OPTIMAL MONETARY POLICY: STICKY WAGES AND THE LINEAR-QUADRATIC APPROACH

NOVEMBER 20, 2008

THE THREE-EQUATION MODEL

- The basic framework

Phillips Curve / Aggregate Supply

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t + u_t$$

IS Curve / Aggregate Demand

$$\hat{y}_t = -\frac{1}{\sigma}\left[\hat{R}_t - E_t \hat{\pi}_{t+1}\right] + E_t \hat{y}_{t+1} + g_t$$

Monetary Policy (interest rate) Rule

$$\hat{R}_t = \delta_\pi \hat{\pi}_t + \delta_y \hat{y}_t$$

The Baseline NK Model

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Optimal Policy in the Baseline NK Model

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  - **Optimize criterion function subject to Phillips Curve and IS curve**
- **What is the correct / natural / interesting / relevant criterion function?**

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- **Optimal policy problem**
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- **What is the correct / natural / interesting / relevant criterion function?**
  - **Typical NK criterion: expected lifetime utility of the representative agent expressed as a second-order approximation → yields a central bank loss function quadratic in inflation gaps and output gaps**

DERIVING CENTRAL BANK OBJECTIVE

- **Lifetime household utility**

Set X = 0 for "cashless" economy

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(n_t) + \chi w(M_t / P_t)\right]$$

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Expand around PARETO-EFFICIENT steady-state (c, n)

Pareto-efficient steady-state "achieved" by assuming

- Long-run inflation = 0

- Sufficient fiscal instruments exist to correct long-run distortions due to monopoly power (in particular, a proportional subsidy to labor income to undo the effects of an inefficient real wage, financed with lump-sum tax)

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…many many many more steps… (see Woodford and Rotemberg 1998 *NBER* Appendix)

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Central Bank Loss Function – depends on only output gaps and inflation gaps

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Unconstrained optimum clearly involves setting all inflation gaps and output gaps to zero: achieves zero loss

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Optimal Policy Problem

- **Minimize** $$E_0 \sum_{t=0}^{\infty} \beta^t \left[\hat{\pi}_t^2 + \hat{y}_t^2 \right]$$

  subject to implementation as a (sticky-price) private-sector equilibrium

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A LINEAR-QUADRATIC PROBLEM

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- **Assume**
  - **Commitment**
  - **Timeless perspective**

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- Assume
  - Commitment
  - Timeless perspective
- Choose δ_π and δ_y to minimize monetary-authority loss function
  - For given processes for the u_t and g_t shocks
  - Analytical results sometimes feasible – because LQ structure
  - Bigger models rely on computational methods

Optimal Policy Results

- Basic features of the optimal policy
  - $\delta_\pi > 1$ – the Taylor principle, ensures determinate equilibrium
    - $\delta_\pi < 1$ recalls the "exogenous-interest-rate-rule indeterminacy" result of Sargent and Wallace (1975)
  - Stabilize inflation in the face of business-cycle magnitude shocks (u_t and g_t)
    - Nearly completely if u_t shocks
    - Completely if no u_t shocks – the King and Wolman (1999) result
    - $\delta_y = 0$ if no u_t shocks (divine coincidence)

See Woodford text (p. 44-55)

OPTIMAL POLICY RESULTS

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- Basic intuition
  - Concavity/symmetry of intermediate-goods aggregator implies output should be (nearly) equated across intermediate firms
  - → Relative prices should be (nearly) equated across intermediate goods
  - → With sticky prices, requires (near-complete) stabilization of nominal price level

OPTIMAL POLICY: FURTHER ISSUES

- Richer interest-rate rules

Data suggests high persistence of monetary-policy interest rates – thus allow for (optimal) "interest-rate smoothing"

$$\hat{R}_t = \delta_r \hat{R}_{t-1} + (1-\delta_r)\left[\delta_\pi \hat{\pi}_t + \delta_y \hat{y}_t\right]$$

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Allow for lagged output and inflation gaps in policy rule?

$$\hat{R}_t = \delta_r \hat{R}_{t-1} + (1-\delta_r)\left[\sum_{i=0}^{K_\pi} \delta_{\pi_i} \hat{\pi}_{t-i} + \sum_{i=0}^{K_y} \delta_{y_i} \hat{y}_{t-i}\right]$$

And / or allow for expected / forecasted output and inflation gaps?

