Optimal Fiscal and Monetary Policy When Money is Essential *

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Abstract

We study optimal fiscal and monetary policy in an environment where explicit frictions give rise to valued money, making money essential in the sense that it expands the set of feasible trades. Our main results are in stark contrast to the prescriptions of earlier flexible-price Ramsey models. The two most important findings that emerge from our work are that the Friedman Rule is typically not optimal and inflation volatility is very low in the face of business-cycle shocks. A departure from the Friedman Rule does not arise because of any incompleteness of the tax system, as can sometimes occur in standard Ramsey models. Rather, by developing a precise notion of margins of adjustment using the familiar notions of MRS and MRT, we show that the tax system in our model is complete. Regarding the optimal dynamic policy, realized (ex-post) inflation is quite stable over the business cycle, in contrast to the very volatile ex-post inflation rates that arise in standard flexible-price Ramsey models. Taken together, these findings turn conventional wisdom from traditional Ramsey monetary models on its head.

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1 Introduction

Monetary theory has made important advances of late, ones that enable researchers interested in applied policy questions to consider explicit frictions that give rise to valued money. In this paper, we build on the work of Lagos and Wright (2005) to study optimal fiscal and monetary policy, in the tradition of Lucas and Stokey (1983) and Chari, Christiano, and Kehoe (1991). Two important findings emerge from our work, both of which are opposite those of earlier flexible-price Ramsey monetary models: the Friedman Rule is typically not optimal and inflation volatility is low in the face of business-cycle magnitude shocks. Our results thus turn conventional wisdom from standard Ramsey monetary models on its head.

The contribution of Lagos and Wright (2005) — hereafter, LW — was to integrate search-based monetary theory, in the spirit of Kiyotaki and Wright (1989, 1993), with standard dynamic general equilibrium macroeconomics. This integration makes the study of policy questions much easier and potentially more relevant than in earlier search-based models. However, these models have been criticized on two grounds. First, they superficially resemble standard cash-in-advance (CIA) or money-in-the-utility-function (MIU) models, making some question whether they really are any deeper than reduced-form models of money. This point has been raised by, among others, Howitt (2003). Second, until now, the policy questions addressed in these new models have been largely confined to the deterministic welfare costs of inflation. When parameterized to seem as close as possible to standard CIA and MIU models, the quantitative answers they have yielded to this question are similar to those obtained with CIA and MIU models, further adding to the sense that these new models simply re-invent CIA or MIU. In this paper, we ask a different policy-relevant question in these new models, and even when we parameterize the model to look very similar to standard reduced-form models of money, we reach conclusions very different from those reached by Chari, Christiano, and Kehoe (1991) and others using typical CIA and MIU frameworks. Our results thus show that the answers to policy questions may indeed be very different once monetary frictions are treated seriously.

We study the canonical Ramsey problem of optimal fiscal and monetary policy using the LW model. Our first main finding is that the nominal interest rate is typically positive because it is optimal to tax activities that require cash.[1] This optimal deviation from the Friedman Rule is not due to any incompleteness of the tax system, as can sometimes occur in Ramsey models. We show

[1] In a different context, one that abstracts from public finance considerations, Rocheteau and Wright (2005) show that a positive nominal interest rate may be optimal because it can correct inefficiencies along the extensive margin of bilateral trading by influencing the relative number of traders on each side of the market. In other micro-founded models of money that also abstract from public finance considerations, Shi (1997), Bhattacharya, Haslag, and Martin (2005), and Head and Kumar (2005) also find that deviations from the Friedman Rule can be optimal.

that our model features a complete set of tax instruments, which means there is always (at least) one policy instrument for each independent margin in the decentralized economy.

Rather, the reason behind the deviation from the Friedman Rule is that, because all final goods should be taxed to some degree as part of an optimal tax system, taxation of cash activities is naturally part of the second-best allocation. This prescription is simply standard Ramsey theory. In the LW environment, the explicit spatial and informational frictions that make money essential (in Kocherlakota's (1998) sense that it expands the set of feasible trades) render inflation the most natural way of taxing activities that require money. Such a prescription does not arise in Chari, Christiano, and Kehoe (1991) — hereafter, CCK — because taxation of labor income indirectly taxes cash activities, making the inflation tax, which would change the effective tax on cash-goods, unnecessary in their environment. As we discuss, our results can be reconciled both technically and conceptually with those of CCK. Interestingly, Kocherlakota (2005) conjectured that the Friedman Rule may not be optimal in a Ramsey problem in search-based models. Our results show his conjecture is correct.

Our second main finding is that realized (ex-post) inflation is quite stable in the face of business-cycle magnitude shocks, which is in contrast to the very volatile ex-post inflation rates that CCK find. Inflation volatility is high in CCK and the related literature because surprise movements in the price level allow the government to synthesize real state-contingent debt payments from nominally risk-free government bonds, without distorting the relative prices of consumption goods. The government then need not change other, distortionary, tax rates much in response to shocks. In our model, in contrast, real activity is distorted by inflation because inflation affects relative prices of goods, in a way that a flexible-price CIA or MIU model cannot articulate. The welfare cost of this relative-price distortion dominates the insurance value of generating state-contingent debt in our model, rendering inflation very stable. The frictions underlying monetary trade thus provide novel justification for the optimality of inflation stability, a prescription that resonates with most central bankers. This result also echoes the long-standing idea in monetary economics that inflation variability is undesirable because it induces relative price shifts and demonstrates that nominal rigidities are not a necessary feature of such a mechanism.

Because our environment features a *complete* set of tax instruments, none of our results is due to a policy instrument that we have assumed the government has available serving as an imperfect proxy for another instrument we have assumed unavailable. As Chari and Kehoe (1999, p. 1679-1680) explain, an incomplete tax system is in place if, for at least one pair of goods in the economy, the government has *no* policy instrument that drives a wedge between the marginal rate of substitution (MRS) between those goods and the corresponding marginal rate of transformation (MRT). Completeness or incompleteness of a tax system is a concept that can be defined in both monetary models and purely real models. In the literature on optimal capital taxation, the examples of Correia (1996), Jones, Manuelli, and Rossi (1997), and Armenter (2008) illustrate that incompleteness of the tax system typically leads to non-zero capital-income taxation. This is because the capital tax ends up imperfectly substituting for the ability to create certain wedges. Similarly, one can easily show that in the monetary models of Chari and Kehoe (1999), if the tax system were incomplete, the Friedman Rule would not be optimal because a positive nominal interest rate serves as an imperfect substitute for the ability to create a wedge in the consumption-leisure margin. In our analysis, the deviation from the Friedman Rule does not arise due to any inability on the part of the government to create wedges between one or more MRS/MRT pairs. Indeed we show that our model features a complete set of policy instruments by demonstrating that the government has one unique policy tool for each wedge between MRS and MRT that it might want to create.

An important technical advantage of the LW framework is that the distribution of money-holdings across agents is simple to track: it simply collapses periodically to a point. At the expense of a heavier computational burden, one may want to think about optimal fiscal and monetary policy when this distribution is non-trivial. Once one goes down that route, an interesting taxation framework to apply may be the Mirrleesian one, in which idiosyncratic shocks and private information become important considerations in shaping optimal policy. However, because even the simpler step of characterizing the Ramsey-optimal policy, which assumes a representative agent, has not been studied in this class of models, we think it makes sense to begin here.

The rest of the paper is organized as follows. Section 2 lays out the baseline LW model in which we study optimal policy. Section 3 presents the Ramsey problem. Section 4 formally demonstrates that the tax system in our model is complete in the sense described above. In Section 5, we characterize the optimal policy. Included here is a proof for a particularly important version of the model in which the Friedman Rule is not optimal, followed by quantitative results that demonstrate that Ramsey-optimal inflation in the face of business cycle shocks is at least an order of magnitude more stable than benchmark Ramsey results. Section 6 provides discussion and interpretation of our results. Section 7 summarizes and offers ideas for future work.

2 Model

We establish our results in a version of the LW model. In this model, the agents in the economy participate in a centralized market (CM) where they trade general consumption goods and assets with the market and in a decentralized market (DM) where they trade specialized consumption goods bilaterally. To enhance comparability with the benchmark cash-credit environment used by CCK, we alter slightly the timing of markets in the original LW model. Specifically, in our version, the CM is the first market in a given period, followed by the DM. We make this alteration because we would like asset markets (which in the LW model meet in the CM) to convene in any period before goods markets (in particular, before goods markets in which money must be used for transactions), which is the timing assumed by CCK. However, we do not see how any of our results depend on the temporal ordering of markets within a period. We proceed by describing the activities of the government, households, and firms in our model.

2.1 Government

Government consumption is assumed to be composed entirely of goods produced in the CM. In nominal terms, the flow budget constraint of the government is

$$M_t + B_t + P_t w_t \tau_t^h H_t = P_t G_t + M_{t-1} + R_{t-1} B_{t-1},$$

which states that the government has three sources of revenues to pay for its consumption: labor income tax revenues, nominal money creation, and nominal debt issuance. The notation is standard: M_t denotes nominal money outstanding at the end of period t, B_t is nominally risk-free government debt outstanding at the end of period t, R_t is the gross nominal interest rate on bonds, τ_t^h is a proportional labor income tax on aggregate hours worked H_t in the CM, P_t is the nominal price level in the CM, and w_t is the real wage in the CM. The nominal return R_t is known at the time B_t is issued and paid in the CM of period $t+1$. We assume that bonds are simply book entries with no tangible proof that one can carry around.

2.2 Households

Households periodically transact in markets for general goods and assets (the CM) and in markets for specialized goods (the DM). In the DM, money is essential in the sense that transactions there are infeasible without money.[2] In the CM, because markets are Walrasian trades can proceed with or without money. We describe first the timing of events in a given period and then present the household's CM and DM problems.

Events unfold for a household in a given period t as follows:

- The household begins the CM with portfolio m_{t-1} and b_{t-1}.
- The uncertainty for the current period is resolved, and the household observes government consumption G_t and the level of technology Z_t. We denote the aggregate state collectively by S_t.
- The household receives the receipts from bond holdings, $R_{t-1} b_{t-1}$.

[2] In a more general model, one can allow a double-coincidence meeting where barter takes place. Doing so does not change any of the properties of the current model and we abstract from it.

- The household chooses its CM consumption x_t, labor supply h_t, portfolio (m_t, b_t) and pays the labor income tax.
- The household enters the DM with m_t.
- Depending on the household's trade in the DM, it exits the DM with $m_t - d_t$, $m_t + d_t$, or m_t money holdings, where d_t is the buyer's payment in bilateral trade.

2.2.1 Household CM Problem

For a household that enters the CM with money holdings m_{t-1} and bond holdings b_{t-1}, the CM problem is

$$W_t\left(m_{t-1}, b_{t-1}, S_t\right) = \max_{x_t, h_t, m_t, b_t} \left\{U(x_t) - Ah_t + V_t(m_t, b_t, S_t)\right\}$$

subject to

$$P_t x_t + m_t + b_t = P_t w_t (1 - \tau_t^h) h_t + m_{t-1} + R_{t-1} b_{t-1}, \tag{1}$$

where $W_t(.)$ denotes the value of entering the CM and $V_t(.)$ denotes the value of entering the DM that convenes after the CM in period t. Note that instantaneous utility in the CM is separable and linear in labor; it is this quasi-linearity in preferences that makes the LW model so tractable because it guarantees a degenerate distribution of money holdings across households after the conclusion of each CM.

