Chapter 19
Optimal Fiscal Policy

We now proceed to study optimal fiscal policy. We should make clear at the outset what we mean by this. In general, "fiscal policy" entails the government choosing its spending (i.e., how much to spend on building roads, bridges, wars, etc.) and some combination of taxes that finance that spending.[153] The convention in most macroeconomic analysis of fiscal policy is to *take as given* government spending; we will adopt this convention. That is, we will not think about *why* the government is choosing a particular level of spending, but rather just focus on, given some amount of spending, the optimal way for the government to pay for that spending. The reasons why the government chooses a particular level of spending are surely interesting to study, but probably take us too far afield into the realms of political economy, the provision of public goods, and political science. Thus, we confine ourselves here to the narrower topic of just the *optimal financing* of a pre-set amount of government spending.

Even given this limitation, we still have a lot to think about. One issue we need to take a stand on right away is what *types* of tax instruments we will assume the government has available to use in order to pay for its spending. For example, the government may be able to levy a labor income tax; it may be able to levy consumption taxes (sales taxes); and it may be able to levy taxes on savings or interest income. As our starting point, we will consider optimal fiscal policy in the context of the one-period consumption-leisure model; as such, a tax on savings or interest income has no meaning because, recall, the basic consumption-leisure model abstracts from time altogether and hence abstracts from savings and interest income. However, we will, once we have developed the core principles of optimal fiscal policy, see to what extent the lessons learned extend to a multi-period economy and what role, if any, taxes on savings or interest income play in the optimal conduct of fiscal policy.

Given our starting point of the one-period consumption-leisure model, it seems we still must decide whether we want to allow our government to have access to a labor income

[153] We are already being a bit loose about the elements of fiscal policy: another component is how much debt to issue. In any given period of time, a government can pay for its spending by either collecting taxes or issuing debt (bonds) – i.e., by borrowing. For our discussion of fiscal policy, we will assume that a government must always run a balanced budget; indeed this assumption can be justified given our restriction (discussed immediately below) here to a one-period setting. Debt only becomes interesting to think about in an explicitly dynamic (multi-period) setting. When we turn to the consideration of *jointly optimal* fiscal and monetary policy, we will do so in an explicitly dynamic setting and allow the government to issue debt. As we shall see (even before turning to that topic), government debt plays a critical role in the *interaction* between fiscal policy and monetary policy; the basic idea behind this comes through in the *consolidated government budget constraint,* which we will study next; but we are not there yet.

tax, a consumption tax, or both. It turns out that in our simple model, it does not matter which one we allow it to have, and in fact if allow it to have both, we run into problems in figuring out what the optimal *mix* of consumption taxes and labor taxes is. Thus, we will assume that it is only labor income taxes that the government has available in order to finance its spending.

As we proceed, the structure of the optimal policy problem should strike you as very familiar. When we studied optimal monetary policy, we laid out a framework that is applicable for *any* optimal policy problem, not just optimal monetary policy. Specifically, we (as the modelers) must specify how consumers make optimal choices. We must specify how production occurs and how firms make optimal (profit-maximizing) choices. We must then consider *simultaneously* the optimal choices of both consumers and firms *along with* the resource constraint of the economy. All of this so far should sound familiar.[154] In studying optimal fiscal policy, there is one more element that becomes critical: the government budget constraint. This is not a new aspect of studying optimal policy problems: indeed, it was present in our study of optimal monetary policy, but there is was not really at center stage.[155] There, it was easy to subsume it into the consumer and firm optimality conditions; here, it is not so easy to subsume it in that way, so it will play a more central role in our analysis. The simultaneous optimal choices of firms, consumers, *along with the government budget constraint,* comprise the **equilibrium** of the economy. Just as before, we will assume the policy-makers (here, fiscal policy-makers) sit "above" the economy, watching how equilibrium unfolds. Fiscal policy-makers understand that for any **given** policy they choose, the private sector (consumers and firms) will make optimal choices that result in *some* equilibrium. The various equilibria that result for any given, arbitrary, policy can be welfare-ranked according to the representative consumer's utility function. The *optimal policy problem* is thus once again a matter of choosing the *best* equilibrium, by choosing some appropriate policy.

We now proceed through the elements of the model, define equilibrium, develop the optimal policy problem, and then consider the results. Because here we are not concerned with monetary issues *at all*, we will not even bother to write things in nominal terms; all prices, quantities, etc. will be written in real terms.

Firms

[154] If not, take a look back at our study of optimal monetary policy.

[155] Recall that the (somewhat trivial) government budget constraint in the CIA model was $\tau_t = g_t M_{t-1}$, where τ was a lump-sum transfer and g was the growth rate of the nominal money supply. The reason this government budget constraint was "trivial" is that there is no government spending in it; what we probably generally mean (in everyday language) by a "government budget constraint" is how a government can/does finance its spending. This is the notion of the government budget constraint we will have here.

