LABOR SEARCH MODELS: EFFICIENCY PROPERTIES

NOVEMBER 13, 2008

LABOR-MATCHING EFFICIENCY

- Social Planning problem
  - Social Planner also subject to matching "technology"

$$\max_{c_t, v_t, N_{t+1}} \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

$$c_t + g_t + \gamma v_t = z_t N_t h_t + (1 - N_t) b$$ Fix $h = 1$

$$N_{t+1} = (1 - \rho^x) N_t + m(u_t, v_t)$$ And $N = 1 - u$

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Efficiency Considerations

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Multipliers

$$c_t + g_t + \gamma v_t = z_t N_t h_t + (1 - N_t) b \qquad \text{Fix } h = 1 \qquad \lambda_t$$

$$N_{t+1} = (1 - \rho^x) N_t + m(1 - N_t, v_t) \qquad \text{And } N = 1 - u \qquad \mu_t$$

  - FOCs

$$u'(c_t) - \lambda_t = 0$$

$$-\lambda_t \gamma + \mu_t m_2(1 - N_t, v_t) = 0$$

$$-\mu_t + \beta E_t \left\{ \lambda_{t+1} \left[z_{t+1} - b \right] \right\} + \beta E_t \left\{ \mu_{t+1} \left[(1 - \rho^x) - m_1(1 - N_t, v_t) \right] \right\} = 0$$

↓ Eliminate multipliers

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$$\frac{\gamma}{m_2(1-N_t,v_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[z_{t+1} - b - \frac{\gamma m_1(1-N_t,v_t)}{m_2(1-N_t,v_t)} + \frac{(1-\rho^x)\gamma}{m_2(1-N_t,v_t)}\right]\right\}$$

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Cobb-Douglas matching

$m(u,v) = u^{\alpha}v^{1-\alpha}$

$m_1(u,v) = \alpha u^{\alpha-1}v^{1-\alpha} = \alpha\theta^{1-\alpha}$

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$k^h(\theta) = \frac{m(u,v)}{u} = m(1,\theta) = \theta^{1-\alpha}$

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KEY IDEAS

Taking the pricing kernel as given, the only unknown process here is θ_t!

Efficiency in job-postings is governed by "getting market tightness right!"

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- **Efficiency in vacancy posting requires $\eta = \alpha$!**

HOSIOS CONDITION

- Cobb-Douglas matching technology + Nash bargaining
  - Pareto-optimal level of job-creation requires $\eta = \alpha$
  - Hosios (1990 *ReStud*)

Efficiency Considerations

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- **Efficiency requires equating private and social returns: $\eta = \alpha$**

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- **Also holds under some more general conditions**
  - **Endogenous search intensity**
  - **Endogenous "vacancy posting intensity" (Pissarides Chapter 5)**
- **Pissarides (2000, p. 198): "..we are not likely to find intuition for it…"**
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- Hosios efficiency emerges endogenously in competitive search equilibrium concept
  - Moen (1997 *JPE*): basic static partial labor search model
  - A well-understood concept in labor theory, but little incorporation into DSGE models

COMPETITIVE SEARCH EQUILIBRIUM (CSE)

- Question: can a "competitive" notion of wage-setting be entertained in a search and matching model?
  - Would get away from the non-genericity of the Hosios bargaining parameterization
  - May be apriori an appealing way of describing labor markets
    - Locating a firm or a worker is costly and time-consuming…
    - …but once matched, wages are more or less determined by "market forces," perhaps with little / no room for "bargaining"

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- Moen (1997 *JPE*) and Shimer (1996) the original implementations of CSE
  - Static partial-equilibrium labor search models

- Will implement in the context of our full DSGE labor-search model
  - Arseneau and Chugh (2007) show how to model in full DSGE framework (goods-search model)

CSE – Basics of Environment

- Need "many markets" and "many firms"
  - To rationalize "competition," so can operationalize decentralized wage-formation process
- Index continuum of labor "submarkets" by *j* – e.g., local labor markets
- Within a submarket *j*, many firms looking to hire workers
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- Idea of firm wage-posting / wage-announcement implementation
  - Define (expected) payoff function to firm *ij* of finding an additional worker
  - Define (expected) payoff function to individual searching for / applying to a job at firm *ij*
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    - i.e., firm *internalizes* the effect of wages on the other side of the market…
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- Internalizing congestion externalities would also be achieved by…
  - Individuals announcing wages taking into account reactions by firms
  - "Market maker" calling out wages taking account reactions by both sides of market

CSE – Implementation

- Firm ***ij*** payoff function described by vacancy-posting decision!

$$\gamma = k^f(\theta_{ijt})E_t\left\{\Xi_{t+1|t}\left(z_{t+1} - w_{ijt+1} + \frac{(1-\rho^x)\gamma}{k^f(\theta_{jt+1})}\right)\right\}$$

Cost of posting a vacancy

Expected benefit of posting a vacancy
= (probability of matching with a worker) x (expected future benefit of an additional worker)

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Note *ij* subscripts:

Matching probability depends on tightness of "applications" at firm *ij*...