Eliminating h_t in the objective function using the budget constraint, the first-order conditions with respect to x_t, m_t, and b_t are

$$U'(x_t) = \frac{A}{w_t(1 - \tau_t^h)}, \tag{2}$$

$$\frac{A}{P_t w_t (1 - \tau_t^h)} = V_{m,t}(m_t, b_t, S_t), \tag{3}$$

$$\frac{A}{P_t w_t (1 - \tau_t^h)} = V_{b,t}(m_t, b_t, S_t), \tag{4}$$

familiar from LW. These optimality conditions imply the usual LW results about degeneracy of asset holdings (m_t, b_t) across households because they are independent of (m_{t-1}, b_{t-1}).[3] All households choose the same portfolio at the end of the CM regardless of the portfolio they entered the market

[3] This result requires a small qualification for bond holdings. There are two parts of the argument in LW. The first part relies on the observation that (m_{t-1}, b_{t-1}) does not appear in (3) and (4). The second part relies on the strict concavity of $V(.)$ or, more specifically, the strict monotonicity of $V_m(.)$ and $V_b(.)$ which means the choice of m_t and b_t is unique. Both parts of the argument go through for money in our environment but only the first part goes through for bonds. This means that in principle there could be multiple values of b_t that households choose, which can create a distribution of bond holdings. Fortunately, such a distribution of bonds holdings is not important for any of our results because bond-holdings will not affect the bargaining problem, as we show below.

with. Thus, the LW result of degeneracy of money holdings readily extends to bond holdings as well. Moreover, we have standard envelope conditions

$$W_{m,t}(m_{t-1}, b_{t-1}, S_t) = \frac{A}{P_t w_t(1-\tau_t^h)}, \tag{5}$$

$$W_{b,t}(m_{t-1}, b_{t-1}, S_t) = \frac{AR_{t-1}}{P_t w_t(1-\tau_t^h)},$$

which show $W_t(.)$ is linear in its arguments. In our derivations below, we use $\chi_t \equiv E_t\left[A/\{P_{t+1}w_{t+1}(1-\tau_{t+1}^h)\}\right]$ which is the marginal value of entering $t+1$ with one extra unit of money.

2.2.2 Household DM Problem

Now we turn to the household's DM problem. Knowing that the distribution of money holdings is degenerate in equilibrium, we will, for notational simplicity, write the household DM problem assuming that when it meets a trading partner, the trading partner has equilibrium money holdings M_t; this allows us to conserve on integrating over all possible money holdings of trading partners that a given household could meet. With probability σ, the household is a buyer in the DM; with probability σ, the household is a seller in the DM; and with probability $1-2\sigma$, the household does not participate in the DM and continues to the CM of the next period without transacting.[4] Buyers consume q in the DM, experiencing utility $u(q)$; sellers produce q in the DM, experiencing disutility, which can be interpreted as the cost of production, $c(q, Z)$, where $c_Z < 0$. We assume throughout that $c(q, Z) = q/Z$.[5]

We can write the problem of a household that enters the DM with portfolio (m_t, b_t) as

$$\begin{aligned} V_t(m_t, b_t, S_t) &= \sigma\left\{u\left[q\left(m_t, M_t, S_t\right)\right] + \beta E_t W_{t+1}\left[m_t - d(m_t, M_t, S_t), b_t, S_{t+1}\right]\right\} \\ &+ \sigma\left\{-c\left[q\left(M_t, m_t, S_t\right), Z_t\right] + \beta E_t W_{t+1}\left[m_t + d(M_t, m_t, S_t), b_t, S_{t+1}\right]\right\} \\ &+ (1-2\sigma)\beta E_t W_{t+1}\left(m_t, b_t, S_{t+1}\right). \end{aligned} \tag{6}$$

The quantity $q(m_{bt}, m_{st}, S_t)$ is the quantity produced and exchanged in a bilateral meeting in the DM, where m_b denotes the money holdings of the buyer, m_s denotes the money holdings of the seller, and $d(m_b, m_{st}, S_t)$ is the amount of money that changes hands. We refer to $[q(.), d(.)]$ as the terms of trade in a single-coincidence meeting. Note that due to the nature of the bonds, neither the buyer's nor the seller's bond holdings will matter for q and d.

[4] This setup can be justified by either the search framework of the original LW model or the preference shocks setup of AWW.

[5] This functional form can be obtained by assuming a linear production function in effort, $q = Ze$, and a linear disutility of effort, $-e$, which is just a normalization. Inverting the production function and substituting into the disutility function gives the cost function $c(q, Z) = q/Z$.

In the DM, we must specify the protocol by which the price and quantity in any bilateral trade are determined — that is, we must define the structure by which the terms of trade are determined. We choose generalized Nash bargaining problem with the bargaining power of buyer given by θ. Denoting the portfolio of the buyer by (m_t, b_t), that of the seller by $(\tilde{m}_t, \tilde{b}_t)$, the generalized Nash bargaining problem is

$$\max_{q_t,d_t} \left[u(q_t) + \beta E_t W_{t+1}\left(m_t - d_t, b_t, S_{t+1}\right) - \beta E_t W_{t+1}\left(m_t, b_t, S_{t+1}\right)\right]^{\theta}$$
$$\times \left[-c(q_t, Z_t) + \beta E_t W_{t+1}\left(\tilde{m}_t + d_t, \tilde{b}_t, S_{t+1}\right) - \beta E_t W_{t+1}\left(\tilde{m}_t, \tilde{b}_t, S_{t+1}\right)\right]^{1-\theta}$$

subject to

$$d_t \leq m_t. \tag{7}$$

where (7) is simply a feasibility condition stating the buyer cannot spend more than he has and the threat points are the values of continuing on to the next CM in period $t+1$. Using the envelope condition (5) and the definition of χ_t

$$\max_{q_t,d_t} \left\{u(q_t) - \beta d_t \chi_t\right\}^{\theta} \left\{-c(q_t, Z_t) + \beta d_t \chi_t\right\}^{1-\theta}$$

subject to (7). In equilibrium, one can show, as LW do, (7) binds and the quantity produced solves

$$\beta \chi_t m_t = g(q_t, Z_t), \tag{8}$$

where

$$g(q, Z) \equiv \frac{\theta c(q, Z) u'(q) + (1-\theta) u(q) c_q(q, Z)}{\theta u'(q) + (1-\theta) c_q(q, Z)} \tag{9}$$

as in LW. The efficient quantity in this bilateral meeting is given by $q^*(Z_t)$ which solves $u'(q) = c_q(q, Z)$. It remains to be seen whether or not the Ramsey equilibrium will feature $q = q^*$. Because the expectation in χ_t is taken with respect to S_t, we denote the bargaining problem outcomes as $q(m_t, S_t)$ and $d(m_t, S_t)$, where the first argument is understood to be the money holdings of the buyer. Substituting this solution into the DM problem (6) and using the envelope conditions for $W_t(.)$, we get

$$V_t(m_t, b_t, S_t) = \sigma \left\{u\left[q_t\left(m_t, S_t\right)\right] - c\left[q\left(M_t, S_t\right), Z_t\right] - \beta \chi_t m_t + \beta \chi_t M_t\right\} + \beta E_t W_{t+1}\left(m_t, b_t, S_{t+1}\right).$$

The relevant envelope conditions for $V_t(.)$ are

$$V_{m,t}\left(m_t, b_t, S_t\right) = \beta \chi_t \left[\sigma \frac{u'(q)}{g_q(q, Z)} + 1 - \sigma\right] \tag{10}$$

$$V_{b_t,t}\left(m_t, b_t, S_t\right) = \beta R_t \chi_t. \tag{11}$$

where we used (8) and that $\partial q_t / \partial m_t = \beta \chi_t / g_q(q_t, Z_t)$.

2.3 Firms

In the CM, a representative firm hires labor in a competitive labor market and operates the linear production technology $Y_t = Z_t H_t$. Profit-maximization therefore implies the wage is $w_t = Z_t$ in equilibrium.

2.4 Private-Sector Equilibrium

Imposing equilibrium ($m_t = M_t$, $x_t = X_t$, etc.) and combining the firms' and households' optimality conditions, we can define the equilibrium as follows. Given policy variables $\{\tau_t^h, R_t\}_{t=0}^{\infty}$, the technology realization $\{Z_t\}_{t=0}^{\infty}$, the government spending realization $\{G_t\}_{t=0}^{\infty}$, and initial condition (M_0, B_0), equilibrium is a set of processes $\{q_t, B_t, M_t, X_t, H_t, P_t\}_{t=0}^{\infty}$ satisfying

$$U'(X_t) = \frac{A}{(1-\tau_t^h)Z_t}, \tag{12}$$

$$\beta M_t E_t \left[\frac{U'(X_{t+1})}{P_{t+1}}\right] = g(q_t, Z_t), \tag{13}$$

$$\frac{U'(X_t)}{P_t} = \beta \left[\sigma \frac{u'(q_t)}{g_q(q_t, Z_t)} + 1 - \sigma\right] E_t \left[\frac{U'(X_{t+1})}{P_{t+1}}\right], \tag{14}$$

$$\frac{U'(x_t)}{P_t} = \beta R_t E_t \left[\frac{U'(x_{t+1})}{P_{t+1}}\right]. \tag{15}$$

$$X_t + G_t = Z_t H_t, \tag{16}$$

$$M_t + B_t + P_t Z_t \tau_t^h H_t = P_t G_t + M_{t-1} + R_{t-1} B_{t-1}.$$

where we used (2) and the definition of χ_t to get $\chi_t = E_t \left[U'(x_{t+1})/P_{t+1}\right]$. Combining (14) and (15) we get a no-arbitrage condition between money and bonds

$$R_t = \sigma \frac{u'(q_t)}{g_q(q_t, Z_t)} + 1 - \sigma, \tag{17}$$

For the Ramsey problem, it will be useful to combine (13) and (14) and rearrange for real money balances,

$$\frac{M_t}{P_t} = \frac{g(q_t, Z_t)}{U'(X_t)} \left[\sigma \frac{u'(q_t)}{g_q(q_t, Z_t)} + 1 - \sigma\right]. \tag{18}$$

Furthermore, in any monetary equilibrium, $R_t \geq 1$ because otherwise households could earn unbounded profits by selling bonds and buying money. We represent this restriction in terms of allocations using (17) as

$$\sigma \left(\frac{u'(q_t)}{g_q(q_t, Z_t)} - 1\right) \geq 0. \tag{19}$$

which we will call the zero-lower-bound (ZLB) constraint.