In this one-period economy, firms hire labor in a perfectly-competitive labor market in order to produce output according the simple linear-in-labor production technology $f(n) = n$.[156] The (real) wage w is taken as given by the firm. The firm maximizes its profit, $f(n) - wn$, by choosing n. Substituting the linear production technology into the profit function and optimizing with respect to n, the result of the firm profit-maximization problem is

$$w = 1. \tag{1.71}$$

Condition (1.71) is one of the equilibrium conditions of the model, and is the only one that arises from the firm (supply) side of the model.

Consumers

The representative consumer has a (one-period) utility function $u(c, 1-n)$, where c stands for consumption, n stands for labor, and $1-n$ is leisure (thus, as we have done before, total time available for all life activities is arbitrarily chosen to be one). The utility function thus depends on consumption and *leisure.*

The budget constraint of the representative consumer (again, we are expressing things in purely real terms here) is

$$c = (1-t)wn, \tag{1.72}$$

where the labor tax rate $t \in (0,1)$ is taken as given, along with the real wage w, by the consumer. That is, the consumer has control over neither the (pre-tax) real wage w nor the labor tax rate the government levies.[157]

The consumer maximizes utility subject to the budget constraint (1.72). We know from our study of this model that at the optimal choice of the representative consumer, the condition

$$\frac{u_2(c, 1-n)}{u_1(c, 1-n)} = (1-t)w, \tag{1.73}$$

holds, where, as always, $u_1(.)$ is the marginal utility of consumption and $u_2(.)$ is the marginal utility of leisure. This completely-standard consumption-leisure optimality

[156] Note there are no time subscripts here because we considering a **one-period** model – there are no distinct time periods.
[157] This part of the model should be familiar from the basic consumption-leisure framework.

condition states, as usual, that if the consumer is making utility-maximizing choices, his MRS between consumption and leisure is equated to the after-tax real wage rate.[158] Condition (1.73) is one of the equilibrium conditions of the model, and is the only one that arises from the consumer (demand) side of the model.

Government

As we mentioned, the government budget constraint plays a central role here. Denote by the variable *govt* government spending. The government must pay for all of its spending through labor income taxes. The total amount of revenue generated by the labor income tax is total wages earned by the consumers in the economy times the tax rate: hence, total labor tax revenue is $t \cdot w \cdot n$. The government budget constraint is thus

$$t \cdot w \cdot n = govt, \tag{1.74}$$

which simply states that total government spending must be covered by total tax revenues. Thus, we assume that the government cannot/does not fail to pay for its spending.

Resource Constraint

As always, the resource constraint describes all of the different uses of total output (GDP) of the economy. In the model here, output is produced by the linear-in-labor production technology and there are *two* uses for output: private consumption (by consumers) and *public consumption* (i.e., government expenditures). Hence the resource constraint here is

$$c + govt = n. \tag{1.75}$$

Despite this being the first time we have introduced government spending into the resource constraint, as always, you should think of the resource constraint as the "GDP accounting equation" from basic macroeconomics.[159]

Private-Sector Equilibrium

[158] And of course, the graphical analysis of this statement is also familiar to you from the basic consumption-leisure framework.

[159] Because we have neither capital (and thus no investment) nor net exports here, all of the output of the economy is used by either consumers or the government, which is what (1.75) states.

Having described the actions of firms, consumers, the government, and the resource constraint, the next step, as always, is to consider *equilibrium.* Specifically, the equilibrium that we describe is the *private-sector equilibrium.* The private sector, which is composed of consumers and firms, of course does not set fiscal policy – that is, agents in the private sector choose neither government spending nor tax rates. Rather, private sector agents take as given both government spending and tax rates when making their optimal choices. Specifically, firms make profit-maximizing decisions that lead to their optimality condition (1.71); consumers make utility-maximizing decisions that lead to their optimality condition (1.73); and all markets clear, including goods markets, which means the resource constraint (1.75) must be satisfied. Thus, conditions (1.71), (1.73), and (1.75) describe the *private-sector equilibrium* of the economy (again, the equilibrium that results for *any* arbitrarily-chosen policy by the government).

In our study of optimal monetary policy, we next proceeded to condense the private-sector equilibrium conditions into a single condition before proceeding to the optimal policy problem. We will do something similar here, but before we do, it turns out that in studying optimal *fiscal* policy it is convenient from a mathematical standpoint to describe the private-sector equilibrium in a slightly different (but completely equivalent) way. Rather than saying the private-sector equilibrium is described by (1.71), (1.73), and (1.75), let's say that the private-sector equilibrium is described by (1.71), (1.73), and (1.72). That is, let's replace in our definition of equilibrium the resource constraint (1.75) with the representative consumer's budget constraint (1.72). We'll return below to why we do so, and why this is justified. For now, let's just proceed to analyze the optimal policy problem supposing that it is conditions (1.71), (1.73), and (1.72) that describe the private-sector equilibrium.

