Motivation

- The current financial crisis.
- Standard New Keynesian (NK) models do not account for financial frictions.
- Standard NK models imply no trade-off between stabilizing inflation and the output gap (the “Divine Coincidence”).
- NK models add ad-hoc cost-push shock to generate a trade-off.
- What are the implications of financial frictions for optimal monetary policy?
Main Results

- Without credit frictions, the monetary authority should stabilize inflation at all dates in response to TFP shocks.
- When credit frictions are present, full stabilization of inflation is not optimal.
- But, inflation stabilization is nearly optimal.
- Following net worth shocks, full stabilization of inflation is not optimal.
- With logarithmic utility function, full stabilization of inflation is optimal in either

The “Divine Coincidence”

**Phillips Curve with no cost-push shock:**
\[
\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{y}_t^g
\]
- Iterating forward: \( \pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t \hat{y}_{t+i}^g \)
- No trade-off between stabilizing inflation and the output gap.

**Phillips Curve with cost-push shock:**
\[
\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{y}_t^g + \nu_t
\]
- Iterating forward: \( \pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t \hat{y}_{t+i}^g + \nu_t \)
- There is a trade-off now.
The Model Economy

- Households: consume and supply two types of labor (u and L).
- Entrepreneurs: Produce intermediate goods.
- The hiring of the L-input by entrepreneurs is subject to a collateral constraint.
- Final good firms.
- There is price rigidity in the final good sector.
- The monetary authority minimizes a loss

Households

- Households’ problem:
  - Max \( U(c_t, L_t, u_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - B_1 \frac{L_t^{1+\theta}}{1+\theta} - B_2 \frac{u_t^{1+\theta}}{1+\theta} \).
  - s.t. \( c_t + Q_s \pi_t + b_t = \frac{R_t b_{t-1}}{\pi_t} + (1 + w_{sub}) w_{f,t} + (1 + r_{sub}) r_{u,t} + s_{\pi,t} (Q_t + D_t) + T_t \)
  - FOCs:
    \[ \frac{U_c(t)}{U_c(t)} = w_t (1 + w_{sub}) \] \( (1) \)
    \[ \frac{U_l(t)}{U_c(t)} = r_t (1 + r_{sub}) \] \( (2) \)
    \[ U_c(t) = E_t \beta U_c(t+1) R_t / \pi_{t+1} \] \( (3) \)
    \[ Q_t U_c(t) = E_t \beta U_c(t+1) [Q_{t+1} + D_{t+1}] \] \( (4) \)
Entrepreneurs

- Entrepreneurs have linear preferences.
- They hire two types of labor inputs ($L$ and $u$) to produce intermediate goods.
- Production Function $x_t = L_t^\alpha u_t^{1-\alpha}$
- The borrowing of the $L$-input is subject to a collateral constraint:

$$w_t L_t \leq g(nw_t, p_t x_t - r_t u_t) = nw_t^b (p_t x_t - r_t u_t)^{1-b}$$

$$nw_t \equiv e_{t-1} (Q_t + D_t)$$

Entrepreneurs, Contd.

- Entrepreneurs’ Problem:
  - $\text{Max } \text{profits}_t = p_t x_t^{\alpha} - w_t L_t - r_t u_t$
  - s.t. the collateral constraint.
  - FOCs:

$$\alpha p_t x_t = w_t L_t (1 + b \phi_t)$$

$$\text{FOC}_t: \quad (1 - \alpha) p_t x_t = r_t u_t$$
Entrepreneurs, Contd.

\[ \text{profits}_t = \alpha_p x_t - w_t L_t = \alpha_p x_t \left( \frac{b\phi_t}{1 + b\phi_t} \right) \]  

(9)

- Entrepreneurs’ Budget Constraint:

\[ c^e_t + e_t Q_t \leq e_{t-1} (Q_t + D_t) + \text{profits}_t \]  

(10)

\[ c^e_t + e_t Q_t \leq \alpha_p x_t \Phi(\phi_t) \]  

(11)

\[ \Phi(\phi_t) = \left[ \left( \frac{b\phi_t}{1 + b\phi_t} \right) + \left( \frac{1}{1 + b\phi_t} \right)^{1/b} \right]. \]

Wage Subsidies

- Introduced to render the deterministic steady state efficient.

\[ B_1 c^\theta_{ss} L^\theta_{ss} = MPL_{ss} \frac{Z_{ss}}{(1 + b\phi_{ss})} (1 + w_{sub}) = MPL_{ss} \]

\[ B_2 c^\theta_{ss} u^\theta_{ss} = MPU_{ss} Z_{ss} (1 + r_{sub}) = MPU_{ss} \]

