.835 (0.045)	0.878 (0.035)	0.878 (0.035)	0.878 (0.035)
	u	1	-0.927 (0.020)	-0.958 (0.012)	-0.958 (0.012)
	v	—	1	0.996 (0.001)	0.996 (0.001)
Correlation matrix	v/u	—	—	1	0.999 (0.001)
	f	—	—	—	1 (0.001)
	p	—	—	—	— 1

All much lower than the data

MODEL SIMULATION RESULTS

☐ Separation-rate shocks alone

Model fails to display a cyclical Beveridge Curve

TABLE 4—SEPARATION RATE SHOCKS

	u	v	v/u	f	s
Standard deviation	0.063 (0.007)	0.059 (0.006)	0.006 (0.001)	0.002 (0.000)	0.075 (0.007)
Quarterly autocorrelation	0.864 (0.026)	0.862 (0.026)	0.732 (0.048)	0.732 (0.048)	0.733 (0.048)
	u	1	0.999 (0.000)	-0.906 (0.017)	-0.906 (0.017)
	v	—	1	-0.887 (0.020)	-0.887 (0.021)
Correlation matrix	v/u	—	—	1	0.999 (0.000)
	f	—	—	—	1 (0.000)
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	<i>v</i>	—	1	-0.887 (0.020)	-0.887 (0.020)	0.888 (0.021)
	<i>cu</i>	—	—	1	1.000 (0.000)	-0.999 (0.000)
	<i>f</i>	—	—	—	1	-0.999 (0.000)
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- Proceeds to dismiss fluctuations in separation rate
- A point of controversy – see Fujita and Ramey (2007)

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MODEL MECHANISM

- Consider a single vacancy-posting decision

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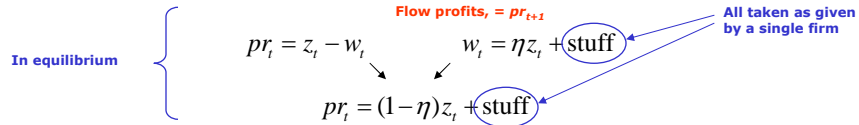
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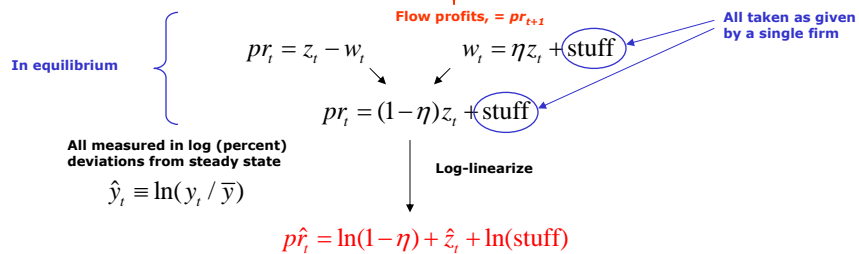
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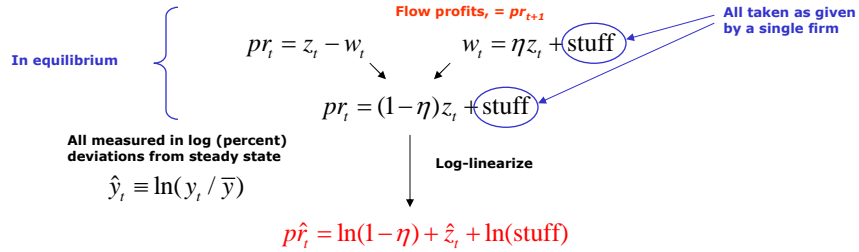
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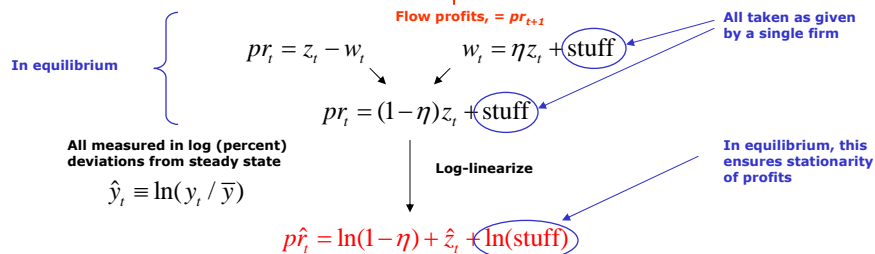


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- High η (worker Nash weight) → most of higher z **not** transmitted to higher pr

$$p\hat{r}_i \approx \ln(1-\eta) + \hat{z}_i$$

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- Interpretation of Shimer (2005) result
 - Wages absorb "too much" of any change in productivity
 - → not much change in firms' vacancy posting incentives
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- The Shimer Puzzle
 - How to address the model shortcoming?
 - Not a criticism of the labor search structure per se – a criticism of the wage-setting mechanism (Nash) used in the model

BEYOND THE BASIC MODEL

- Hall (2005): a “social norm” under which w doesn’t change in response to cyclical fluctuations
 - Permissible as an equilibrium DUE TO the “bargaining interval” between z and b
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- **Key insight: in basic RBC model, “gap” between social value of market work (z) and value of non-market activity (b) equals ZERO**

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- **Hagedorn and Manovskii (2008)**
 - Use data on vacancy posting costs
 - Use data on elasticity of wages with respect to productivity
 - (Recall from basic RBC: quite low)
 - Consider effects of taxes (which affects the receipt of labor income by households)

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$\eta = 0.05$
(much smaller than Shimer and
typical labor literature)

↓ Back out values of η (worker Nash bargaining weight) and b (flow value of unemployment)

$b = 0.95$
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Table 4: Results from the Calibrated Model.

Simulations of the Hagedorn and Manovskii calibration: matches data well

	u	v	v/u	p
Standard Deviation	0.145	0.169	0.292	0.013
Quarterly Autocorrelation	0.830	0.575	0.751	0.765
	u	1	-0.866	-0.966
	v	—	1	0.966
Correlation Matrix	v/u	—	—	1
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High value of b more important than low value of η because it essentially "makes wages rigid"/makes profits more responsive to z shocks

But low η helps too...

FULL MACRO MODELS

- **Full DSGE macro models that take on the Shimer Puzzle**
 - Krause and Lubik (2005): job-to-job transitions
 - Gertler and Trigari (2006): "staggered (Calvo) Nash bargaining"
 - Rotemberg (2006): monopolistic competition and markup shocks
 - Acemoglu and Hawkins (2006): Shapley-value as model of bargaining
 - Krusell et al (2007), Nakajima (2007): heterogenous risk-averse households (hence no consumption insurance)
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- **Pre-Shimer: the effects of labor matching frictions on basic RBC model dynamics?**
 - Andolfatto (1996 AER)
 - Merz (1995 JME)
 - den Haan, Ramey, Watson (2000 AER)