...but future asset value of employee depends on market *j* conditions (i.e., replacement value depends on (sub-)market conditions)

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$$U(w_t) = b + \beta E_t\left\{k^h(\theta_t)W(w_{t+1}) + (1-k^h(\theta_t))U(w_{t+1})\right\}$$

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With probability $1-k^h(\theta_{ijt})$, individual gets this payoff

- With individuals (households) optimally directing their search, the expected payoff of searching for / applying to a job at firm ***ij*** is

$$k^h(\theta_{ijt}) W(w_{ijt}) + (1-k^h(\theta_{ijt})) U(w_t) = \underbrace{X}$$

Payoff of searching at another firm or another submarket independent of *ij*

CSE – IMPLEMENTATION

- Firm *ij* maximizes

$$\gamma = k^f(\theta_{ijt})E_t\left\{\Xi_{t+1|t}\left(z_{t+1} - w_{ijt+1} + \frac{(1-\rho^x)\gamma}{k^f(\theta_{jt+1})}\right)\right\}$$

taking as constraint

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- First-order conditions

$$-k^f(\theta_{ijt}) - \varphi_{ijt}k^h(\theta_{ijt})W'(w_{ijt}) = 0$$

$$\frac{\partial k^f(\theta_{ijt})}{\theta_{ijt}}E_t\left\{\Xi_{t+1|t}\left(z_{t+1} - w_{ijt+1} + \frac{(1-\rho^x)\gamma}{k^f(\theta_{jt+1})}\right)\right\} - \varphi_{ijt}\frac{\partial k^h(\theta_{ijt})}{\theta_{ijt}}\left[W(w_{ijt}) - U(w_t)\right] = 0$$

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= $J(\theta_{ijt})$ if firms are optimizing

Cobb-Douglas matching

$m(u,v)=u^\alpha v^{1-\alpha}$

$$k^h(\theta)=\frac{m(u,v)}{u}=m(1,\theta)=\theta^{1-\alpha}$$

$$k^f(\theta)=\frac{m(u,v)}{v}=m(\theta^{-1},1)=\theta^{-\alpha}$$

AND

$$\frac{\partial k^h(\theta)}{\partial\theta}=(1-\alpha)\theta^{-\alpha}$$

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Combine and rearrange

Efficiency Considerations

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Exactly the Nash-bargaining sharing rule with endogenous emergence of Hosios condition ($\eta = \alpha$)!!!

Inserting value equations and solving explicitly for wage obviously gives same outcome as Nash-bargained wage with $\eta = \alpha$...

CSE – INTEREPRETATIONS

- Mortensen and Pissarides (1999 ***Handbook Chapter*** p. 2589-2592)
  - "Price of time" priced efficiently by markets in CSE
  - "Price of time" generically mispriced in bargaining equilibrium
  - ("Price of time" = matching probabilities, which reflect congestion externalities)

  - Bargaining equilibrium features a particular type of market incompleteness: workers and firms cannot contract on efficient surplus sharing before meeting
  - CSE effectively fills in this missing market…

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    - (i.e., each side of a job-match would have an incentive to try to "renegotiate" the "posted" wage once they actually meet)
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    - (i.e., each side of a job-match would have an incentive to try to "renegotiate" the "posted" wage once they actually meet)
    - An open question in search theory
- CSE in principle an alternative equilibrium concept in search models
  - But turns out to be equivalent to bargaining equilibrium with Hosios condition
  - (At least in simple environments….will equivalence hold in richer environments?...)
- Little explored in DSGE contexts
  - Question: Would some types of market frictions, tax issues, etc break the equivalence between CSE and Nash-Hosios bargaining?...

Relevance of Hosios Condition in DSGE

- Optimal policy (monetary and/or fiscal) will depend on whether or not $\eta = \alpha$
  - Yet another distortion (if $\eta = \alpha$ not satisfied) for policy to respond to
  - Deviation from Friedman Rule can be used to correct search externalities (Cooley and Quadrini (2004 *JET*), Arseneau and Chugh (2008 *JME*), Faia (2008 *JEDC*))

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- Model dynamics can depend (noticeably) on whether or not $\eta = \alpha$
  - Walsh (2005 *RED*)
- Hosios issues arise in any DGE model with any type of search market
  - Money search models
    - Rocheteau and Wright (2005 *Econometrica*)
    - Aruoba and Chugh (2007)
  - Product search models
    - Hall (2007)
    - Arseneau and Chugh (2007)

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Summary

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  - An appealing description of labor markets
  - Maybe of other markets also
- Tractable to incorporate in DSGE models because of assumption of aggregate matching function
- Too ad-hoc or "reduced-form" because of assumption of (black box) aggregate matching friction?
- The Shimer Puzzle and attempted answers continue…
- …as do New Keynesian modelers' incorporation of labor search structure
  - Provides "cover" for talking "meaningfully" about the tradeoffs between inflation and unemployment…
  - …i.e., seemingly resuscitates the original Phillips Curve, not the NK Phillips Curve (which links inflation to marginal costs…)