As we did in our study of optimal monetary policy, then, let's try to condense as far as possible these private-sector equilibrium conditions. It is easy to substitute (1.71) into (1.73); the expression

$$\frac{u_2(c,1-n)}{u_1(c,1-n)}=(1-t) \tag{1.76}$$

condenses them. Next, we can substitute the consumer budget constraint (1.72) (with $w=1$ substituted into it…) into (1.76) to eliminate the c, which gives us

$$\frac{u_2((1-t)n,1-n)}{u_1(1-t)n,1-n)}=(1-t). \tag{1.77}$$

Condition (1.77) condenses the entire description of the private-sector equilibrium of the economy down to a single expression (again, we will describe below the reason why it is valid in our analysis here to replace condition (1.75) with condition (1.72)). In terms of prices and quantities being determined by the interaction of consumers and firms in markets, the only unknown in (1.77) is labor. n. Thus, we may think of condition (1.77) as defining a function $n(t)$.[160] Condition (1.77) is crucial for the optimal policy problem.

Formulation and Solution of Optimal Policy Problem

In terms of setting up the optimal policy problem, condition (1.77) defines the optimal quantity of labor in the economy *as a function of the labor tax rate t.* To emphasize this functional dependence, we will write $n(t)$ to summarize the content of expression (1.77).

The policy problem is to choose such a tax rate that maximizes the representative consumer's utility taking into account the function $n(t)$ *and* the government budget constraint. The government budget constraint was presented in expression (1.74). As has always been the case in our consideration of optimal policy problems, the government understands everything about the private-sector equilibrium of the economy: specifically, it understands that $w=1$ and that the function $n(t)$ describes how labor in the economy responds to tax rates. In the government budget constraint (1.74), then, *because we are now at the stage of formulating the optimal policy problem,* we may substitute these two conditions. Doing so allows us to express the government budget constraint as

$$t\cdot n(t)=govt\,. \tag{1.78}$$

Note that here (again, *at the stage of formulating the optimal policy problem)*, the labor tax rate appears *twice* in the government budget constraint: once directly, and once through its influence on the private-sector equilibrium choice of labor.

The government's policy problem here thus boils down to the government choosing a tax rate t that maximizes the representative consumer's utility $u((1-t)n(t),1-n(t))$ subject to its (the government's) budget constraint (1.78). Here we must seemingly formulate a Lagrangian for the optimal policy problem. The Lagrangian for the optimal policy problem is

$$u((1-t)n(t),1-n(t))+\lambda[govt-t\cdot n(t)], \tag{1.79}$$

where λ is the Lagrange multiplier for the government budget constraint. As already stated, we are assuming here that $govt$ is pre-determined, thus the only policy choice the government must make is that of the optimal labor tax rate.

[160] This idea is completely analogous to the idea that the consumer's optimality condition implicitly defined a function $c(g)$ when we studied optimal monetary policy.

Thus far, we have been proceeding in very close analogy with our analysis of optimal monetary policy. However, at this point we can depart from this close analogy. In our monetary policy problem, there was no government budget constraint to consider because there was no government spending that needed to be financed; here, there is a government budget constraint. The equilibrium version of the government budget constraint, (1.78), has only one unknown in it – the labor tax rate t. We have one equation (the government budget constraint) with one unknown (t) to be chosen. Hence, *there is no need to set up a Lagrangian for the optimal policy problem.* That is, we can simply solve equation (1.78) for the labor tax rate!

Before doing so, first suppose that $govt = 0$. In this case, there would obviously be no need to levy taxes (there is no expenditure that the government needs to finance), and indeed the government budget constraint immediately tells us that the government should levy $t = 0$. With $t = 0$, the consumer ends up choosing consumption and leisure in such a way as to make the MRS between consumption and leisure equal to one – this follows simply from plugging in $t = 0$ (along with $w = 1$) in condition (1.76). Having studied economic efficiency, we can draw some further conclusions. We saw earlier in the one-period consumption-leisure model that *economic efficiency* requires the MRS between consumption and leisure to be one. That is *exactly* the same outcome that we have here in the optimal policy problem with $govt = 0$. The government we have been thinking about here is *not* a Social Planner that gets to direct all the resources of the economy by fiat. The government we have been thinking about here *does,* in contrast to the Social Planner, explicitly consider markets and prices and equilibrium when making the optimal policy choice. Yet, if $govt = 0$, the government ends up choosing a policy that implements the economically-efficient outcome.