2.5 Efficient Allocations

Before we turn to the Ramsey problem, we consider the problem of a social planner, which will be useful for understanding the margins of adjustment in our economy. The social planner chooses $\{X_t, H_t, q_t, e_t\}$ to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(X_t) - AH_t + \sigma \left[u(q_t) - \frac{q_t}{Z_t} \right] \right\} \tag{20}$$

subject to the CM resource constraint

$$X_t + G_t = Z_t H_t \tag{21}$$

and the DM production function (or DM resource constraint)

$$q_t = Z_t e_t, \tag{22}$$

where e_t is the effort exerted by sellers in the DM, which underlies the reduced-form cost function $c(q, Z) = q/Z$ as we explain in footnote 5. Efficient allocations thus are characterized by

$$u'(q_t) = \frac{1}{Z_t}, \tag{23}$$

$$U'(X_t) = \frac{A}{Z_t}, \tag{24}$$

and

$$X_t + G_t = Z_t H_t. \tag{25}$$

These efficiency conditions make clear that there are two, and only two, margins in this model; these margins are between the pairs (e_t, q_t) and (X_t, H_t).[6] This observation allows us to state the following:

Proposition 1. *The marginal rate of substitution (MRS) and marginal rate of transformation (MRT) for the pairs* (e_t, q_t)*,* (X_t, H_t) *are defined by*

$$\begin{aligned} MRS_{e_t,q_t} \equiv -u'(q_t) \qquad MRT_{e_t,q_t} &\equiv -\frac{1}{Z_t} \\ MRS_{X_t,H_t} \equiv -\frac{A}{U'(X_t)} \qquad MRT_{X_t,H_t} &\equiv -Z_t \end{aligned} \tag{26}$$

One can easily verify that one obtains (23) and (24) when one sets the MRS and MRT equal for the two pairs because socially optimal allocations are of course described by zero wedges between the MRS and MRT for all pairs of goods. The Ramsey problem, in contrast, is all about choosing the optimal pattern of wedges between the MRS and MRT for all pairs of goods. We describe next the Ramsey problem.

[6] Due to the linear utility function for effort in the DM, the marginal utility of DM effort is unity and thus it does not appear in (23).

3 Ramsey Problem

As is common in the Ramsey literature, we adopt the primal approach and cast the Ramsey problem as that of a planner that chooses allocations subject to feasibility and the need to raise exogenous government revenue, making sure the resulting allocations are implementable as a monetary equilibrium. We prove the following in Appendix A.1:

Proposition 2. *The allocations in a monetary equilibrium satisfy (16), (19), and the present-value implementability constraint (PVIC),*

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U'(X_t)X_t - AH_t + \sigma g(q_t, Z_t) \left(\frac{u'(q_t)}{g_q(q_t, Z_t)} - 1 \right) \right] = U'(X_0) \left[\frac{M_{-1} + R_{-1}B_{-1}}{P_0} \right]. \quad (27)$$

In textbook Ramsey problems, implementability constraints typically take the form $E_0 \sum_t \beta^t \sum_i U_i(x_{1t}, ..., x_{Nt})x_{it} = a_0$, where $\{x_{it}\}_{i=1}^N$ is the set of N goods the agent consumes at time t.[7] At first glance, (27) does not seem to conform to this general form because the term related to the DM, $\sigma g(q_t, Z_t)(u'(q_t)/g'(q_t) - 1)$ does not look like marginal utility of a good times the quantity of that good. However, this term does indeed have such an interpretation; we can show that the term in the PVIC is simply the product of money balances and its marginal utility.

To see this, note that from the bargaining problem and (8), $S_b(q) \equiv u(q) - g(q, Z)$ is the surplus of the buyer and therefore $S_b'(q) \equiv u'(q) - g_q(q, Z)$ is the marginal surplus of the buyer. Moreover, money has no use in the DM unless the household is a buyer, which occurs with probability σ. Thus, the marginal utility of money can be expressed as $\sigma S_b'(q)\partial q/\partial m$. From (8) we have $m = g(q, Z)/\beta\chi$ and $\partial q/\partial m = \beta\chi/g_q(q, Z)$. Combining these, we obtain the third term under the summation in the PVIC. With this interpreration, one may argue that our model looks like a MIU model, which would have a term mU_m in the PVIC. In our context, though, the marginal utility of money is linked to the fundamentals of the economy — allocations and technology — and it is not an arbitrary function.

If $\sigma = 0$, the DM shuts down and our PVIC collapses to the usual CCK PVIC *in a real model*. That is, the model collapses to a purely real model. This is due to the fact that when $\sigma = 0$, the only source of money demand shuts down in our model and the only equilibrium of the model is the nonmonetary equilibrium.

We assume the Ramsey planner is able to commit at time zero to a policy for $t \geq 1$. We thus sidestep here the potentially interesting issue of time-inconsistency in this model. The Ramsey

[7] In the CCK model, for example, instantaneous utility is defined over cash goods, credit goods, and labor, $u(c_1, c_2, l)$, and the PVIC takes the form $\sum_{t=0}^{\infty} \beta^t [u_{1t}c_{1t} + u_{2t}c_{2t} + u_{lt}l_t] = A_0$, with A_0 a function of initial money and bonds. See Chari and Kehoe (1999, p. 1676-1686) for more discussion of optimal taxation problems in general.

problem is thus to choose $\{X_t, H_t, q_t\}$ to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \{U(X_t) - AH_t + \sigma [u(q_t) - c(q_t, Z_t)]\} \tag{28}$$

subject to the CM resource constraint

$$X_t + G_t = Z_t H_t,$$

the PVIC (27), and the ZLB constraint (19), taking as given $\{G_t, Z_t\}$. In the Ramsey objective functon (28), $\sigma [u(q_t) - c(q_t, Z_t)]$ arises because the planner aggregates over the measure σ of buyers in the DM (each of whom experiences $u(q_t)$), the measure σ of sellers in the DM (each of whom experiences $c(q_t, Z_t)$), and the measure $1 - 2\sigma$ of households that do not trade in the DM. In Appendix A.2, we list the conditions that characterize the solution to this problem, along with the conditions that allow us to construct the policies and prices that support the Ramsey allocation. Thus, as we already noted, our approach is a straightforward application of Ramsey theory.

4 Completeness of the Tax System

As discussed in the introduction, an important issue in models of optimal taxation is whether or not the assumed tax instruments constitute a *complete* tax system.[8] In this section, we establish that the tax system is complete in our model. Establishing this is important for two reasons. First, at a technical level, proving completeness reaffirms that the Ramsey problem as formulated in Section 3, in which the only constraints are the sequence of CM resource constraints and the single PVIC, is indeed the correct Ramsey problem.[9] As shown by Chari and Kehoe (1999, p. 1680), Correia (1996), Armenter (2008), and many others, incompleteness of the tax system requires imposing additional constraints that reflect the incompleteness. Incompleteness is not an issue in our model, therefore we do not need to impose additional constraints. Second, it is well-known in Ramsey theory that incomplete tax systems can lead to a wide range of "non-standard" policy prescriptions in which some instruments imperfectly stand in for the ability to create certain wedges that *cannot*, by assumption of the available tax instruments, be created in a decentralized economy. Proving completeness therefore establishes that none of our results is due to any policy instrument serving as imperfect proxies for other, unavailable, instruments.

[8]For convenience, we restate the definition of Chari and Kehoe (1999, 1679-1680) that an *incomplete* tax system is in place if, for at least one pair of goods in the economy, the government has *no* policy instrument that drives a wedge between the marginal rate of substitution (MRS) and the corresponding marginal rate of transformation (MRT). If this is not the case, then the tax system is instead said to be *complete.*

[9]For the purpose of establishing completeness, the ZLB constraint is irrelevant because it stems from the need to implement a monetary equilibrium and has nothing to do with completeness/incompletness of the tax system.

As we showed in Section 2.5, there are two independent MRS/MRT pairs in our environment. Completeness of the tax system requires that each of these margins is affected by (at least) one policy instrument. To establish completeness, we first express explicitly in terms of MRS/MRT pairs the private-sector equilibrium conditions that are the analogs of the efficiency conditions (23) and (24). We do this for the case when $\theta = 1$, (which also corresponds to price-taking in the DM) because this is the important case for which we prove all our results. The same ideas would apply to the case $\theta < 1$.

The relevant equilibrium conditions for this argument, (12) and (17), simplify to

$$U'(X_t) = \frac{A}{(1-\tau_t^h)Z_t} \tag{29}$$

and

$$R_t = \sigma Z_t u'\left(q_t\right) + 1 - \sigma \tag{30}$$

because, assuming $\theta = 1$, we have $g_q\left(q, Z\right) = c_q\left(q, Z\right) = 1/Z$. Using the definitions of MRSs and MRTs presented in Proposition 1, we have

$$\frac{MRS_{e_t,q_t}}{MRT_{e_t,q_t}} = 1 + \frac{R_t - 1}{\sigma} \tag{31}$$

and

$$\frac{MRS_{X_t,H_t}}{MRT_{X_t,H_t}} = 1 - \tau_t^h. \tag{32}$$

These two conditions demonstrate that there is a unique policy instrument for each of the two independent margins in our economy. The nominal interest rate, R_t affects the (e, q) margin while the labor income tax affects the (X, H) margin. Hence, the tax system is complete in the typical sense understood in the Ramsey literature, and any result we obtain regarding the optimal R (in particular the deviation from the Friedman Rule) is not due to incompletness of the tax system.

Next, we express in the same way the first-order conditions of the Ramsey planner (which are derived in Appendix A.2); doing so gives

$$\frac{MRS_{e_t,q_t}}{MRT_{e_t,q_t}} = 1 - \frac{\xi}{1+\xi}\left[q_t u''(q_t) Z_t\right] - \frac{\iota_t}{1+\xi}\left[Z_t^2 u''(q_t)\right] \tag{33}$$

and

$$\frac{MRS_{X_t,H_t}}{MRT_{X_t,H_t}} = 1 + \left(\frac{\xi}{1+\xi}\right)\frac{U''(X_t)X_t}{U'(X_t)}, \tag{34}$$

where ξ and ι_t are the Lagrange multipliers of the Ramsey problem associated, respectively, with the PVIC and the sequence of ZLB constraints.

To conclude, we show that the Ramsey planner chooses allocations $\{q_t, e_t, X_t, H_t\}$ in order to create wedges between the MRS and MRT of (e_t, q_t) and (X_t, H_t) as given in (33) and (34). Turning to (31) and (32), these wedges can be uniquely implemented by the two policy instruments $\left(R_t, \tau_t^h\right)$ that the government has access to. Any prescriptions for R and τ^h that we derive thus are not due to incompleteness of the tax system.

5 Optimal Policy

One of our central results is that for a range of values for θ, the optimal nominal interest rate is positive. We can establish this analytically for the case $\theta = 1$, which we do next. The case $\theta = 1$ is an especially important one because Rocheteau and Wright (2005) show that for this case, bargaining yields the same outcomes as if there were competitive forces in the DM, making DM trades look less non-standard from the point of view of modern DGE theory. The $\theta = 1$ case is as conceptually close as this class of models can get to a standard CCK-type environment. For $\theta < 1$, analytical solutions are not as easy to obtain, and we resort to numerical solutions.

