Chapter 14
Money in the Intertemporal Model

We have for the most part ignored the role of money and thus monetary policy in our study so far. This is because the main issues we have been studying – in particular, the idea of optimal decision-making by the representative agent along the consumption-leisure and consumption-savings margins – do not require explicit consideration of money. This is also in part due to the fact that it has proven somewhat difficult to model money in modern macroeconomic models. However, our discussion of RBC theory and New Keynesian theory is the right opportunity to try to think about money in our micro-founded environment because it illustrates a fundamental difference between the two theories and a fundamental split in modern macroeconomic theory.

There are many ways to introduce "money" in a modern macroeconomic model. Indeed, there is much debate about what the most illuminating way of introducing money a model economy is. The general point we wish to make here does not depend on how we model money, so we use one of the most-widely used approaches, the **money-in-the-utility function (MIU)** approach. Very simply put, this approach simply inserts (real) money as an argument to the representative consumer's utility function.

To develop the basic idea, let's use a simple dynamic setting reminiscent of our intertemporal consumption-leisure model.[94] We now augment the utility function to include money as an argument. Suppose the representative consumer's period-t utility function is

$$u\left(c_t, \frac{M_t}{P_t}, l_t\right),$$

in which M_t / P_t is the consumer's real money balances, with M_t nominal money holdings. The period-t budget constraint of the consumer is

$$P_t c_t + M_{t+1} = M_t + W_t n_t,$$

where, as usual, P_t is the nominal price of consumption and W_t is the nominal hourly wage. Notice the timing of the budget constraint: in period t, the consumer chooses nominal money holdings to carry into period $t+1$.[95] Thus, the timing of the model is the following: the consumer begins period t with nominal money holdings M_t, and then makes choices about how much to work and how much to consume in period t and how much money to carry into period $t+1$. At the time period-t decisions are made, the price P_t and the nominal interest rate i_t are known. However – and this is an important

[94] Strictly speaking, we should use the consumption-savings model because it has a more well-defined notion of "inflation," namely, the rate of change

[95] Mechanically, we know this because it is M_{t+1}, rather than M_t, that appears on the left-hand-side of the budget constraint, and the left-hand-side represents "outlays" in period t.

point – money is not only a choice variable, but also has a random (shock) component because the nominal money supply is controlled by the central bank. Thus, because monetary policy is at least somewhat unexplainable – and, at the very least, outside the control of the representative consumer – it is useful to think of money as a shifter. The specific way we will think of money as a shifter is the following: after period-t decisions have been made by the consumer (including an optimal choice of M_{t+1}), the central bank announces how much money will **actually** be in the economy in period $t+1$, which may or may not be equal to what the consumer's optimal choice of M_{t+1} is.

The main issue we are interested in studying is how changes in M_{t+1} by the central bank affect the nominal price level of the economy as well as consumption (and, by implication, GDP) in the economy. To study this main issue, we can actually ignore the consumption-leisure margin, so including leisure and labor earnings in the model is only a nod towards realism here. It is only the consumption-money margin that interests us here.

As with any consumer choice problem, at the optimal choice, the marginal rate of substitution between the objects being chosen equals the (relevant) price ratio between the two objects. Denote by i_t the nominal interest rate between period-t and period-$t+1$ on some **alternative asset** in the economy, which is not explicitly modeled in the above budget constraint.[96] As mentioned above, i_t is set at the time period-t decisions are made. Note well that i_t is **not** an interest rate on money. Indeed, we are assuming that money pays no interest.[97] With this, it can be shown[98] that the optimality condition along the consumption-margin is

$$\frac{u_{m_{t+1}}}{u_{c_{t+1}}} = i_t \tag{1.1}$$

where $u_{c_{t+1}}$ denotes, as usual, the marginal utility of period-$t+1$ consumption, and now $u_{m_{t+1}}$ denotes the marginal utility of period-$t+1$ **real money holdings** (lower-case m_t denotes real money – that is, $m_t \equiv M_t / P_t$**).** Note well the time subscripts in this optimality condition – because of our timing assumption that the central bank determines M_{t+1} at the end of period t, it is the consumption-money margin in period $t+1$ that is potentially affected by the monetary policy action.

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[96] For example, it could be simply the nominal asset A_t that we included in our original simple two-period model, except now we would interpret A_t to be some aggregate asset **excluding** money – whereas earlier, the asset A_t in principle could have been money.