- The subsidy for \( u \):

\[ r_{sub} = \frac{1}{\varepsilon - 1} \]

- The subsidy for \( L \):

\[ w_{sub} = \frac{1 + \varepsilon \phi_{ss}}{\varepsilon - 1} \]
Final Good Firms

- They purchase intermediate goods and package them into final goods.
- Production function: \( y_{t,j} = a_t x_{t,j} \)
- Price adjustment cost: \( \phi \left[ \frac{(p_{t,j} - p_{t-1,j}) / p_{t-1,j}}{2} \right] y_t \)
- Profits of final good firms are paid as dividends to shareholders: households and entrepreneurs.
- Dividends: \( D_t = (1 - z_t) y_t - \frac{\phi}{2} (\pi_t - 1)^2 y_t \)

Breaking Down the “Divine Coincidence”

The Phillips Curve (No financial friction):

\[
\hat{\pi}_t = \lambda \hat{\pi}_t + \beta E_t \hat{\pi}_{t+1} + \lambda \epsilon_t^n \tag{12}
\]

- The output gap:

\[
\hat{y}_t^g = \frac{1}{\sigma + \theta} \hat{\pi}_t - \frac{\alpha}{(\sigma + \theta)} b \hat{\phi}_t \tag{17}
\]

The Phillips Curve (With financial friction):

\[
\hat{\pi}_t = \lambda (\sigma + \theta) \hat{y}_t^g + a \lambda b \hat{\phi}_t + \beta E_t \hat{\pi}_{t+1} + \lambda \epsilon_t^n \tag{25}
\]

- Fluctuations in the risk premium lead to fluctuations in inflation. No “Divine Coincidence”, even in the absence of markup shocks. The risk premium behaves as a cost-push shock.
Some Important Relationships

\[ \hat{\nu}_t^\beta = \frac{1}{\sigma + \theta} \hat{\nu}_t + \frac{\alpha(\sigma - 1)}{(\sigma + \theta)(1 + \theta)} b \hat{\phi}_t \]  
(15)

\[ \hat{L}_t^\theta = \frac{1}{\sigma + \theta} \hat{L}_t - \left[ \frac{\sigma(1 - \alpha)(\sigma + \theta)}{(\sigma + \theta)(1 + \theta)} \right] b \hat{\phi}_t \]  
But, if \( \alpha = 0 \), the risk premium term does not drop!  
(16)

\[ \hat{z}_t^\beta = \frac{1}{\sigma + \theta} \hat{z}_t - \frac{\alpha}{(\sigma + \theta)} b \hat{\phi}_t \]  
(17)

\[ \frac{\partial \Phi}{\partial \alpha_t} = \frac{\beta(1 - \sigma)(1 + \theta)(1 - \rho_a)}{(1 - \beta \rho_a)(ab + \theta + \sigma(1 - ab))} \]

So, the response of the risk premium to TFP shocks depends on the value of \( \sigma \). The risk premium might be procyclical!

Optimal Monetary Policy Problem

\[ \max_{\{\theta_t, \Phi_t\}} \sum_{t=0}^{\infty} \beta^t \left[ \frac{E_t \Sigma_t^\infty}{\lambda} \right] + \left( \sigma + \theta \right) (\hat{\eta}_t^\beta)^2 + \frac{\alpha(1 - \alpha)}{1 + \theta} b (\hat{\phi}_t)^2 \],  
(30)

subject to the constraints

\[ \hat{\pi}_t = \lambda (\sigma + \theta) \hat{\eta}_t^\beta + a \lambda b \hat{\phi}_t + \beta E_t \hat{\pi}_{t+1} + \lambda \hat{\pi}_t \]  
(31)

\[ \hat{\eta}_t = \frac{1}{\beta} (\hat{\eta}_t - \epsilon(1 - \beta)(\sigma + \theta) \hat{\eta}_t^\beta + [1 + \beta \Lambda - \alpha \epsilon(1 - \beta)b] \hat{\pi}_t + n_t] \],  
(32)

\[ \hat{\phi}_t = \frac{(1 + \theta)}{(1 + \Lambda)} E_t \Delta \hat{\pi}_{t+1} - \frac{(1 - \alpha b)}{(1 + \Lambda)} E_t \Delta \hat{\phi}_{t+1} - \frac{(\sigma - 1)(1 + \theta)}{(\sigma + \theta)(1 + \Lambda)} E_t \Delta \hat{\eta}_{t+1} - \frac{1}{(1 + \Lambda)} E_t n_{t+1} \]  
(33)

Captures the concerns of central banks about credit market tightness!