5.1 Optimal Deviation from the Friedman Rule

5.1.1 Proof for $\theta = 1$ (Buyer-Take-All Bargaining)

The Friedman Rule is not optimal if $\theta = 1$, as we now show:

Proposition 3. ***(Optimal Deviation from the Friedman Rule)*** *If $\theta = 1$, the optimal policy features a strictly positive net nominal interest rate in every period $t \geq 1$. Furthermore, if $u(.)$ is CRRA (constant relative risk aversion) then the optimal nominal interest rate is constant over time.*

Proof. Let ξ be the multiplier on the PVIC (27) in the Ramsey problem, and consider the Ramsey problem with the ZLB constraint dropped. The first-order condition of this problem with respect to q_t for $t \geq 1$ is given in Appendix A.2. With $\theta = 1$, we have that $g(q, Z) = c(q, Z) = q/Z$, so this FOC simplifies considerably,

$$u'(q_t) - \frac{1}{Z_t} = -\left(\frac{\xi}{1+\xi}\right) q_t u''(q_t). \tag{35}$$

Because u is strictly concave, the multiplier $\xi > 0$ under any interesting Ramsey allocation, and of course $q_t > 0$ $\forall t$ in a monetary equilibrium, the right hand side of the first order condition above is strictly positive. This implies $u'(q_t) > 1/Z_t$, which in turn implies

$$\sigma \frac{u'(q_t)}{g_q(q_t, Z_t)} + 1 - \sigma > 1,$$

imposing $g_q(q, Z) = c_q(q, Z) = 1/Z$ because $\theta = 1$. But this implies, by the equilibrium condition (17), that $R_t > 1$, so we have established that the Friedman Rule is not optimal.

Next, suppose $u(q) = q^{(1-\eta)}/(1-\eta)$. Looking at (17), we see that for R_t to be constant over time, $u'(q_t)/g_q(q_t, Z_t)$ has to be constant. With $\theta = 1$, this requires that $Z_t u'(q_t)$ is constant. The CRRA utility function has the property $q_t u''(q_t) = -\eta u'(q_t)$. Imposing this in (35) and collecting the $Z_t u'(q_t)$ terms, we have

$$Z_t u'(q_t) = \left[1 - \eta\left(\frac{\xi}{1+\xi}\right)\right]^{-1},$$

which shows that $Z_t u'(q_t)$ is constant. □

Deviations from the Friedman Rule have been obtained in other Ramsey models, as well. For example, Schmitt-Grohe and Uribe (2004a) show that a positive nominal interest can tax producers' monopoly profits, and Chugh (2006) shows that it can tax monopolistic labor suppliers' rents. We know from Ramsey theory that taxing rents is optimal because it is non-distorting. However, the deviations from the Friedman Rule in Schmitt-Grohe and Uribe (2004a) and Chugh (2006) are instances of the Ramsey planner using a positive nominal interest rate to *indirectly* tax some rent — in neither case is activity requiring money the ultimate object the Ramsey planner seeks to tax. In Section 6, we offer a rent-seizing interpretation of our result and also connect it to the results of CCK.

The solution to the Ramsey problem, independent from its actual implementation, requires q_t to be smaller than the socially efficient level. From the perspective of the results in LW and much of the ensuing related work, which invariably find that $q = q^*$ is optimal, it is surprising to entertain the idea that $q < q^*$ is *optimal* in any sense. However, as we mentioned at the outset, a Ramsey problem — which is one about financing of government activities — is inherently one about creating optimal inefficiencies. A standard result in public finance is that such inefficiencies ought to be spread across *all* final goods. Because q is of course a final good, we have $q < q^*$. In order to decentralize this feature of the Ramsey allocation, what is needed is the ability to create a wedge between the MRS and MRT of the pair (q, e) in (33). Expression (31) shows that there is a policy instrument — the nominal interest rate — that achieves precisely this. Thus, a strictly positive net nominal interest rate (i.e., $R_t > 1$) creates exactly the wedge between MRS and MRT that the Ramsey allocation requires. The deviation from the Friedman Rule does not arise as an imperfect substitute for some other instrument. In other words, as we proved the completeness of the tax system in Section 4, this result has nothing to do with incompleteness of the tax system.[10]

Left to still consider is the quantitative degree of the departure from the Friedman Rule. The rent-seizing argument we alluded to above and discuss further in Section 6 would suggest that the optimal inflation rate should be one that confiscates the entire rent, but this would imply $q = 0$. Thus, the optimal inflation rate must balance the motive to seize the money rent versus pushing q too low. Our numerical results, presented next for both $\theta = 1$ and the more general case $\theta < 1$, confirm this intuition.

[10]In Section 6 we entertain the idea of introducing a sales tax in the DM. As should be obvious from the preceding discussion, this creates an indeterminacy between its use and the use of the inflation tax.

5.1.2 Numerical Results

Obtaining analytic solutions for $\theta < 1$ is not as easy, so we study the optimal steady-state policy for this case numerically, using the full set of non-linear Ramsey conditions. Before presenting numerical results, we briefly describe the parameterization of the model. To the extent possible, we use the parameters and functional forms that LW provide, whose model is calibrated at an annual frequency to match some long-run features of the US economy.

The DM utility function is

$$u(q) = \frac{(q+b)^{1-\eta} - b^{1-\eta}}{1-\eta},$$

with $b = 0.0001$, which is a parameter that forces $u(0) = 0$, which can occur in the DM if a household does not meet another agent with whom to trade. In the CM, instantaneous utility is $B\ln(X) - H$.

We consider two cases: buyer-take-all in the bargaining problem ($\theta = 1$), which is equivalent to price-taking, and $\theta < 1$. For the former case we use $(\eta, B, \sigma) = (0.27, 2.13, 0.31)$ and for the latter case we use $(\eta, B, \sigma, \theta) = (0.39, 1.78, 0.5, 0.34)$.

The exogenous government spending and TFP processes each evolve as an AR(1) in logs,

$$\ln G_{t+1} = (1-\rho_G)\ln \bar{G} + \rho_G \ln G_t + \epsilon^G_{t+1},$$

$$\ln Z_{t+1} = \rho_Z \ln Z_t + \epsilon^Z_{t+1},$$

with $\epsilon^G \sim N(0, \sigma^2_{\epsilon^G})$ and $\epsilon^Z \sim N(0, \sigma^2_{\epsilon^Z})$. We calibrate $\bar{G} = 0.4$, so that government purchases constitute about 18 percent of total GDP in steady-state.[11] In line with Schmitt-Grohe and Uribe (2004b) and the RBC literature, we set the parameters of the stochastic processes $\sigma_{\epsilon^G} = 0.03$, $\sigma_{\epsilon^Z} = 0.023$, $\rho_G = 0.9$, and $\rho_Z = 0.82$. We set the annual subjective discount factor $\beta = 0.962$, which delivers an annual real interest rate of about 4 percent. Finally, we choose the level of steady-state government debt, an object not pinned down by the model, so that it is 45 percent of steady-state output, consistent with the parameterizations of CCK and Schmitt-Grohe and Uribe (2004b).

The solid line in Figure 1 shows the steady-state Ramsey policy and key allocation variables as functions of θ. At $\theta = 1$, the optimal nominal interest rate is about 2 percent at an annual rate; the associated optimal inflation rate is thus -1.6 percent, higher than the Friedman rate of deflation, which would be -3.4 percent in our model.

As θ falls below unity, the optimal nominal interest rate falls. This is due to a combination of the holdup problem associated with holding money when $\theta < 1$ discussed by LW and the nonmonotonicity of the Nash bargaining solution discussed by Aruoba, Rocheteau and Waller

[11] Real GDP takes into account both CM and DM output: $\sigma M/P + ZH$.

(2007). The former effect predicts that if $\theta < 1$, the buyer does not obtain the full benefit from a match, which reduces his incentives to hold money (i.e. he is held-up by the seller), causing the equilibrium q to fall. The latter affect arises from the fact that when $\theta < 1$, the level of real money balances that maximize the buyer's surplus is lower than the socially optimal level. In Section 6.1.4 we disentangle the two effects by considering a bargaining solution that satisfies individual monotonicity, and we show that for any buyer's surplus share, the Friedman Rule is not optimal. Taking into account both of these (dis)incentives, the Ramsey planner reduces the inflation tax as θ falls below unity – this balances the planner's desire to tax the buyer's surplus against the desire to reduce the effects of the holdup and nonmonotonicity problems. Because seignorage revenue (not shown) falls along with the nominal interest rate, the government's revenue shortfall must be made up with the labor tax, causing the labor tax rate to rise, as the top right panel of Figure 1 shows.

The associated responses of the allocation variables q and X are easy to understand as well. We again emphasize that q is below its Pareto-optimal level, which, given all the particulars of the LW environment, is $q^* = 1$ at the steady state. Finally, and as is intuitive, as the labor income tax rate rises with the fall in θ, hours worked and hence consumption in the CM decline.

If θ falls far enough the ZLB constraint binds, making the Friedman Rule the optimal policy. For our calibration, the ZLB constraint binds if $\theta \in (0, 0.62)$, as can be seen by the fact that the net nominal interest rate is zero over that interval. The kink when the ZLB constraint binds leads to kinks in the labor tax rate and allocations as well.

The dotted line in Figure 1 shows the allocations and implied R and τ^h that emerge from the Ramsey problem with the ZLB constraint (19) dropped. The results for $\theta \in (0.62, 1)$ are of course identical because in that region the ZLB constraint did not bind anyway. With the ZLB constraint dropped and $\theta \in (0, 0.62)$, we see that the Ramsey planner would like to implement, if it were consistent with monetary equilibrium, a negative net nominal interest rate, apparently to boost q. Of course, deflation faster than the Friedman Rule is inconsistent with a monetary steady-state equilibrium. Hence, the Friedman Rule becomes the constrained optimal policy.

5.2 Optimal Inflation Stability

We now turn to the dynamics of the Ramsey policy, which reveals our second central result: optimal inflation is very stable in the face of business-cycle magnitude shocks. To investigate the dynamic behavior of our model, we solve for the dynamic Ramsey equilibrium and simulate the model. We conduct 1000 simulations of 500 periods each and discard the first 100 periods. As in Khan, King, and Wolman (2003) and others, we assume that the initial state of the economy is the asymptotic Ramsey steady state. For each simulation, we then compute first and second moments and report the averages of these moments over the 1000 simulations. We offer some details and observations regarding solving for the dynamics of our model and then present results.

As we explain in Appendix A.2, given $\{Z_t, G_t\}$, the first-order conditions of the Ramsey problem and the feasibility condition for the CM characterize the allocations $\{q_t, X_t, H_t\}$ and the multiplier on the ZLB constraint, $\{\iota_t\}$. Then we can use the equilibrium conditions to back out $\{\tau_t^h, R_t, \pi_t\}$. To reduce computational time, we approximate both the functions $q(Z_t, G_t)$ and $X(Z_t, G_t)$, as well as $\pi(Z_t, G_t)$. Our strategy is to construct global nonlinear approximations of these functions because of the presence of the potentially occasionally-binding ZLB constraint.[12] Of interest to many practitioners, however, should be our (unreported) findings that, for the versions of the model in which we know for sure the ZLB constraint is always slack, first-order and second-order local approximations yielded results virtually identical to our global approximation.[13] To construct the approximations, we use as the functional equations the first-order conditions of the Ramsey problem with respect to q_t and X_t and the equilibrium condition (14). We use the remaining equations to solve for the other variables of interest.

