Department of Economics University of Maryland

Economics 701
Advanced Macroeconomics I
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Outline – September 4

Solving Business Cycle Models Computationally – An Introduction

Use the (efficient) RBC model as basic laboratory

- No distortionary taxes (i.e., set $\tau_t^n = 0 \forall t$ and $\tau_t^k = 0 \forall t$), so First Welfare Theorem applies → can solve Social Planning problem
- Recall basic Social Planning problem for RBC economy

$$\max_{c_t, n_t, k_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$

subject to

$$c_t + k_{t+1} - (1-\delta)k_t + g_t = z_t f(k_t, n_t)$$

- Construct infinite-period Lagrangian

- Compute first-order conditions with respect to c_t, n_t, and k_{t+1}
- Combine to yield consumption-leisure efficiency condition and consumption-investment efficiency condition

- Together with aggregate resource constraint, these define **time-invariant mappings** from the state of the economy S_t to period-t consumption and labor and period-$t+1$ capital stock: $c_t = c(S_t), n_t = n(S_t), k_{t+1} = k(S_t)$
  - o Question: Do we need to be careful about to which agent (i.e., which household, or which firm, etc?) these decision rules apply?

Approximation Methods

- In general **cannot** solve for these decision rules analytically; must resort to computational/numerical/approximation methods
    - Thus need to construct functions $c^{approx}(S_t), n^{approx}(S_t), k^{approx}(S_t)$ that are hopefully!...) close to the "true" functions $c(S_t), n(S_t), k(S_t)$

- Two "types" of solution methods for constructing $c^{approx}(S_t), n^{approx}(S_t), k^{approx}(S_t)$:
    - **Global Approximations –** approximated functions are close to the true functions "everywhere" (over a very broad range of states)
        - Several popular methods
            - Chebyshev polynomials
            - Finite-element methods
        - Hard to implement for medium- and large-scale models given current hardware capacity

    - **Local Approximations –** approximated functions are close to true functions only in a relatively small range of states
        - Virtually all based on Taylor-series approximations
            - Linear (first-order) the most widely used
            - Quadratic (second-order) becoming much more commonly used
            - Cubic (third-order) and beyond quite uncommon – but becoming easier and easier to implement
        - Biggest virtue: relatively easy to implement, even for medium- and large-scale models
        - For business cycle dynamics, all available evidence suggests not much accuracy is lost compared to using global approximations (see Aruoba, Fernandez-Villaverde, and Rubio-Ramirez (2006))

Local Approximation

General Idea

- Must pick a point around which to conduct the approximation
    - Recall Taylor expansions are conducted *around a point…*
- Construct linear (or higher-order) approximation using the derivatives (first or higher-order, corresponding to the choice of order of approximation) **of the equilibrium conditions with respect to**

Implementation (Linear) for the RBC Model

- Will follow the algorithm/implementation of Schmitt-Grohe and Uribe (2004 *JEDC*)

- (Many) alternative algorithms/implementations exist, but they differ only in the exact route (set of manipulations) used to arrive at the linear approximation (see Aruoba, Fernandez-Villaverde, and Rubio-Ramirez (2006), footnote 4).

- **The natural point around which to center the approximation: the deterministic steady state of the model**

- Computing the deterministic steady state?
  - Assume all real variables reach a constant after a sufficiently long period of time during which **no shocks** affect the economy

  - **Mechanically, can thus drop the time indices from the consumption-leisure efficiency condition, consumption-investment efficiency condition, and aggregate resource constraint:**

  - Three (non-linear) equations in the three unknowns *c*, *n*, and *k*

→ SOLVE!!! (either by hand or numerically (i.e., Project 0))