[97] Some forms of money do, of course pay interest – for example, checkable deposits, a major component of the M1 notion of money, usually do pay interest. We instead are thinking here of money as simply cash, which pays zero interest.

[98] The derivation, which relies on Lagrangian methods, is presented at the end of this section.

The reason that the nominal interest rate i_t between period t and $t+1$ is the relevant relative price between money and consumption is that it represents an opportunity cost of holding money. Any resources (wealth) that are held in the form of money are resources that are not held in the form of the alternative asset that pays interest.[99] Interest income is foregone due to money holdings and thus represents an opportunity cost – hence, a relevant price – of money.

When the period-t consumption choice is made, there is an implicit decision for period-$t+1$ consumption made. To gain intuition for this idea, recall our simple two-period model. Because of the LBC, once c_1 was decided, c_2 was simultaneously decided. Now think of an economy with arbitrary many periods as an economy in which there are an infinite number of overlapping two-period timeframes. The first two-period timeframe is the period-1/period-2 margin. The second two-period timeframe is the period-2/period-3 timeframe. The third two-period timeframe is the period-3/period-4 timeframe. And so on. With arbitrary many periods, the consumer thus is able to re-set his period-$t+1$ consumption after period-t decisions are made, but because the period-t decision represented an optimal choice along the period-t/period-$t+1$ margin, the consumer does not want to have to re-set his choice unless he must.

Now suppose after period-t decisions are made, the central bank decides to change the money supply M_{t+1}. For concreteness, let's say the central bank increases the money supply M_{t+1}. This change in monetary policy must affect the consumption-money optimality condition (1.1) somehow. The interest rate i_t, as mentioned already, is known at the time period-t decisions are made, so it will not adjust. So the entire adjustment must occur in the MRS – meaning, because the MRS is ultimately just a function of choices, choices and/or prices will have to change.

To illustrate how this necessary adjustment may occur, suppose the utility function is $u\left(c_t, \frac{M_t}{P_t}, l_t\right) = \ln(c_t) + \ln\left(\frac{M_t}{P_t}\right) + v(l_t)$, in which we leave the sub-utility function over leisure arbitrary because it does not affect the issue we are studying. The marginal utility functions with respect to consumption and real money are thus $u_{c_t} = \frac{1}{c_t}$ and $u_{m_t} = \frac{1}{M_t / P_t}$. The MRS at time $t+1$ is thus

$$\frac{u_{m_{t+1}}}{u_{c_{t+1}}} = \frac{1/(M_{t+1}/P_{t+1})}{1/c_{t+1}} = \frac{P_{t+1}c_{t+1}}{M_{t+1}} \tag{1.2}$$

[99] For example, suppose the only alternative asset is a nominal bond that pays the interest rate i_t. Every dollar of wealth held as money then entails foregoing i_t dollars of interest if it had been held instead as a bond.

Expression (1.2) makes clear the nature of the adjustment that must take place when M_{t+1} unexpectedly rises: either P_{t+1} must rise (by the same percentage as the change in M_{t+1}) or c_{t+1} must rise (by the same percentage as the change in M_{t+1}) or both must rise (each by some percentage less than the change in M_{t+1}). The conclusion about which will rise is the fundamental debate between RBC theory and New Keynesian theory.

Recall the idea developed above that consumers in period t have already, in effect, made an optimal consumption decision for period $t+1$.[100] But they now unexpectedly have more money for period $t+1$. Feeling flush with cash, consumers will raise their demand for (all) goods. Facing this increased demand, market forces will put upward pressure on (all) goods' prices. In RBC theory, prices are completely flexible – that is, prices can adjust costlessly and immediately. Thus, in the RBC view, the entire adjustment of the economy due to the money injection can be borne by a change in P_{t+1} and no change in c_{t+1}. So in the RBC view, no change in the consumption allocation (c_t, c_{t+1}) across time will occur. Hence, all monetary policy affects is the inflation rate in the economy and does not affect at all consumption (and, by extension, GDP) in the economy. Money is thus said to be **neutral** in RBC theory – monetary policy is **neutral** when it affects only nominal prices in the economy and has no effect at all on consumption/output.