Before turning to simulations, we make a few observations by inspecting the first-order conditions of the Ramsey problem. First, government spending affects only CM hours because none of the first-order conditions for q_t, X_t, and ι_t (the multiplier on the ZLB constraint) involve G_t. Once X_t is determined, H_t adjusts according to the shocks to G_t. This result follows from the quasi-linearity of preferences in the CM. Because households essentially have risk-neutral preferences over hours, fluctuations in G_t are fully reflected in H_t.[14] Second, and related, the dynamics of q_t and X_t follow the dynamics of the technology shock as the latter is the only driving force for the former. Third, for the particular utility function we choose in the CM – in fact for any CRRA utility function — the labor income tax rate is constant over time.[15] This can be viewed as the extreme case of the usual consumption-smoothing motive as spelled out in, say, Barro (1979).

Table 1 presents simulation-based moments for the key allocation and policy variables for $\theta = 1$ (which, again, is equivalent to price-taking) and for $\theta < 1$. Let us first discuss the results for $\theta = 1$. The first three rows show the dynamics of realized inflation, the labor income tax rate, and the

[12] We approximate these functions using linear combinations of Chebyshev polynomials, following Judd (1992). Results from Aruoba, Fernandez-Villaverde and Rubio-Ramirez (2006) and Aruoba, Waller, and Wright (2007) indicate that this approximation method is very accurate. While our algorithm allows the ZLB to be an occasionally binding constraint, which means the multiplier $\iota(Z_t, G_t)$ may have one or more kinks in it, our quantitative results indicate that for the parameterizations we use the ZLB either always binds or never binds.

[13] Of course, this statement only holds for sufficiently-small driving shocks; the business-cycle magnitude shocks that we assume are apparently small enough.

[14] To make this point more clear, if we shut down the technology shock, then all variables except for H_t will remain at their steady state values, and H_t will fluctuate in line with G_t.

[15] This follows from the fact that $Z_t U'(X_t)$ is constant. This can be seen easily from the first-order condition of the Ramsey planner for X_t.

net nominal interest rate under the Ramsey policy. We hone in first on the result that the optimal inflation rate is quite smooth over time, with a standard deviation of about only about 75 basis points (at an annual rate) around a mean deflation rate of 2 percent. The stable inflation rate is in sharp contrast to the extremely volatile optimal inflation rate first found by CCK in a flexible-price Ramsey model and recently verified in, among others, the flexible-price versions of Schmitt-Grohe and Uribe (2004a, 2004b), Siu (2004), and Chugh (2006, 2007).[16]

In these baseline Ramsey monetary models, inflation does not distort the relative prices of goods. It is easiest to see this in a cash-credit economy: the nominal price of both cash and credit goods is P, and the relative price depends only on the nominal interest rate, reflecting the opportunity cost of the money used to purchase the cash good. In other words, given a nominal interest rate, dynamic fluctuations in the price level do not alter the relative price between cash and credit goods and therefore do not affect equilibrium allocations. In these baseline models, then, the driving force behind price-level dynamics is just the (desirable) ability of price-level fluctuations to tailor the real returns on nominal government debt, thus avoiding the need to change other distortionary taxes in the face of shocks to the government budget. Quantitatively, assuming business-cycle magnitude shocks, realized inflation turns out to be very volatile.[17]

With money essential, this result is overturned because inflation affects the relative price of DM and CM goods. We discuss the mechanism behind the optimality of inflation stability more fully in Section 6, but the basic idea is that inflation affects the relative price between DM and CM goods in a way that simply does not exist in a baseline Ramsey model. In a baseline Ramsey model, “cash goods” and “credit goods” are assumed be technologically identical goods and have a relative price of unity. As such, variations in inflation affect both nominal prices equally. In contrast, in our model “cash goods” (q) and “credit goods” (X) do not have a fixed unit relative price. Hence, variations in inflation have the potential to distort their relative price, which would reduce the welfare of consumers. Our quantitative results show that this channel is quantitatively powerful.

Other Ramsey models make the prediction that inflation stability is optimal, most notably

[16]From their simulation experiments, CCK report a mean inflation rate of -0.44 percent with a standard deviation of 19.93; Schmitt-Grohe and Uribe (2004a) report a mean inflation rate of -3.39 percent with a standard deviation of 7.47 percent; Siu (2004) reports a mean inflation rate of -2.59 percent with a standard deviation of 5.08 percent; and Chugh (2006) reports a mean inflation rate of -4.01 percent with a standard deviation of 6.96 percent. Each of these models is calibrated in a slightly different way from the others, but the general result that comes through is clear: with flexible-prices, the Ramsey inflation rate is quite volatile.

[17]We also point out that with the assumption of full commitment on the part of the Ramsey planner, the use of state-contingent inflation is not a manifestation of time-inconsistent policy. The “surprise” in surprise inflation is due solely to the unpredictable components of government spending and technology, and not due to a retreat on past promises.

Schmitt-Grohe and Uribe (2004b), Siu (2004) and Chugh (2006). The basic mechanism behind their inflation stability results is also a relative-price distortion caused by inflation; however, these models all rely on nominal rigidities to generate the relative-price effect. We emphasize that in our model, prices are fully flexible and yet inflation causes relative price distortions. The real frictions underlying monetary exchange are behind our result.

An important feature of inflation dynamics is that it displays high persistence. In the benchmark CCK model, which assumed fixed capital, inflation persistence is virtually zero no matter how persistent are the driving shocks. Chugh (2007) shows that allowing for capital accumulation or habit formation in preferences generates optimal inflation persistence, but clearly here we have that result with neither of these features. The high persistence of Ramsey-optimal inflation is also helpful in understanding the low volatility of inflation, as we discuss in Section 6.

Finally, consider the results for $\theta < 1$, reported in the second panel of Table 1. The means of the variables of interest are of course in line with the steady state results. Compared to the price-taking case ($\theta = 1$), the average labor income tax rate is higher and average consumption (both CM and DM) and GDP are lower. The Friedman rule is optimal with an average deflation equal to the rate of time preference. In our simulations, which are driven by business-cycle-magnitude shocks, we find that the optimal nominal interest rate is once again constant over time.[18] We also find that q_t is less volatile if $\theta < 1$, which in turn causes GDP to be less volatile and the correlations of other variables with GDP to be lower than what we find when $\theta = 1$. In short, we find that except for the expected changes in the means, the dynamic behavior of the Ramsey problem with $\theta < 1$ is qualitatively identical to the case with $\theta = 1$.

6 Discussion

Here, we expand on the analysis and intuition behind our main results. We divide our discussion into two parts, first further analyzing the non-optimality of the Friedman rule and then further analyzing the optimality of stable inflation.

6.1 Optimal Deviation from the Friedman Rule

6.1.1 Alternative Tax Instruments

We proved in Section 4 that our environment features a complete set of tax instruments, and the Ramsey planner has exactly one tax instrument to create a wedge between the MRS and MRT of

[18]However, this result is not robust to large shocks. In simulations not reported here, we considered very large negative technology shocks (a standard deviation of more than 60 percent of the average). When hit by these large shocks, the Ramsey solution includes small deviations from the Friedman rule.

each margin in the economy. By definition of completeness, allowing the government any additional policy instruments necessarily creates redundancies across policy instruments, which means there is no model-based justification for preferring the use of one policy instrument over another. This point is implied by the discussion of Chari and Kehoe (1999, p. 1679) regarding the availability of *more* than one policy instrument along at least one of the margins in the economy. Nonetheless, in this section we briefly consider introducing an additional instrument, a sales tax in the DM, which of course leads to redundancies across policy instruments. Indeed, as we show, introducing a sales tax generates an indeterminacy of the decentralization of the Ramsey policy, and there is no model-based justification for preferring one decentralization over another.[19]

We introduce a DM sales tax in the following way: after a buyer turns over to a seller $\tilde{P}_t q_t$ units of money in a DM trade ($\tilde{P}_t$ denotes the nominal price of DM goods), the seller must remit $\tau_t^d \tilde{P}_t q_t$ to the government in the next CM, which, given our timing assumptions, occurs in period $t+1$.[20] After re-solving the model with this modification, one of the key equilibrium conditions, (17), is replaced by

$$R_t = \sigma(1 - \tau_t^d) Z_t u'(q_t) + 1 - \sigma, \tag{36}$$

along other related changes in other equations. We then find that the PVIC and therefore the Ramsey problem and its solution (in terms of allocations) are unchanged by this modification.[21] What can now differ, of course, is the precise way in which the Ramsey allocation is decentralized. Given the Ramsey allocation, an indeterminacy arises between the nominal interest rate and the DM sales tax, as condition (36) shows. In particular, any non-negative nominal interest rate can be supported with an appropriate sales tax/subsidy in the DM. One particular policy would be to set the Friedman Rule ($R = 1$) along with whatever DM sales tax rate is required. While this is certainly model-admissible, there is no justification within the context of the model for this particular decentralization. Within the context of the model, the sales tax and the nominal interest rate are *perfectly-substitutable* policy instruments. Thus, if one were to prefer this "restoration of the Friedman Rule," it must be driven by considerations outside the scope of the model.

One could just as easily step outside the cash-good/credit-good model of CCK in order to justify the optimality of a *deviation* from the Friedman Rule. In particular, consider the following

[19] We have the case where $\theta = 1$ in mind in our discussion below, which is the case we prove our results above. Detailed derivations are available upon request.

[20] Thus, we assume that it is the sellers that pass along the sales tax receipts to the government; assuming that it is buyers that remit taxes would formally lead to the same conclusion. Regarding the timing, we can suppose that the government receives the revenue in the DM but waits until the next CM to spend it. Because asset markets are not open in the DM, the government cannot invest this extra revenue in an interest-bearing asset (nor can sellers, for that matter).

[21] The PVIC is the same as in (27), except for the fact that τ_{-1}^d appears as part of the constant term on the right-hand-side. Because optimization begins in period zero, we treat τ_{-1}^d as fixed and, in particular, equal to zero.

modification to CCK's environment: in addition to the standard policy tools of the nominal interest rate and the labor tax rate (which together constitute a complete tax system in their framework), allow a separate tax/subsidy on cash-good consumption. One would obtain an equilibrium condition very similar to (36) (modified for the particular environment, of course). The Ramsey allocation in this altered version of the CCK model would be identical to that in the original specification. Regarding decentralization, then, the Ramsey allocation could be supported by *any* non-negative nominal interest rate along with an appropriate tax/subsidy on cash goods. If one were to prefer this "deviation from the Friedman Rule" predicted by this innocuously-altered version of the CCK model, it must be driven by considerations outside the scope of the model.

Thus, one can indeed obtain optimality of a deviation from the Friedman Rule in the CCK environment. But if one's contention is that introducing redundant policy instruments in the CCK model as a way of obtaining this result is uninteresting, then the same logic leads to the conclusion that "restoring the Friedman Rule" by introducing redundant policy instruments in our model is also uninteresting.[22]

6.1.2 Comparison of Results with CCK

Our conclusion that the Friedman Rule is not optimal of course differs from that of CCK. At a technical level, it can be reconciled with their result by considering basic principles of public finance. In CCK, optimality of the Friedman Rule depends on a certain class of utility functions. In particular, CCK require cash goods and credit goods to enter the utility function homothetically and separably from leisure. Similarly, in Chari and Kehoe's (1999) MIU model, money and consumption must enter utility homothetically and separably from leisure in order for the Friedman Rule to be optimal. These results are essentially an application of the uniform taxation result of Atkinson and Stiglitz (1980), requiring cash-good consumption and credit-good consumption (or money and consumption) to be taxed uniformly; a deviation from the Friedman Rule would mean that cash goods are taxed more heavily than credit goods, hence cannot be optimal.