But, what if \( \alpha = 1 \)? The risk premium term drops as well!  
The weight of the risk premium falls with \( \alpha \) when \( \alpha > 0.5 \)!
Alternative Policies

• Full stabilization of inflation.
• A Taylor Rule: \( \hat{R}_t = \tau \hat{r}_t + \tau_g \hat{g}_t \)
  \( \tau = 1.5, \tau_g = 0.5 \)
• A Taylor Rule with response only to inflation (Taylor rule without a gap).
• But, what about a Taylor-type rule where the monetary authority responds to asset prices or the risk premium?

Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Benchmark</th>
<th>Alternative</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>( \beta )</td>
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<td>0.99</td>
</tr>
<tr>
<td>( \sigma )</td>
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<td>2.0</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.47</td>
<td>2.0</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>( b )</td>
<td>2/3</td>
<td>2/3</td>
</tr>
<tr>
<td>( \phi )</td>
<td>173.08</td>
<td>173.08</td>
</tr>
</tbody>
</table>

CFP assume price duration of 5 quarters!
Most recent evidence suggests significantly shorter duration (1.5-3.5 quarters).
(Bils and Klenow, 2004; Ravenna and Walsh, 2006; Sbordone, 2002)
Impulse Responses to TFP Shock - Benchmark

Impulse response to technology shock

Note: The x-axis measures quarters after the shock. The y-axis measures percentage deviations from the steady state for all variables except for the premium, inflation and the nominal rate where it measures the deviations in annualized basis points.

Impulse Responses to TFP Shock - Alternative Calibration

Note: See Figure 1A.
Impulse Responses to Net Worth Shock - Different Inflation Weights

Welfare Losses - Benchmark Calibration
Table 1A (σ = 0.16, θ = 0.47)

Welfare Losses under Different Monetary Policies
The 3 entries in each cell correspond to α = 0.5, α = 0.25, and α = 0.01.

<table>
<thead>
<tr>
<th></th>
<th>Technology shocks</th>
<th>Net Worth Shocks</th>
<th>Markup Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal Commitment</td>
<td>19σ^2_2</td>
<td>116σ^2_2</td>
<td>297σ^2_2</td>
</tr>
<tr>
<td></td>
<td>0σ^2_2</td>
<td>3σ^2_2</td>
<td>9σ^2_2</td>
</tr>
<tr>
<td>Inflation Stabilization</td>
<td>22σ^2_2</td>
<td>13σ^2_2</td>
<td>34σ^2_2</td>
</tr>
<tr>
<td></td>
<td>11σ^2_2</td>
<td>53σ^2_2</td>
<td>38σ^2_2</td>
</tr>
<tr>
<td></td>
<td>0σ^2_2</td>
<td>2σ^2_2</td>
<td>41σ^2_2</td>
</tr>
<tr>
<td>Taylor Rule with gap</td>
<td>152σ^2_2</td>
<td>152σ^2_2</td>
<td>536σ^2_2</td>
</tr>
<tr>
<td></td>
<td>66σ^2_2</td>
<td>41σ^2_2</td>
<td>384σ^2_2</td>
</tr>
<tr>
<td></td>
<td>16σ^2_2</td>
<td>2σ^2_2</td>
<td>572σ^2_2</td>
</tr>
<tr>
<td>Taylor Rule without gap</td>
<td>150σ^2_2</td>
<td>155σ^2_2</td>
<td>489σ^2_2</td>
</tr>
<tr>
<td></td>
<td>100σ^2_2</td>
<td>58σ^2_2</td>
<td>449σ^2_2</td>
</tr>
<tr>
<td></td>
<td>98σ^2_2</td>
<td>2σ^2_2</td>
<td>422σ^2_2</td>
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</tbody>
</table>

(We assume that the TFP follows an AR(1) process with autoregressive coefficient 0.91, whereas net worth and markup processes are assumed to slightly less persistent with AR coefficients 0.9. Welfare losses are scaled by the variance of the innovation in the exogenous processes for TFP (σ_T^2), net worth (σ_N^2) and markup (σ_M^2), respectively. Each table entry is multiplied by 100.)
**Welfare Losses- Alternative Calibration**

**Table 1B (σ = 2, θ = 2)**

**Welfare Losses under Different Monetary Policies**

The 3 entries in each cell correspond to α = 0.5, α = 0.28, and α = 0.01.