In contrast, consider New Keynesian theory, a central assumption of which is price stickiness. Consider the most extreme form of price-stickiness possible: nominal prices can never change (because of, say, infinite menu costs that always render price changes prohibitive). In this case, the change necessitated by the monetary policy action will be borne by an adjustment of consumption c_{t+1} as consumers raise their demand for (all) goods because they are flush with cash. Of course, New Keynesian theory does not hold that prices never change, but rather that it takes some time for all prices in the economy to change – because of staggered price-setting, for example, as we have studied. Thus, in New Keynesian theory, inflation does not fully adjust when the monetary policy action occurs, so the quantity of consumption must bear some of the adjustment as well. **Eventually,** when all firms have been allowed to change prices, inflation will have fully absorbed the monetary policy action. But until that happens, monetary policy is **non-neutral** in the New Keynesian view because it does affect consumption/output.[101]

[100] Think in terms of the two-period model here: once consumption in period 1 is chosen, consumption in period 2 is pinned down by the LBC (that is, there is no other possible choice of consumption in period 2 possible that satisfies the LBC).

[101] Another way to state this is that in the **short-run** in New Keynesian theory (that length of time during which at least some firms have not yet adjusted prices), monetary policy is non-neutral, even though in the **long-run** it is neutral. In contrast, in RBC theory, monetary policy is neutral in both the short-run and in the long-run.

Derivation of Consumption-Money Optimality Condition

Here we show how to derive the consumption-money optimality condition in the MIU model. The derivation relies on Lagrangian techniques. To simplify the presentation, we drop leisure and labor income completely in this derivation because, as we saw in the main discussion, it plays no role in the main issue.

The goal of the consumer is to maximize lifetime utility

$$\sum_{t=0}^{T} u\left(c_t, \frac{M_t}{P_t}\right),$$

where T is the (finite) lifetime of the representative consumer,[102] subject to the budget constraint,

$$P_t c_t + M_{t+1} = M_t .$$

The period-t choice variables are c_t and M_{t+1}. Construct the lifetime Lagrangian,

$$L = \sum_{t=0}^{T}\left[u\left(c_t, \frac{M_t}{P_t}\right) + \frac{\lambda_t}{P_t}\left(M_t - P_t c_t - M_{t+1}\right)\right],$$

where we have introduced λ_t / P_t as the Lagrange multiplier on the period budget constraint. For any $t \in (0, T-1)$, the first-order-conditions with respect to c_t and M_{t+1} are, respectively:

$$u_{c_t} - \lambda_t = 0$$

$$u_{m_{t+1}} \cdot \frac{1}{P_{t+1}} - \lambda_t \cdot \frac{1}{P_t} + \lambda_{t+1} \cdot \frac{1}{P_{t+1}} = 0 .$$

Combining these two (along with the analogous first-order-condition on consumption in period $t+1$), we get

$$u_{m_{t+1}} \cdot \frac{1}{P_{t+1}} - u_{c_t} \cdot \frac{1}{P_t} = -u_{c_{t+1}} \cdot \frac{1}{P_{t+1}} .$$

Multiply this expression through by P_{t+1} and use the fact that $P_{t+1} / P_t = 1 + \pi_{t+1}$ to get

$$u_{m_{t+1}} - u_{c_t} \cdot (1 + \pi_{t+1}) = -u_{c_{t+1}} .$$

Finally, move the second term on the left-hand-side to the right-hand-side and divide the entire expression by $u_{c_{t+1}}$ to get

$$\frac{u_{m_{t+1}}}{u_{c_{t+1}}} = \frac{u_{c_t}}{u_{c_{t+1}}} \cdot (1 + \pi_{t+1}) - 1 .$$

From the consumption-savings model, we know that $u_{c_t} / u_{c_{t+1}} = 1 + r_t$, so we can write this expression as

[102] We solve the finite-life problem, rather than the more conventional infinite-horizon version, to avoid introducing time-discounting to render the solution stationary (it is a trivial extension to allow this, but is irrelevant for our main derivation).

$$\frac{u_{m_{t+1}}}{u_{c_{t+1}}} = (1+r_t)(1+\pi_{t+1}) - 1,$$

which is the optimality condition we examined (following application of the Fisher equation) in the main discussion. Using the exact Fisher equation, we can instead express this optimality condition as

$$\frac{u_{m_{t+1}}}{u_{c_{t+1}}} = i_t,$$

as desired.