With $\theta = 1$, the instantaneous social utility function (the one that the Ramsey planner maximizes) in our model takes the form $\mathcal{U}(q, X, e, h) = \sigma [u(q) - e] + U(X) - AH$ (e denotes the effort of sellers in the DM). If we interpret q as the cash good and X as the credit good, q and X must enter $\mathcal{U}$ homothetically to satisfy the CCK requirement. Our Proposition 3 admits this case. For example, we can set $u(.) = U(.)$ and Proposition 3 of course still holds. However, realize that, given the structure of the LW model, $e = q/Z$. The *reduced-form* social utility function thus has

[22] We also considered a direct tax on money balances levied in the CM. Not surprisingly, it leads to the same kind of indeterminacy of policy as the sales tax in the DM. Finally, as it is well understood from CCK, a consumption tax in the CM would create an indeterminacy between this tax and the labor income tax in the CM and will not affect the results regarding the optimality of the Friedman rule.

the form $\tilde{\mathcal{U}}(q, X, h) = \sigma\left[u(q) - q/Z\right] + U(X) - AH$, and q and X will in general not enter $\tilde{\mathcal{U}}(q, X, h)$ homothetically. In other words, even though we have homothetic preferences in terms of the primitives, the reduced-form representation, which is the one relevant for the Ramsey planner, does not feature homothetic preferences. Our results thus reconcile in a technical sense with those of CCK.

Given the lack of homotheticity of the social welfare function, there is no presumption that the CCK result carries over to our environment. If we had simply started with the reduced-form social welfare function, one may have easily guessed the CCK result would not hold; doing so, of course, would have begged the question of how such social preferences arise. In our setting, the social welfare function arises from the primitives of the LW environment. As we show below, the suboptimality of the Friedman Rule also holds in some natural and existing extensions of the LW environment. Before turning to those extensions, however, we offer a more conceptual reconciliation of our results with those of CCK and standard Ramsey theory.

6.1.3 Rents Associated with DM Activity

Given the fundamental need to tax activities requiring money, we think one useful way of considering the deviation from the Friedman Rule is that it stems from the presence of a rent associated with DM activity. To make ideas as clear as possible, consider again the case $\theta = 1$. Recall that the entire surplus of a DM trade is obtained by the buyer with $\theta = 1$. We noted above that the instantaneous social welfare function in our model takes the form $\sigma[u(q) - e] + U(X) - AH$. Define $W(q, X) \equiv \sigma[u(q) - e] + U(X)$. The e term in $W(.)$ can be thought of as a scarce, or fixed, factor in the social utility function. More precisely, from the perspective of a *buyer*, e is inelastic with respect to any of *his* actions because e represents the (utility) cost to the *seller*. The social welfare function aggregates preferences over both households that turn out to be buyers and those that turn out to be sellers. With $\theta = 1$, the full surplus of DM trades accrues to buyers, a feature of equilibrium that the Ramsey planner of course understands; hence, maximization of the social welfare function can be interpreted as maximization of simply buyers' utility. From the Ramsey point of view, the e term can be therefore viewed as a fixed factor in preferences.

With this way of thinking about the optimal policy problem, our results and interpretation fit squarely into something pointed out by Chari and Kehoe (1999, p. 1734-1735): if preferences exhibit decreasing returns in cash goods (our q) and credit goods (our X) because of a scarce factor that affects preferences of cash goods, then the Friedman Rule is not optimal.[23] Their literal interpretation was that the fixed factor was something supplied inelastically by the representative

[23] As another exercise, we solved numerically for the optimal long-run policy in a basic CCK model assuming that preferences over cash goods and credit goods took the ad-hoc form $v(c_1, c_2) = \sigma[u(c_1) - c_1/Z] + U(c_2)$. Even with $u(.) = U(.)$, we found, not surprisingly, that the optimal nominal interest rate was positive.

household. In our model, the latter part of this intuition is modified to something inelastically supplied by *some* household because there is no representative household in the DM – rather, ex-post, there are three types of households. The Ramsey planner, though, aggregates over all types, so the "reduced-form" preferences feature this type of scarce factor in preferences. Note that this not an artifact of some strange aggregation scheme: the Ramsey planner considers the ex-ante utility of all agents, and households *ex-ante* do not know their DM type.

Fixed factors are the source of rents. In our model, the scarce factor and hence the source of rents is fundamentally related to DM activity. Hence arises our model's prescription to tax DM activity, be it through inflation, which we think is quite natural, through the DM sales taxes we described above, through the less natural direct taxation of money balances we also described above, or any combination of these. Of course, in our model, the fixed factor is not something we arbitrarily introduce into preferences to obtain a deviation from the Friedman Rule — rather, it arises from the primitives of the environment.

6.1.4 Alternative Model Specifications

We consider two alternative specifications for which we can prove variants of our Proposition 3 above. First, we drop the within-period separability assumption for utility and suppose the household's utility function is given by $U(X, q)$. We are able to prove that if $U_{qx}(X, q) < 0$, i.e. if q and X are Edgeworth substitutes, then a deviation from the Friedman rule continues to be optimal.

Second, we consider a different pricing scheme in the DM. We assume that the buyer and the seller split the surplus with the share θ received by the buyer. As Aruoba, Rocheteau and Waller (2007) show, the only change in the equilibrium is that the function $g(q, Z)$ in (9) is replaced by

$$g(q, Z) = (1 - \theta)u(q) + \theta c(q, Z)$$

with all equilibrium conditions and Ramsey optimality conditions unchanged. We are able to prove that for any $\theta > 0$ the Friedman rule is not optimal. This substantiates our intuition in Section 5.1.2 that the non-monotonicity of the Nash bargaining solution is the reason underlying the Friedman rule becoming the constrained optimal policy as θ falls.

6.2 Optimal Inflation Stability

Here, we offer some other perspectives on our inflation stability result.

6.2.1 Relative Price Distortions

The fact that inflation exhibits quite low volatility in the face of shocks seems to stem from stabilization of the relative price $\tilde{P}_t/P_t$ of DM and CM goods (where, as above, $\tilde{P}_t$ denotes the nominal price of a DM good). Using equilibrium conditions, we can show that this relative price, in an arbitrary private-sector equilibrium, can be expressed as

$$\frac{\tilde{P}_t}{P_t} = \frac{R_t}{Z_t U'(X_t)}.$$

We showed in Proposition 3 that for CRRA preferences the Ramsey equilibrium features $R_t = R$ for all t. Furthermore, in the Ramsey equilibrium, $Z_t U'(X_t)$ is also constant over time for CRRA preferences (see footnote 13). Thus, the relative price $\tilde{P}_t/P_t$ is constant in the Ramsey equilibrium.

The issue thus becomes how or why (near-complete) stabilization of inflation is associated with or required for (complete) stabilization of this relative price. Another way to express the equilibrium value of this relative price is, using (14) and (18),

$$\frac{\tilde{P}_t}{P_t} = \frac{g(q_t, Z_t)}{\beta E_t \left[U'(X_{t+1})/\pi_{t+1} \right]}.$$

Due to covariance with $U'(X_{t+1})$, the term in the denominator on the right-hand-side is of course not simply expected future inflation, but our intuition for how movements in π_t affect $\tilde{P}_t/P_t$ stems from how movements in π_t transmit into movements in $E_t\pi_{t+1}$.

We can track the dynamic behavior of $E_t\pi_{t+1}$ in our model by forecasting, using our approximated decision rules, the expected one-period-ahead Ramsey inflation rate along our simulations. Across all our simulations, the correlation between π_t and $E_t\pi_{t+1}$ is extremely high, at 0.99. More importantly, the volatilities of π_t and $E_t\pi_{t+1}$ are virtually identical. These two observations lead to the conclusion that the dynamics of π_t and $E_t\pi_{t+1}$ are virtually identical under the Ramsey plan in our model, which we can also verify simply by inspecting time-series plots of these variables from our simulations.

This is the opposite of what occurs in a basic CCK model. In a basic CCK model, π_t is very volatile, yet $E_t\pi_{t+1}$ is very stable, always remaining very close to its unconditional (i.e., deterministic steady-state) mean. The fact that the dynamics of π_t and $E_t\pi_{t+1}$ are extremely similar in our model is consistent with the finding we noted above that Ramsey-optimal inflation is highly persistent in our framework. The fact that the dynamics of π_t and $E_t\pi_{t+1}$ are extremely dissimilar in a basic CCK model is consistent with the CCK result that Ramsey-optimal inflation displays virtually zero correlation

The upshot of this analysis is that unanticipated inflation leaves an imprint on expected future (one-period-ahead) inflation in our model, and low volatility in the latter is required to maintain a stable relative price across the DM and the CM. This kind of policy transmission channel simply does not exist in a CCK model because contemporaneous inflation is so little correlated with conditional expectations of future inflation and hence does not distort activity.

6.2.2 Dissipation of Exogenous Volatility

Our settings for the standard deviations of the shocks to TFP and government purchases are in line with CCK, Schmitt-Grohe and Uribe (2004b), Siu (2004), and Chugh (2006). In all these models, relatively low volatility of inflation goes hand-in-hand with relatively high volatility of labor income tax rates. Given that our model predicts that *both* inflation *and* labor income tax rates exhibit very low volatility, it is of interest to know where the volatility of the exogenous driving forces "goes" in our model.

One mechanism through which volatility can be dissipated is through quasi-linearity of preferences. Because households are essentially risk-neutral with respect to their CM level of hours worked, some volatility of aggregate H_t is tolerable from the Ramsey point of view. In the first row of Table 2, we report the volatility of key Ramsey policy and allocation variables in a version of the CCK model featuring linear-in-labor preferences.[24] The volatility of inflation, at 4.25 percent, is a bit lower than the CCK benchmark (see footnote 16 above), and the volatility of H_t, at 1.24 percent, is a bit higher than the CCK benchmark. Thus, quasi-linearity of preferences and the attendant rise in volatility of equilibrium labor explains part of the dissipation of volatility. But it clearly does not explain all of it, given that our model's inflation volatility is still about an order of magnitude smaller.

To further understand the result, we also tabulate the volatilities of Ramsey-equilibrium real money balances and government debt outstanding at the end of period $t-1$. By the latter, we mean the nominal debt issued by the government in period $t-1$, valued at the price level of period $t-1$; that is, we define $b_{t-1} \equiv B_{t-1}/P_{t-1}$. Important to realize is that b_{t-1} is distinct from the real debt *repayments* made by the government in period t — in terms of the price level in the period of repayment, real repayments are $R_{t-1}b_{t-1}/\pi_t$. As the second row of Table 2 displays, in the baseline calibration of our model, the volatilities of both real money balances and real government debt outstanding are an order of magnitude larger than in the quasi-linear CCK model. Apparently, the Ramsey planner's ability to generate volatility in these policy instruments permits low volatility of inflation.