Optimal Policy in the Baseline NK Model

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- **Relevant "long-run targets?" – i.e., how to define $\hat{x}_t$?**
  - **Target a constant (steady-state)?**
  - **Target the "flexible-price" time-varying level of inflation / output?**
    - **Underlying RBC model provides natural benchmark time-varying inflation / output – then define gaps relative to these**

Nominal Wage Rigidity

- Nominal rigidities originally discussed in terms of wages, not prices
- But somehow dismissed as modern macro literature evolved
  - Basic reason (roughly): are wage payments allocative?
    - Goodfriend and King (2001): "…potential allocative inefficiencies from infrequent setting of nominal wages are likely to be offset in the context of long-term employment relationships.." and "…unlikely to influence recommendations for policy
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- EHL: introduce sticky nominal wages in modern DSGE NK model
  - Adapt Calvo framework
  - Main results
    - Sticky nominal wages introduce an endogenous shifter in the price-Phillips curve (i.e., a source of the "cost-push" shock u_t)
    - Optimal policy
      - Stabilize a weighted average of output gap, nominal price inflation, and nominal wage inflation
      - Complete price inflation stability no longer optimal
- EHL formulation of sticky wages has become the standard in DSGE models – i.e., Christiano, Eichenbaum, and Evans (2005 *JPE*)

NOMINAL WAGE RIGIDITY

- Dixit-Stiglitz sticky-price structure
  - Continuum of differentiated goods
    - Each goods producer has monopoly power
    - Each goods producer faces exogenous probability of re-setting nominal price
  - "Retailer" Final goods producer "packages" intermediate goods and sells composite final good

Erceg, Henderson, Levin (2000 JME)

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  - "Retailer" Final goods producer "packages" intermediate goods and sells composite final good
- Dixit-Stiglitz sticky-wage structure (as first formulated by EHL)
  - Continuum of differentiated labors
    - Each labor supplier has monopoly power
    - Each labor supplier faces exogenous probability of re-setting nominal wage
  - "Employment agency" Final labor supplier "packages" intermediate labors and sells composite final labor
- Dixit-Stiglitz-Calvo machinery adapted to the labor market

EHL Model – Employment Agency

- Representative employment agency (perfectly-competitive)
  - Aggregates individual households' labors

$$N_t = \left[\int_0^1 n_{it}^{\frac{\varepsilon^w - 1}{\varepsilon^w}} di \right]^{\frac{\varepsilon^w}{\varepsilon^w - 1}}$$

ε^w the elasticity of substitution between different types of labor

  - Sells N_t to intermediate-goods firms (i.e., composite labor needed for goods production)

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- Employment agency profit-maximization problem

$$\max_{n_{it}} W_t N_t - \int_0^1 W_{it} n_{it} di$$

<u>Revenues</u>: aggregate wage payments from firms

<u>Costs</u>: individual wage payments to differentiated households

$\downarrow$ profit maximization

$$n_{it} = \left[\frac{W_{it}}{W_t} \right]^{-\varepsilon^w} N_t$$

Usual Dixit-Stiglitz demand functions (for each type of labor *i*)

EHL MODEL – HOUSEHOLDS

Exogenous probability **of *not* being able to (re-)set wage**

Set *X* = 0 for "cashless" economy

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left[u(c_{it+s}) + v(n_{it+s}) + \chi w(M_{it+s} / P_{t+s}) \right]$$

Full set of state-contingent securities allows each hh *i* to insure against its own idiosyncratic wage – i.e., full consumption insurance

Households *i*'s individual labor income

Time *t* budget constraint

$$P_t c_{it} + M_{it} - M_{it-1} + B_{it} - R_t B_{it-1} = (1+\tau^w) W_{it} n_{it} + P_t pr_t + T_t$$

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hh *i* also takes as given demand function for *its* labor

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Substitute using demand function

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Note time subscripts!!!

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- **FOCs with respect to**
  - c_{it}
  - W_{it}
  - **And assets**

Erceg, Henderson, Levin (2000 JME)

EHL Model – Optimal Wage-Setting

FOC wrt W_{it}

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mrs (between consumption and leisure) of hh *i*

relative wage set by hh *i*

A Dixit-Stiglitz pricing result

$$\boxed{-\frac{\varepsilon^w}{\varepsilon^w - 1} mrs_{it} = \left(\frac{W_{it}}{W_t}\right)}$$

Relative wage is a markup over mrs

EHL MODEL – OPTIMAL WAGE-SETTING

FOC wrt W_{it}

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left\{ \left(\frac{W_{it}}{W_{t+s}}\right)^{-\varepsilon} N_{t+s} \left[-\varepsilon^w v'(n_{it+s}) \left(\frac{W_{it}}{W_{t+s}}\right)^{-1} + (1-\varepsilon^w)(1+\tau^w) u'(c_{it+s}) \right] \right\} = 0$$

Rewrite: multiply by $1/(\varepsilon^w - 1)$

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left\{ \left(\frac{W_{it}}{W_{t+s}}\right)^{-\varepsilon} N_{t+s} \underbrace{\left[\frac{-\varepsilon^w}{\varepsilon^w - 1} v'(n_{it+s}) \left(\frac{W_{it}}{W_{t+s}}\right)^{-1} - (1+\tau^w) u'(c_{it+s}) \right]} \right\} = 0$$