MIU – An Extended Look

Here, we take a more in-depth look at the money-in-the-utility function (MIU) model, in which we modify the basic model we already examined in four ways. The most important way in which we extend the model is to explicitly introduce two assets, a **nominal bond** and a real asset which carries a real interest rate, which allows us to apply the asset-pricing framework we studied earlier. A consequence of this extension is that we will be able to formally derive the exact Fisher relation. We also modify the MIU model in three more cosmetic ways: we now allow an infinite horizon, rather than a finite horizon; we modify the timing of the MIU setup slightly to facilitate analysis with a nominal bond in the environment; and we right away drop the leisure component of the model, since, as we saw before, the main issue we wish to highlight is the intertemporal role of money and nominal interest rates.

Government Bond Market

First, let's consider the bond market. We will assume that the bonds in our model are government bonds.[103] Moreover, we assume all bonds are nominal bonds, meaning that they pay back a fixed amount of currency.[104] We will speak of a single government bond market within a country, even though there are many different types of bonds issued by governments, distinguished primarily by their maturity length and face value. A bond's **maturity length** is the time from issuance until the full value of the bond is repaid to the bond-holder, while a bond's **face value** is the full value that is repaid upon maturity. For example, the U.S. government issues one-month Treasury bills, three-month Treasury

[103] There exist also corporate bonds (bonds issued by companies) and hence markets for corporate bonds, which are important markets. However, for our purposes, it is irrelevant which types of bonds exist, so we will ignore corporate bond markets.

[104] The U.S. government, along with many other countries, also issues inflation-protected (inflation-indexed) bonds, whose dollar payoff at maturity depends on the realized rate of inflation between issuance of the bond and maturity of the bond. Thus, this type of bond has a currency payoff which is unknown at the time of issuance, in contrast to a conventional (unindexed) bonds. Inflation-protected bonds are a relatively recent development in bond markets, despite their seeming obviousness as a valuable financial asset.

bills, six-month Treasury bills, two-year Treasury notes, three-year Treasury notes, five-year Treasury notes, and ten-year Treasury notes of various face values.[105]

Bonds are simply loans, a point that is often misunderstood. Regardless of a bond's maturity length and face value, a government bond is simply a loan that a bond-holder makes to the government to be repaid at a later date with interest. The amount to be repaid at the pre-specified date is the bond's face value.[106]

Because the face value is not repaid until some time in the future, the amount that a bond-holder would be willing to pay for a bond of face value FV dollars is something less than FV dollars. The reason for this is simply the time-discounting of future values that you encountered in introductory microeconomics and introductory macroeconomics. For example, \$100 one year from now is likely worth less than \$100 to you right now – in other words, you are likely to be willing to accept something less than \$100 at this instant in lieu of receiving nothing now and \$100 one year from today.

Because of time-discounting, the period-t price of a bond, denoted P_t^b, is related to its face value FV and the nominal interest rate i_t, which represents the nominal interest rate between period t and period t+1, in the following way:

$$P_t^b = \frac{FV}{1+i_t} \tag{1.3}$$

The way this expression is written makes it seem that it defines the price of a bond. In fact, this relationship instead defines the nominal interest rate i because at any point in time a bond's face value and the amount investors are willing to pay are known. Thus knowing P_t^b and FV defines i_t -- we can emphasize this relationship by simply solving expression (1.3) for the nominal interest rate:

$$i_t = \frac{FV}{P_t^b} - 1 \tag{1.4}$$

Expressions (1.3) and (1.4) are obviously equivalent to each other. Implicit in the way we write the above expressions is that bonds are held for "one period." That is, the maturity length of bonds in our theoretical model will always be one period. For the

[105] And until a few years ago, a 30-year Treasury bond (commonly referred to in financial circles as the "long bond"), although it was recently announced that the 30-year bond will be returning. You can find an introduction to various types of Treasury securities available at http://www.publicdebt.treas.gov/com/comintro.htm.

[106] There are two main types of bonds – coupon bonds and zero-coupon bonds. A coupon bond is one that makes interest payments (called coupon payments) to the bond-holder at specified times before a final payment of the face value at the maturity date, while a zero-coupon bond offers no intermediate payments before the payment of the face value at the maturity date. For convenience, we will suppose that all bonds are zero-coupon bonds.

main issues we want to consider, this simplification does not matter. We could of course assume longer holding periods.