As the third, fourth, and fifth rows of Table 2 show, the Ramsey-optimal volatility of real money balances is sensitive to η, which governs curvature of DM preferences. To conceptually cast things

[24]Because this result was unavailable in the literature — all results reported in the studies we have cited feature curvature in preferences with respect to leisure — we implemented a quasi-linear version of the basic CCK model ourselves.

in terms of a MIU model, higher values of η mean household preferences are more risk-averse in real money holdings; a decline in volatility of M_t/P_t as η rises naturally follows. However, notice that real government debt obligations remain three to four times as volatile compared to the quasi-linear CCK model, while inflation variability remains around one percent.

The way we understand these findings, then, is that the Ramsey government essentially has two ways of engineering a particular time-path of government debt: keep real debt *issuance* (valued at the price level of the period of issuance) relatively state-non-contingent and then use state-contingent inflation to vary the debt returns; or make real debt issuance (valued at the price level of the period of issuance) itself somewhat state contingent, which then requires less state-contingency in inflation to vary the debt returns.

We view this analysis as perhaps suggesting a new way to view existing Ramsey results: what ultimately matters for the Ramsey planner is volatility in $R_{t-1}b_{t-1}/\pi_t$. The literature has typically understood the mechanism to be one in which the required volatility must come through π_t — for example, see the discussion in Chari and Kehoe (1999, p. 1741-1742). However, due to the forward-looking nature of (perhaps quite complicated) equilibrium relationships, it is conceivable that the required volatility can be engineered through variations in b_t itself, along with the Ramsey planner's manipulation of appropriate covariances, etc., which we do not investigate. The forward-looking nature of equilibrium relationships seems to admit this possibility. Thoroughly parsing out such effects is beyond the scope of our work; we thus leave to future research a more thorough disentangling of such effects.

7 Conclusion

We view our work and results as a first step in taking more seriously the new class of micro-founded models of money as a laboratory for studying policy questions. Given the general properties of the environment we study, our central findings are that the Friedman Rule is typically not the optimal policy and that inflation fluctuates very little over time. These findings are opposite those of the workhorse CCK flexible-price Ramsey model. Despite the flexibility of prices in our model, our inflation-stability result is much more in line with results from models featuring nominal rigidities than models featuring flexible prices.

In a companion paper, Aruoba and Chugh (2008), we also study optimal capital taxation in the Aruoba, Waller, and Wright (2007) extension of the Lagos and Wright (2005) model. The findings we report here all carry over to the environment with capital; the main new finding is that the optimal policy calls for a subsidy to capital accumulation, counter to the standard Ramsey prescription of setting zero long-run capital taxes. In light of recent results regarding asset taxation in the new dynamic public finance literature — for example, Golosov, Kocherlakota, and

Tsyvinski (2003) and Albanesi and Sleet (2006) — and Albanesi and Armenter's (2007) attempt at reconciling them with standard Ramsey results, it may be interesting to know how or whether the capital-taxation implications of a micro-founded model of monetary exchange square with this growing body of knowledge.

There are of course a number of ways one might want to modify our framework. Monopoly power in goods and labor markets are thought by many to be important realistic features. It would be straightforward to introduce monopoly power in the CM. The results of Schmitt-Grohe and Uribe (2004a) and Chugh (2006) suggest that inflation in such an environment would be partly a direct tax on the money rent we identify and partly an indirect tax on producers' and labor suppliers' rents. It may be interesting to know quantitatively how these direct and indirect uses of the inflation tax interact.

Once one has monopoly power in the CM, one could go further in adding elements monetary policy makers often think are important, such as nominal rigidities in prices and wages. For example, Aruoba and Schorfheide (2007) show that when one replaces the typical "cashless" assumption of a Calvo-type model with micro-founded frictions for the use of cash, welfare implications are altered significantly. Investigating both long-run and short-run optimal policy — be it monetary alone or monetary and fiscal jointly — in the presence of both temporary nominal rigidities and deep-rooted frictions underlying monetary trade also seems likely to yield new insights.

Pushing our first step in different directions, another interesting issue to study may be the nature of and solution to the time-inconsistency problem of the Ramsey policy in this sort of environment. It is not clear how the time-consistency results of, say, Alvarez, Kehoe, and Neumeyer (2004) or Persson, Persson, and Svensson (2006), would extend to our environment. Neither is it clear how the emerging results in the aforementioned new dynamic public finance literature, which places at center stage distributional concerns, might extend to a version of our environment in which money holdings were allowed to differ across households.

This paper is also part of a larger effort underway in the literature studying the policy implications of deep-rooted, non-Walrasian frictions in goods markets, money markets, and labor markets. A central focus of this larger project has been to think about what sorts of departures from typical Walrasian frameworks impinge importantly on conventional policy prescriptions derived from standard models. Much progress has recently been made using micro-founded models of labor market transactions — for example, Walsh (2005), Trigari (2006), Lubik and Krause (2007), Faia (2008), and Arseneau and Chugh (2008), to name just a few. We think much progress is in the offing using micro-founded models of money as well.

Recent developments in understanding the micro-foundations of monetary exchange are sometimes viewed as simply having provided justification for the reduced-form models of money commonly used in practice, not least of all because they superficially end up resembling the reduced-form models. Our results throw in to question the conclusion that they must therefore yield the same answers to interesting questions as existing models. We think it may be worthwhile to re-examine a number of issues in monetary policy using this now-tractable framework.

A The Ramsey Problem

A.1 Proof of Proposition 2

That allocations from a monetary equilibrium should satisfy the CM feasibility condition (16) and the zero-lower-bound constraint (19) is obvious.

Using the household optimality conditions (12), (14), and (15) along with the equilibrium conditions, we now derive the present-value implementability constraint the Ramsey planner must respect. We begin by summing the budget constrains of the three types of agents (buyer, sellers and nonparticipants in the previous DM) to get

$$P_t X_t + B_t + M_t = P_t w_t (1-\tau_t^h) H_t + M_{t-1} + R_{t-1} B_{t-1}. \tag{37}$$

To construct the present-value implementability constraint, begin by multiplying the flow budget constraint by $\beta^t U'(X_t)/P_t$ and summing from $t = 0..\infty$,

$$E_0 \sum_{t=0}^{\infty} \beta^t U'(X_t) X_t + \sum_{t=0}^{\infty} \beta^t U'(X_t) \frac{B_t}{P_t} + \sum_{t=0}^{\infty} \beta^t U'(X_t) \frac{M_t}{P_t} =$$
$$\sum_{t=0}^{\infty} \beta^t U'(X_t)(1-\tau_t^h) w_t H_t + \sum_{t=0}^{\infty} \beta^t U'(X_t) \frac{M_{t-1}}{P_t} + \sum_{t=0}^{\infty} \beta^t U'(X_t) \frac{R_{t-1}B_{t-1}}{P_t}.$$

We point out that, as usual in a dynamic Ramsey problem assuming commitment to the time-zero policy, any E_t terms that appear in intermediate expressions are eliminated by the law of iterated expectations because the entire implementability constraint is conditioned on the time-zero information set, hence the E_0. For ease of exposition, we therefore proceed dropping E_t operators that would appear in intermediate expressions as well as the E_0 operator because it is understood to be present in all subsequent expressions.

Substitute into the second term on the left-hand-side using expression (15) to get

$$\sum_{t=0}^{\infty} \beta^t U'(X_t) X_t + \sum_{t=0}^{\infty} \beta^{t+1} U'(x_{t+1}) \frac{R_t B_t}{P_{t+1}} + \sum_{t=0}^{\infty} \beta^t U'(X_t) \frac{M_t}{P_t} =$$
$$\sum_{t=0}^{\infty} \beta^t U'(X_t)(1-\tau_t^h) w_t H_t + \sum_{t=0}^{\infty} \beta^t U'(X_t) \frac{M_{t-1}}{P_t} + \sum_{t=0}^{\infty} \beta^t U'(X_t) \frac{R_{t-1}B_{t-1}}{P_t}.$$

The second summation on the left-hand-side cancels with the the last summation on the right-hand-side to leave only the initial bond position,

$$\sum_{t=0}^{\infty} \beta^t U'(X_t) X_t + \sum_{t=0}^{\infty} \beta^t U'(X_t) \frac{M_t}{P_t} = \sum_{t=0}^{\infty} \beta^t U'(X_t)(1-\tau_t^h) w_t H_t + \sum_{t=0}^{\infty} \beta^t U'(X_t) \frac{M_{t-1}}{P_t} + U'(x_0) \frac{R_{-1}B_{-1}}{P_0}.$$

Next, substitute into the second term on the left-hand-side using (14) to get

$$\sum_{t=0}^{\infty}\beta^t U'(X_t)X_t + \sum_{t=0}^{\infty}\beta^{t+1}U'(x_{t+1})\frac{M_t}{P_{t+1}}\left[\sigma\frac{u'(q_t)}{g'(q_t)}+1-\sigma\right] =$$
$$\sum_{t=0}^{\infty}\beta^t U'(X_t)(1-\tau_t^h)w_t H_t + \sum_{t=0}^{\infty}\beta^t U'(X_t)\frac{M_{t-1}}{P_t} + U'(x_0)\frac{R_{-1}B_{-1}}{P_0}.$$

Expand the second summation on the left-hand-side to get

$$\sum_{t=0}^{\infty}\beta^t U'(X_t)X_t + \sum_{t=0}^{\infty}\beta^{t+1}U'(x_{t+1})\frac{M_t}{P_{t+1}} + \sigma\sum_{t=0}^{\infty}\beta^{t+1}U'(x_{t+1})\frac{M_t}{P_{t+1}}\left[\frac{u'(q_t)}{g'(q_t)}-1\right]$$
$$= \sum_{t=0}^{\infty}\beta^t U'(X_t)(1-\tau_t^h)w_t H_t + \sum_{t=0}^{\infty}\beta^t U'(X_t)\frac{M_{t-1}}{P_t} + U'(x_0)\frac{R_{-1}B_{-1}}{P_0}.$$

Cancel the second summation on the left-hand-side with the second summation on the right-hand-side to leave only the initial money holdings,

$$\sum_{t=0}^{\infty}\beta^t U'(X_t)X_t + \sigma\sum_{t=0}^{\infty}\beta^{t+1}U'(x_{t+1})\frac{M_t}{P_{t+1}}\left[\frac{u'(q_t)}{g'(q_t)}-1\right] = \sum_{t=0}^{\infty}\beta^t U'(X_t)(1-\tau_t^h)w_t H_t + U'(x_0)\left[\frac{M_{-1}+R_{-1}B_{-1}}{P_0}\right].$$

Using (12), we can substitute into the first term on the right-hand-side to get

$$\sum_{t=0}^{\infty}\beta^t U'(X_t)X_t - \sum_{t=0}^{\infty}\beta^t AH_t + \sigma\sum_{t=0}^{\infty}\beta^{t+1}U'(x_{t+1})\frac{M_t}{P_{t+1}}\left[\frac{u'(q_t)}{g_q(q_t,Z_t)}-1\right] = U'(x_0)\left[\frac{M_{-1}+R_{-1}B_{-1}}{P_0}\right].$$