If wages are completely flexible (i.e., if $\alpha_w = 0$):

$$-\frac{\varepsilon^w}{\varepsilon^w - 1} \underbrace{\frac{v'(n_{it})}{u'(c_{it})}} = (1+\underset{=0}{\cancel{\tau^w}}) \underbrace{\left(\frac{W_{it}}{W_t}\right)}$$

mrs (between consumption and leisure) of hh *i*

relative wage set by hh *i*

A Dixit-Stiglitz pricing result

$$\boxed{-\frac{\varepsilon^w}{\varepsilon^w - 1} mrs_{it} = \left(\frac{W_{it}}{W_t}\right)}$$

Relative wage is a markup over mrs

mrs is the household's "marginal cost" of producing/supplying labor

Erceg, Henderson, Levin (2000 JME)

EHL Model – Optimal Wage-Setting

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Rewrite: multiply each term by W_{t+s}/W_{t+s}, multiply each term by $u'(c_{it+s})$ / $u'(c_{it+s})$, and multiply entire expression by W_{it}

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left\{ u'(c_{it+s}) W_{t+s} \left(\frac{W_{it}}{W_{t+s}} \right)^{-\varepsilon} N_{t+s} \left[\frac{-\varepsilon^w}{\varepsilon^w - 1} mrs_{it+s} - (1+\tau^w) \frac{W_{it}}{W_{t+s}} \right] \right\} = 0$$

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PDV of nominal marginal cost (ie, mrs) until next wage change

$$P_t x_t^2 \equiv E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left\{ u'(c_{it+s}) W_{t+s} \left(\frac{W_{it}}{W_{t+s}} \right)^{-\varepsilon} N_{t+s} \left[-(1+\tau^w) \frac{W_{it}}{W_{t+s}} \right] \right\}$$

PDV of nominal marginal revenue (ie, labor income) until next wage change

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EHL Model – Optimal Wage-Setting

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PDV of nominal marginal revenue (ie, labor income) until next wage change

Next step: express x^1 and x^2 recursively -> WAGE PHILLIPS CURVE expressed compactly as $x^1 = x^2$

EHL Model – Equilibrium

- Price Phillips Curve (i.e., FOC of sticky-price firm)
- Household Euler equation
- Aggregate resource constraint
  - Deadweight loss from both sticky prices and sticky wages

  But EHL set τ^w such that no deadweight loss
- Wage Phillips Curve (aka consumption-leisure optimality condition)
- Law of motion for real wage
  - A non-trivial equilibrium condition in models with sticky nominal wages + sticky nominal prices *and / or* explicit money demand

$$\frac{w_t}{w_{t-1}} = \frac{\pi_t^w}{\pi_t}$$

$$\text{Growth of real wage} = \frac{\text{Nominal wage inflation}}{\text{Nominal price inflation}}$$

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    - Sticky nominal price-setting and / or money demand influences price inflation
    - Sticky nominal wage-setting influences wage inflation
    - Technology influences real wage growth
    - No reason that all three of these are compatible with each other (simple example: w_t = TFP every period)

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In sticky-wage economy, this condition is an allocative condition, not simply an identity – hence part of the definition of equilibrium

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See Chugh (2006 *RED* p. 691-692)

EHL Model – Optimal Policy

- **Optimal policy problem – maximize representative household's lifetime utility subject to all private-sector equilibrium conditions**

- **In LQ form (see EHL Appendix B), minimize**

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\hat{\pi}_t^2 + \hat{y}_t^2 + \theta \hat{\pi}_t^{w2} \right]$$

Relative importance of stabilizing *wage* inflation versus stabilizing *price* inflation depends on relative stickiness of prices and wages – NO divine coincidence here

subject to (linearized) private-sector equilibrium conditions

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  - EHL p. 298
- Main Result
  - Fully stabilizing price inflation NOT optimal
  - Stabilize a weighted average of price inflation and wage inflation

NK Optimal Policy – Where Next?

- LQ analysis around distorted steady state
  - Benigno and Woodford (2003 ***NBER Macroeconomics Annual***)
  - Drop assumption of "sufficiently-rich fiscal instruments…"
    - Implies log-linearization (much) more difficult…
    - …but now demonstrated can be handled
  - Importance: more realistic policy analysis – in particular, allows role for monetary stabilization policy around distorted long-run growth path

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- Optimal policy in NK model featuring labor search frictions?
  - Thomas (2008 ***JME***), Faia (2008 ***JEDC***)
- Optimal policy in open-economy NK models?
  - Not very much work exists…
- Optimal policy in NK model with fundamental role for money?