Timing

As mentioned above, we alter the timing of the model a bit compared to our initial look at the MIU model. Suppose the representative consumer begins period t with M_{t-1} units of currency, a_{t-1} units of a "real" asset (think of these as "stocks" from our study of stock-pricing models), and B_{t-1} units of nominal bonds.[107] Without loss of generality, we will assume that the face value of each bond is $FV = 1$. In reality, of course, bonds come in different denominations (i.e., a $100 bond, a $1,000 bond, etc.), but this is simply a scaling issue. That is, if we want to speak of "one $100 bond," we can simply consider "100 $1 bonds." With a face value of 1, from the above we immediately have that in our model the price of a bond is $P_t^b = \frac{1}{1+i_t}$.

In period t, the consumer receives some nominal income Y_t over which he has no control (recall, we are dropping the leisure aspect of the model). The consumer also receives the proceeds of his bond holdings carried into period t, which is $FV \cdot B_{t-1}$ (that is, each unit of bonds carried into the period yields FV units of currency), **and** the proceeds of stock holdings. As before, we will denote proceeds of stock holdings as $(S_t + D_t)a_{t-1}$. As we stated above, we will assume without loss of generality that $FV = 1$, in which case the proceeds of maturing bonds is simply B_{t-1}. In period t, the consumer must choose his consumption c_t for period t as well as how much nominal money M_t, how many nominal bonds B_t, and how many stocks a_t he wishes to carry into period $t+1$. Each unit of bonds the consumer purchases in period t to carry into the following period costs P_t^b units of currency, and each unit of stock costs S_t units of currency in period t.

With this timing, the period-t flow budget constraint of the representative consumer is

$$P_t c_t + P_t^b B_t + M_t + S_t a_t = Y_t + M_{t-1} + B_{t-1} + (S_t + D_t)a_{t-1}.$$

As always, an analogous expression holds in every period of the economy – for example, the period t+1 flow budget constraint is $P_{t+1}c_{t+1} + P_{t+1}^b B_{t+1} + M_{t+1} + S_{t+1}a_{t+1} = Y_{t+1} + M_t + B_t + (S_{t+1} + D_{t+1})a_t$, and so on.

[107] Rather than M_t units of money as in our previous timing. Thus, here M_{t-1} and B_{t-1}, which are both stock variables, are holdings of money and bonds, respectively, at the very end of period $t-1$ or, equivalently, at the very beginning of period t.

Preferences

The representative consumer's instantaneous utility function is defined over consumption and real money balances,

$$u\left(c_t, \frac{M_t}{P_t}\right).$$

With an infinite-period model, we re-introduce the notion of (constant) time discounting, so that lifetime discounted utility from period t forward is

$$u\left(c_t, \frac{M_t}{P_t}\right) + \beta u\left(c_{t+1}, \frac{M_{t+1}}{P_{t+1}}\right) + \beta^2 u\left(c_{t+2}, \frac{M_{t+1}}{P_{t+2}}\right) + \ldots,$$

where β (as always, a numerical value between zero and one) again represents the subjective discount factor. Note that, as stated above, we have dropped leisure from the model completely. Also note that M_t / P_t, the consumer's **real** money balances, enters the utility function, whereas it is M_t, the consumer's **nominal** money holdings, the consumer is free to choose. Nominal money holdings, which is under the consumer's control, divided by the price level, **which is outside the individual's control,** is what enters the instantaneous utility function. This has the technical implication that when we differentiate through the Lagrangian, we have to use the chain rule from calculus when taking the first-order-condition with respect to M_t.

The last modification we make to our model is to eliminate the "randomness" in monetary policy we allowed in our initial look. This is because our focus here is on the relationship between money, bonds, and stocks as alternative assets, rather then on issues of neutrality of money.

Optimal Choices

Setting up the sequential lifetime Lagrangian in the usual way,[108] we have the following first-order conditions with respect to c_t, a_t, M_t, and B_t, respectively:

[108] You should confirm that you are able to set up the problem – the formulation follows very closely that in our infinite-period stock-pricing model.

$$u_1\left(c_t, \frac{M_t}{P_t}\right) - \lambda_t P_t = 0,$$
$$-\lambda_t S_t + \beta\lambda_{t+1}(S_{t+1} + D_{t+1}) = 0,$$
$$\frac{u_2\left(c_t, \frac{M_t}{P_t}\right)}{P_t} - \lambda_t + \beta\lambda_{t+1} = 0,$$
$$-\lambda_t P_t^b + \beta\lambda_{t+1} = 0$$

The first first-order condition states the usual result that the marginal utility of consumption equals the Lagrange multiplier (scaled by the price level P_t). The second first-order condition is our familiar stock-pricing equation. The third first-order condition is that on nominal money holdings; the first term on the left-hand-side involves the term $1/P_t$ because, as stated above, we need to use the chain rule from calculus because it is **nominal** money the consumer chooses but it is **real** money that enters the utility function. The fourth first-order condition is that on bond holdings.