<table>
<thead>
<tr>
<th></th>
<th>Technology Shocks</th>
<th>Net Worth Shocks</th>
<th>Markup Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Optimal Commitment</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2σ²</td>
<td>35σ²</td>
<td>68σ²</td>
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<tr>
<td>1σ²</td>
<td>19σ²</td>
<td>60σ²</td>
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</tr>
<tr>
<td>0.5σ²</td>
<td>1σ²</td>
<td>61σ²</td>
<td></td>
</tr>
<tr>
<td><strong>Inflation Stabilization</strong></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>2σ²</td>
<td>36σ²</td>
<td>61σ²</td>
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</tr>
<tr>
<td>1σ²</td>
<td>19σ²</td>
<td>64σ²</td>
<td></td>
</tr>
<tr>
<td>0.5σ²</td>
<td>1σ²</td>
<td>66σ²</td>
<td></td>
</tr>
<tr>
<td><strong>Taylor Rule with gap</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>945σ²</td>
<td>414σ²</td>
<td>1575σ²</td>
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</tr>
<tr>
<td>945σ²</td>
<td>114σ²</td>
<td>1514σ²</td>
<td></td>
</tr>
<tr>
<td>960σ²</td>
<td>1σ²</td>
<td>1517σ²</td>
<td></td>
</tr>
<tr>
<td><strong>Taylor Rule without gap</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1536σ²</td>
<td>90σ²</td>
<td>369σ²</td>
<td></td>
</tr>
<tr>
<td>1517σ²</td>
<td>3σ²</td>
<td>366σ²</td>
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<tr>
<td>1498σ²</td>
<td>1σ²</td>
<td>276σ²</td>
<td></td>
</tr>
</tbody>
</table>

(We assume that the TFP follows an AR(1) process with autoregressive coefficient 0.95, whereas net worth and markup processes are assumed to slightly less persistent with AR coefficients 0.9. Welfare losses are scaled by the variance of the innovation in the exogenous processes for TFP (σ²), net worth (σ_n²) and markup (σ_m²), respectively. Each table entry is multiplied by 100.)

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**The Role of Monetary Policy**

- Monetary policy responds to movements in the risk premium in order to ease the borrowing of entrepreneurs (i.e. lower the tightness of the collateral constraint).
- Monetary policy affects dividends and share prices by affecting profits.
- There is an interplay between price stickiness and the collateral constraint.
Conclusions

- Agency costs act as endogenous mark-up shocks.
- Under the presence of credit frictions, full stabilization of inflation is not optimal following TFP and net worth shocks.
- However, near inflation stabilization is optimal, since the weight of inflation in the loss function is considerably higher than the weights of the output gap and the risk premium.

Optimal Long-Run Inflation Rate

- The question: What is the optimal long-run inflation rate (the optimal inflation target)?
- Conjecture: a positive inflation rate.
- Why: a precautionary motive in the part of policy makers.
- In the background: The Debt-Deflation Theory.
Methodology

- How: replacing the *always-binding* constraint in CFP by *occasionally-binding* constraint.
- The method: The “penalty function” approach.
- The entrepreneur’s profit function:

\[ \Pi = p l^a w^{1-a} - w l - r < - \frac{\gamma}{\gamma} \exp\left[ - w d (t-1)(Q + Q - w l) \right] \]

Preliminary Results

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope of PC</td>
<td>0.80</td>
<td>0.70</td>
<td>0.60</td>
<td>0.50</td>
<td>0.40</td>
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<tr>
<td>Price Rigidity $\varphi$</td>
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<td>7.14</td>
<td>8.33</td>
<td>10.00</td>
<td>12.50</td>
<td>16.67</td>
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<tr>
<td>Lower Bound</td>
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<td>0.83</td>
<td>0.57</td>
<td>0.35</td>
<td>0.17</td>
<td>0.05</td>
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<tr>
<td>Mean</td>
<td>1.71</td>
<td>1.32</td>
<td>0.96</td>
<td>0.67</td>
<td>0.43</td>
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<tr>
<td>Upper Bound</td>
<td>2.27</td>
<td>1.80</td>
<td>1.35</td>
<td>0.99</td>
<td>0.69</td>
<td>0.43</td>
</tr>
</tbody>
</table>

1. The optimal inflation rate is *decreasing* in the degree of price rigidity.
2. Tension between the credit friction channel and the price stickiness channel.
Thank You 😊