Writing $\frac{M_t}{P_{t+1}} = \frac{M_t}{P_t}\frac{P_t}{P_{t+1}}$, express this as

$$\sum_{t=0}^{\infty}\beta^t U'(X_t)X_t - \sum_{t=0}^{\infty}\beta^t AH_t + \sigma\sum_{t=0}^{\infty}\beta^{t+1}U'(x_{t+1})\frac{M_t}{P_t}\frac{P_t}{P_{t+1}}\left[\frac{u'(q_t)}{g_q(q_t,Z_t)}-1\right] = U'(x_0)\left[\frac{M_{-1}+R_{-1}B_{-1}}{P_0}\right].$$

Use (18) to substitute for M_t/P_t,

$$\sum_{t=0}^{\infty}\beta^t U'(X_t)X_t - \sum_{t=0}^{\infty}\beta^t AH_t + \sigma\sum_{t=0}^{\infty}\beta^{t+1}\frac{U'(x_{t+1})}{P_{t+1}}\frac{P_t}{U'(X_t)}g(q_t,Z_t)\left[\sigma\frac{u'(q_t)}{g_q(q_t,Z_t)}+1-\sigma\right]\left[\frac{u'(q_t)}{g_q(q_t,Z_t)}-1\right]$$
$$= U'(x_0)\left[\frac{M_{-1}+R_{-1}B_{-1}}{P_0}\right].$$

Finally, from (14), we can make the substitution $\beta E_t\left[\frac{U'(x_{t+1})}{P_{t+1}}\right] = \frac{U'(X_t)}{P_t}\left[\sigma\frac{u'(q_t)}{g_q(q_t,Z_t)}+1-\sigma\right]^{-1}$ in the third summation on the left-hand-side. Cancelling terms and reintroducing the E_0 operator leaves us with

$$E_0\sum_{t=0}^{\infty}\beta^t\left[U'(X_t)X_t - AH_t + \sigma g(q_t,Z_t)\left(\frac{u'(q_t)}{g_q(q_t,Z_t)}-1\right)\right] = U'(x_0)\left[\frac{M_{-1}+R_{-1}B_{-1}}{P_0}\right],$$

which is the present-value implementability (PVIC) constraint for the Ramsey problem in the LW model. Any allocation that satisfies this restriction, the resource constraint, and the ZLB constraint can be supported as a monetary equilibrium; furthermore, the allocations from any monetary equilibrium can be described by these three conditions.

A.2 The Solution to the Ramsey Problem

The Kuhn-Tucker conditions for the problem in Section 3 are

$$
\begin{gathered}
[u'(q_t) - c_q(q_t, Z_t)] \\
+\xi \left[g_q(q_t, Z_t) \left(\frac{u'(q_t)}{g_q(q_t, Z_t)} - 1 \right) + g(q_t, Z_t) \left(\frac{u''(q_t) g_q(q_t, Z_t) - u'(q_t) g_{qq}(q_t, Z_t)}{[g_q(q_t, Z_t)]^2} \right) \right] \\
+\iota_t \left[\frac{u''(q_t) g_q(q_t, Z_t) - u'(q_t) g_q q(q_t, Z_t)}{g_q(q_t, Z_t)^2} \right] = 0,
\end{gathered} \tag{38}
$$

$$U'(X_t) - \frac{A}{Z_t} + \xi \left[U''(X_t) X_t + U'(X_t) - \frac{A}{Z_t} \right] = 0,$$

$$X_t + G_t = Z_t H_t$$

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U'(X_t) X_t - A H_t + \sigma g(q_t, Z_t) \left(\frac{u'(q_t)}{g_q(q_t, Z_t)} - 1 \right) \right] = U'(x_0) \left[\frac{M_{-1} + R_{-1} B_{-1}}{P_0} \right]$$

$$\iota_t \left[\frac{u'(q_t)}{g_q(q_t, Z_t)} - 1 \right] = 0, \text{ and } \iota_t \geq 0$$

We can represent the right-hand side of the PVIC in terms of allocations as

$$U'(X) \left[\frac{g(q, 1)}{\beta U'(X)} + \frac{\mathcal{B}}{\beta} \right]$$

where $\mathcal{B}$ is the steady state real bond balances and variables without subscripts are steady state values.

With these FOCs in hand, we proceed as follows. Imposing steady state on these conditions, we solve for the steady state values of allocations and the multiplier ξ. Next, given ξ and $\{Z_t, G_t\}$, the conditions above characterize $\{q_t, X_t, \iota_t\}$ and (16) defines $\{H_t\}$. Finally, we back out policies $\{\tau_t^h, R_t\}$ from (12) and (17) statically, and inflation can be obtained from solving (14) dynamically.

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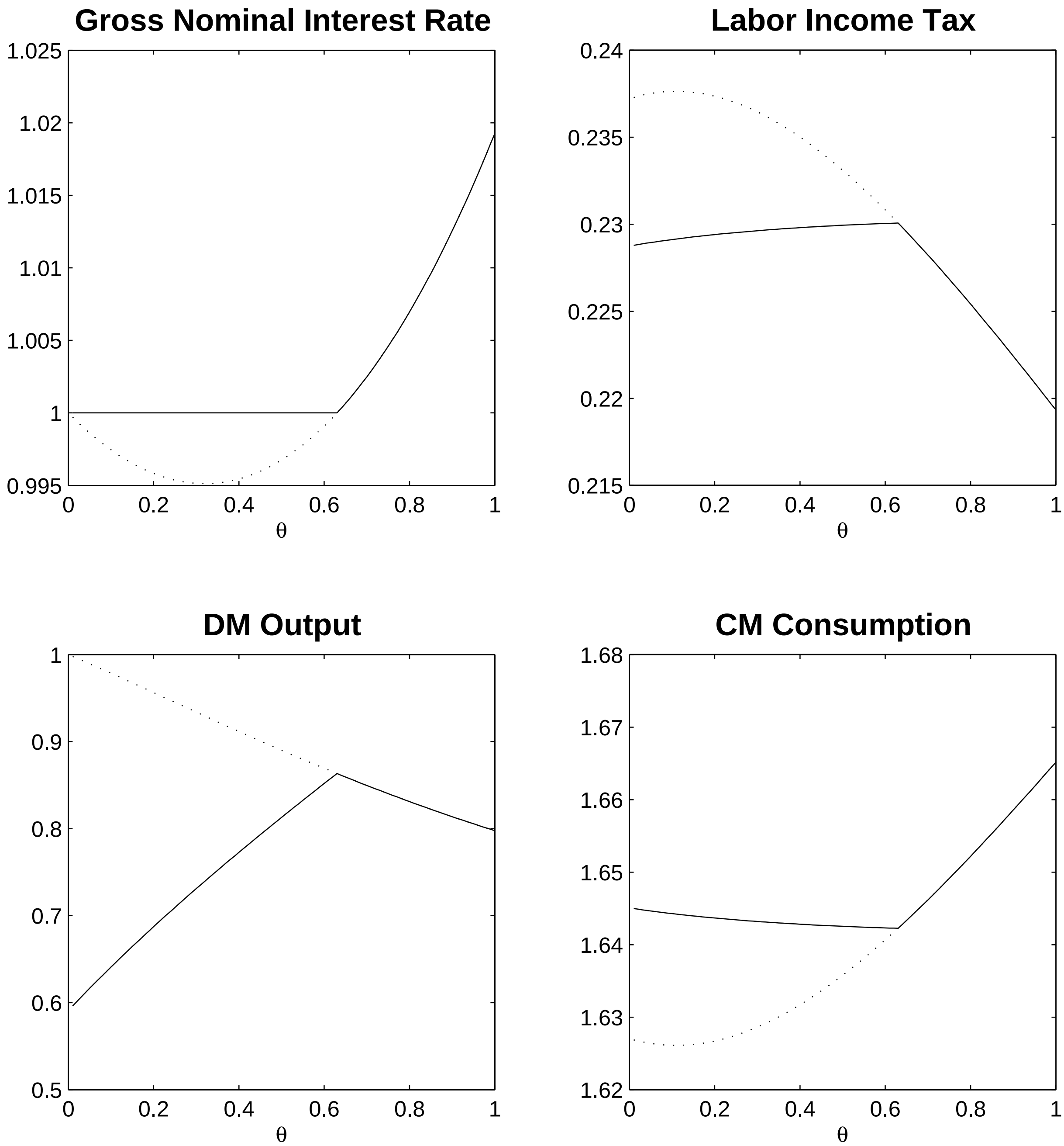
Figure 1 - Ramsey Steady-State in the Basic Model



Notes : Ramsey steady-state policy and allocation as a function of θ with the ZLB constraint (solid line) and without the ZLB constraint (dotted line).

Table 1 - Simulation Results

(a) Price-Taking / Bargaining ($\theta = 1$)

Variable	Mean	Std. Dev.	Auto corr.	Corr(x, Y)	Corr(x, Z)	Corr(x, G)
$\pi - 1$	-1.937	0.761	0.810	0.960	0.998	0.004
τ^h	0.220	0	—	—	—	—
$R - 1$	1.931	0	—	—	—	—
q	0.807	0.120	0.810	0.960	0.997	0.004
X	1.666	0.066	0.811	0.961	1.000	0.005
H	2.066	0.031	0.870	-0.246	-0.499	0.859
GDP	2.267	0.099	0.817	1	0.961	0.274
P_{DM}/P	0.795	0	—	—	—	—

(b) Bargaining ($\theta < 1$)

Variable	Mean	Std. Dev.	Auto corr.	Corr(x, GDP)	Corr(x, Z)	Corr(x, G)
$\pi - 1$	-3.795	0.746	0.810	0.951	0.998	0.004
τ^h	0.277	0	—	—	—	—
$R - 1$	0	0	—	—	—	—
q	0.576	0.058	0.811	0.952	0.999	0.004
X	1.288	0.051	0.811	0.952	1.000	0.005
H	1.688	0.031	0.870	-0.216	-0.499	0.859
GDP	2.026	0.089	0.819	1	0.952	0.304
P_{DM}/P	1.173	0	—	—	—	—

Notes: Simulation-based moments. Inflation and nominal interest rate reported in percentage points.

Table 2 - Volatility of Ramsey Policies and Allocations

Model	M_t/P_t	B_{t-1}/P_{t-1}	GDP_t	H_t	π_t	Long-run π
CCK, quasi-linear preferences	1.96	0.008	3.60	1.24	4.252	-3.87
Baseline model ($\eta = 0.27$)	15.08	0.070	4.33	1.51	0.761	-2.20
Model with $\eta = 1$	3.93	0.037	3.44	1.49	0.894	3.55
Model with $\eta = 2$	1.97	0.033	3.37	1.44	0.961	11.31
Model with $\eta = 5$	0.80	0.031	3.31	1.35	1.095	26.77

Notes: Simulation-based volatilities of key Ramsey policy and allocation variables for the case where $\theta = 1$. All volatilities reported in terms of percentage standard deviations, except end-of-period bond obligations and inflation, which are reported as absolute standard deviations. Inflation statistics reported in percentage points.