We again delve into a bit of finance theory. We can rearrange the first-order condition on bond holdings to get

$$P_t^b = \frac{\beta\lambda_{t+1}}{\lambda_t}.$$

Recall from our study of stock-pricing that $\beta\lambda_{t+1}/\lambda_t$ is the pricing kernel of the economy. What the above expression states is that the price of a nominal bond equals the pricing kernel **times one.**[109] Or, stated from the opposite perspective, the pricing kernel of an economy equals the price of a nominal bond. There is thus a crucial link between bond prices and stock prices – stock prices can thus be said to be keyed (partially) off of bonds prices. Note that the above expression is of the same general form as the stock-pricing equation we encountered earlier – the price of an asset (P_t^b) depends on a pricing kernel and a future payoff (which is simply $FV = 1$). Bonds are thus priced using the general type of asset-pricing equation we used to price stocks. The big-picture, finance-theoretic, lesson to take away here is that asset-pricing equations invariably have the same general form, regardless of what specific type of asset is being considered.

Continuing, the first-order condition on a_t gives us

$$S_t = \frac{\beta\lambda_{t+1}}{\lambda_t}(S_{t+1} + D_{t+1}),$$

[109] The "one" here is simply the payoff of the nominal bond in our model – that is, we assumed that the face value, hence the payoff, of the bond is $FV = 1$.

which is our usual stock price condition. From what we now know, we can alternatively express the stock-price as

$$S_t = P_t^b (S_{t+1} + D_{t+1}),$$

which explicitly demonstrates the link between bond prices and stock prices.

Fisher Relation

We can derive the exact Fisher equation as an implication of optimal choices in this model, rather than as a relationship which is imposed on the model as we have done thus far. To see this, begin with the last expression, $S_t = P_t^b (S_{t+1} + D_{t+1})$. Next, divide this expression through by the price level P_t (which is distinct from the price of a bond P_t^b), to get

$$\frac{S_t}{P_t} = P_t^b \frac{(S_{t+1} + D_{t+1})}{P_t}.$$

Next, on the right-hand-side, multiply and divide by P_{t+1} / P_{t+1} (which is of course just multiplying by one, which is always a valid operation to conduct…) to arrive at

$$\frac{S_t}{P_t} = P_t^b \frac{(S_{t+1} + D_{t+1})}{P_{t+1}} \cdot \frac{P_{t+1}}{P_t}.$$

The **real** price of stock purchased in period t is S_t / P_t (because it is divided by the current price level), while the **real** payoff in period t+1 of the stock purchased in period t+1 is $(S_{t+1} + D_{t+1}) / P_{t+1}$ (because it is divided by the future price level). The period-(t+1) real payoff divided by the period-t real price is the **real return** on the asset – that is, it is the object we have heretofore been calling the real interest rate.[110] Letting r_t denote the real interest rate between period t and period t+1, we therefore have that

$$1 + r_t = \frac{(S_{t+1} + D_{t+1}) / P_{t+1}}{S_t / P_t}.$$

With this, we can write the previous expression as

[110] Stocks are considered to be "real" assets because their payoff is generally not fixed in currency terms, whereas bonds are considered to be "nominal" assets because their payoff is generally fixed in currency terms (non-indexed bonds, at least).

$$\frac{1}{P_t^b} = (1+r_t)\cdot\frac{P_{t+1}}{P_t}.$$

Only one more step remains in deriving the exact Fisher relation from first principles. To finish the derivation, note that, by construction and our definitions, $1/P_t^b = 1+i_t$, and $P_{t+1}/P_t = 1+\pi_{t+1}$, so that the previous expression becomes

$$1+i_t = (1+r_t)(1+\pi_{t+1}),$$

which **is** the exact Fisher relation. Thus, using a model with a nominal asset (the bond) and a real asset (the stock), we have derived the exact Fisher relation as an implication of optimal choices on the part of the representative consumer, rather than simply assuming it is a relationship that holds when we initially encountered it in our study of the two-period consumption-savings model. In short, the Fisher equation emerges naturally in a model featuring both nominal assets and real assets.

The economic intuition behind the Fisher equation is that it links the returns available on nominal assets (nominal bonds) and the returns available on real assets (stocks). The linkage is through inflation; once the nominal returns of bonds are adjusted by inflation, **their returns on nominal bonds are exactly equal to the returns on stocks.** This type of idea – that, once returns are converted into comparable units, they are equalized when consumers are making optimal choices – goes by the terminology of **no-arbitrage** in finance theory. No-arbitrage relationships are key building blocks of more advanced finance theory; we defer richer consideration of issues stemming from such relationships to a more advanced course on finance theory.

Nominal Interest Rate as the Opportunity Cost of Holding Money

Finally, let's consider how the nominal interest rate i_t affects the consumption-money margin. Rewrite the first-order condition on money holdings from above as

$$\frac{u_2\left(c_t, \frac{M_t}{P_t}\right)}{P_t} - \lambda_t = -\beta\lambda_{t+1}.$$

The first-order condition on bonds gives us $\beta\lambda_{t+1} = \lambda_t P_t^b$, so inserting this in the previous expression, we have

$$\frac{u_2\left(c_t, \frac{M_t}{P_t}\right)}{P_t} - \lambda_t = -\lambda_t P_t^b.$$

Dividing through by λ_t,

$$\frac{u_2\left(c_t, \frac{M_t}{P_t}\right)}{\lambda_t P_t} - 1 = -P_t^b.$$

Next, we can use the first-order condition on consumption to replace the $\lambda_t P_t$ term on the left-hand-side, giving us

$$\frac{u_2\left(c_t, \frac{M_t}{P_t}\right)}{u_1\left(c_t, \frac{M_t}{P_t}\right)} = 1 - P_t^b.$$

The term on the left-hand-side now is just the MRS between money and consumption – i.e., it is the ratio of the marginal utility of (real) money to the marginal utility of consumption. As for the right-hand-side of this expression, with $P_t^b = 1/(1+i_t)$, it reduces to

$$\frac{u_2\left(c_t, \frac{M_t}{P_t}\right)}{u_1\left(c_t, \frac{M_t}{P_t}\right)} = 1 - \frac{1}{1+i_t}.$$

One final algebraic simplification gives us

$$\frac{u_2\left(c_t, \frac{M_t}{P_t}\right)}{u_1\left(c_t, \frac{M_t}{P_t}\right)} = \frac{i_t}{1+i_t},$$

which states that the MRS between period-t real money and period-t consumption equals a function of the nominal interest rate at the representative agent's optimal choice. We will refer to this as the **consumption-money optimality condition,** in complete with the consumption-leisure optimality condition and the consumption-savings optimality condition with which we have become familiar. The consumption-money optimality condition states that when consumers are making their optimal choices, they choose consumption and **real** money holdings in such a way as to equate their MRS between consumption and real money to a function of the **nominal** interest rate. Except for the fact that it is the period-t MRS, not the period-(t+1) MRS, this condition is the same as the one we saw in our first look at the MIU model. The difference in timing of the MRS is simply a consequence of the different in timing/notation we've assumed here regarding money holdings.

Steady-State and the Monetarist Link Between Money Growth and Inflation

We have been considering an infinite-period model. As we were able to do in our earlier, simpler, infinite-period model absent money, it is useful to consider steady-states. In our explicitly monetary model here, considering the steady-state will starkly reveal a relationship important to all of monetary theory, a relationship between inflation and the rate of growth of the nominal money supply of the economy. This way of thinking about inflation goes under the name of "monetarism."[111]

To facilitate our consideration of this issue, let's specialize our utility function to

$$u\left(c_t, \frac{M_t}{P_t}\right) = \ln c_t + \ln \frac{M_t}{P_t}.$$

None of the conclusions we will reach depend on this particular functional form, but it will allow us to conduct our algebra quite simply, hence we invoke it.

The marginal utility functions associated with this utility form are obviously $u_1\left(c_t, \frac{M_t}{P_t}\right) = \frac{1}{c_t}$ and $u_2\left(c_t, \frac{M_t}{P_t}\right) = \frac{1}{M_t / P_t}$. This means that the period-t consumption-money optimality condition can be written as

$$\frac{c_t}{M_t / P_t} = \frac{i_t}{1+i_t},$$

or, after a bit of rearranging,

$$\frac{M_t}{P_t} = \left(\frac{1+i_t}{i_t}\right) c_t.$$

Of course, a completely-analogous condition holds in period t-1 (or period t-2, or period t+1, etc.):

[111] One of the most-often quoted sayings by the late Milton Friedman, the 1976 Nobel laureate in economics, is that "inflation is everywhere and always a monetary phenomenon," which has commonly been interpreted to mean that it is the actions of the central bank of an economy (in particular, how the central bank manages the money supply of an economy) that **alone** determine the rate of inflation in the economy. As we are about to see, precisely speaking, only in the steady state (i.e., in the "long run" or "on average") is inflation a purely monetary phenomenon.

$$\frac{M_{t-1}}{P_{t-1}} = \left(\frac{1+i_{t-1}}{i_{t-1}}\right)c_{t-1}.$$

Let's combine these time-t and time-(t-1) versions of the consumption-money optimality condition by dividing one by the other; doing so gives us

$$\frac{M_t / P_t}{M_{t-1} / P_{t-1}} = \frac{c_t}{c_{t-1}}\left(\frac{1+i_t}{i_t}\right)\left(\frac{i_{t-1}}{1+i_{t-1}}\right).$$

Reorganizing terms a bit, we have

$$\frac{M_t}{M_{t-1}}\frac{P_{t-1}}{P_t} = \frac{c_t}{c_{t-1}}\left(\frac{1+i_t}{i_t}\right)\left(\frac{i_{t-1}}{1+i_{t-1}}\right).$$

From our usual definition of inflation, we have that $\frac{P_{t-1}}{P_t} = \frac{1}{1+\pi_t}$. Let's define the *growth rate of nominal money* in an analogous way. Specifically, let's define $\mu_t = \frac{M_t}{M_{t-1}} - 1$ as the growth rate of the nominal money stock of the economy between period t-1 and period t. Obviously, $1+\mu_t = \frac{M_t}{M_{t-1}}$, and thus, for example, if the nominal money supply does not change between period t-1 and period t, we would have $\mu_t = 0$.

Using our definitions of the inflation rate and the money growth rate in the last expression, then, we have

$$\frac{1+\mu_t}{1+\pi_t} = \frac{c_t}{c_{t-1}}\left(\frac{1+i_t}{i_t}\right)\left(\frac{i_{t-1}}{1+i_{t-1}}\right).$$

Now let's consider the steady-state. Recall our definition of a steady-state as a state of the economy in which all real variables settle down to constant values over time, but nominal variables need not do so. Let's make the latter part of this concept a bit more precise than we did earlier: it is only nominal *level* variables that need not settle down to constant values in the long run. For example, the nominal price *level* of the economy need not settle down to a constant value in the long run. The same is true of the *level* of the nominal money supply of the economy. On the other hand, nominal *growth rate* variables *do* settle down to constant values in the long run. That is, **the growth rate of a nominal variable is considered to be a real variable.** Moreover, interest rates, regardless of real or nominal, are considered to settle down to constant values in the steady-state.

Applying this more precise concept of a steady-state to the last expression derived above, we see that *all* of the variables contained in it settle down to constant values in the long-run: that is, $c_{t-1} = c_t = \bar{c}$, $i_{t-1} = i_t = \bar{i}$, $\mu_{t-1} = \mu_t = \bar{\mu}$, and $\pi_{t-1} = \pi_t = \bar{\pi}$. Imposing these steady-state values in that expression and canceling terms, we obtain

$$\frac{1+\bar{\mu}}{1+\bar{\pi}} = 1,$$

or, more simply,

$$\bar{\pi} = \bar{\mu}. \qquad (1.5)$$

Expression (1.5) captures the essence of the monetarist school of thought within macroeconomics, stating that (in the long run – i.e., in the steady state) the inflation rate of the economy is governed by the rate of growth of the money supply. The rate of growth of the money supply is of course controlled by an economy's central bank because it is ultimately an economy's sole (legal) supplier of money. The higher is the growth rate of money in an economy, the higher is (in the long-run) the economy's inflation rate.

We will soon examine much further the conduct of monetary policy and some of its effects, with a special focus on the interactions between monetary policy and fiscal policy; this monetarist linkage will be in the background of many of the causes and effects we discuss there.