Things are different when $govt > 0$, in which case the government necessarily has to levy a $t > 0$. The question before us is: what is the t the government should pick? We just answered this question above in very general terms: the government must pick a t that satisfies (1.78) – there is no need to formulate a Lagrangian. However, we cannot make any more progress on computing the optimal labor tax rate without making some particular assumptions about the utility function. That is, without any knowledge of the function $n(t)$ (and ultimately it is the utility function that pins down the function $n(t)$), there is nothing further to say than "the government should pick a t that satisfies expression (1.78)." We will assume a particular functional form for utility in a moment.

The Laffer Curve

Before we zoom in on a special case, let's generalize our analysis a bit. The government budget constraint equates tax revenues with government expenditures. The two objects are conceptually distinct from each other – that is, in concept, tax revenues and government expenditures need not have anything to do with each other. It is only the fact that governments (generally?...) try to make their revenues roughly line up with their expenditures that the two end up being related.

So let's define total tax revenue as a function of the tax rate, $TR(t)$. The left-hand-side of the equilibrium version of the government budget constraint, expression (1.78), reveals that

$$TR(t) = t \cdot n(t), \tag{1.80}$$

meaning total tax revenue collected depends not only on the tax rate directly, but also on the equilibrium labor supply induced by a particular tax rate. Thus, there are two effects of any given tax rate t on tax revenues, TR. These two effects are revealed by differentiating TR with respect to t; using the chain rule,

$$TR'(t) = n(t) + t \cdot n'(t), \tag{1.81}$$

which states that any *change* in the tax rate has two effects on tax revenue: it directly raises revenue (the first term on the right-hand-side of (1.81)) and it also leads to a *fall* in revenue provided $n'(t) < 0$ (the second term on the right-hand-side of (1.81)). That is, if the equilibrium quantity of labor in the economy falls when the tax rate rises (which is what $n'(t) < 0$ means), then a rise in the tax rate has a depressing effect on total tax revenue. Overall, whether tax revenues rise or fall when the tax rate increases (that is, whether $TR'(t) > 0$ or $TR'(t) < 0$) depends on which of the two effects on the right-hand-side of (1.81) is larger. For many utility functions (and hence specifications of the function $n(t)$), the first effect dominates for small values of t and the second effect dominates for large values of t. Thus, for many utility functions, the behavior of $TR(t)$ as a function of t is as depicted in the following figure:

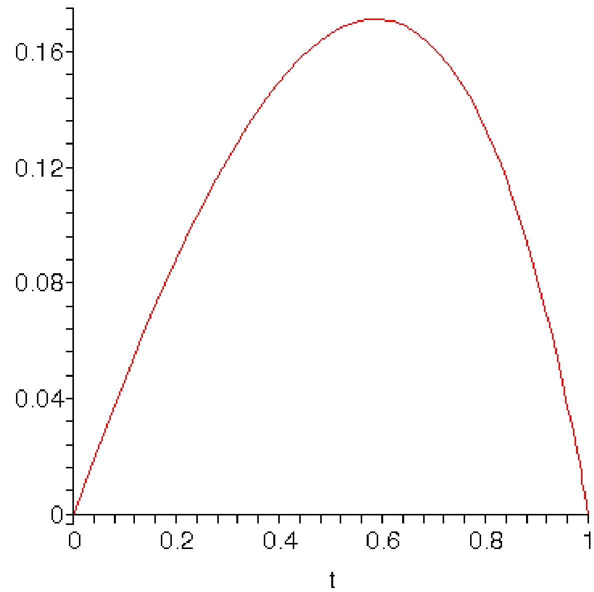

Figure 84. The Laffer Curve, with total tax revenue plotted on the vertical axis and the labor tax rate plotted on the horizontal axis.

The labor tax rate naturally takes on values only between zero and one, hence we limit the horizontal axis to this range. As Figure 84 shows, total tax revenue rises until some threshold tax rate and then falls. This concept, known as the **Laffer Curve,** is one you should be familiar with from basic macroeconomics.

A Numerical Example

To take a concrete example, suppose the utility function is $u(c,1-n)=2\sqrt{c}+2\sqrt{1-n}$. This means the marginal utility functions are $u_1 = 1/\sqrt{c}$ and $u_2 = 1/\sqrt{1-n}$. With this functional form for utility, we next need to determine the function $n(t)$. Recall from our definition of the private-sector equilibrium that it is condition (1.77) that we must use to figure out the function $n(t)$. With our assumed utility function, condition (1.77) takes on the form

$$\frac{\sqrt{1-t}\sqrt{n}}{\sqrt{1-n}} = 1-t, \tag{1.82}$$

which you should verify you can obtain. Going through the few steps of algebra to solve this for n, we have that the function $n(t)$ for this particular utility function is

$$n(t) = \frac{(1-t)^2}{1-t+(1-t)^2}. \tag{1.83}$$

Plotting this, we find

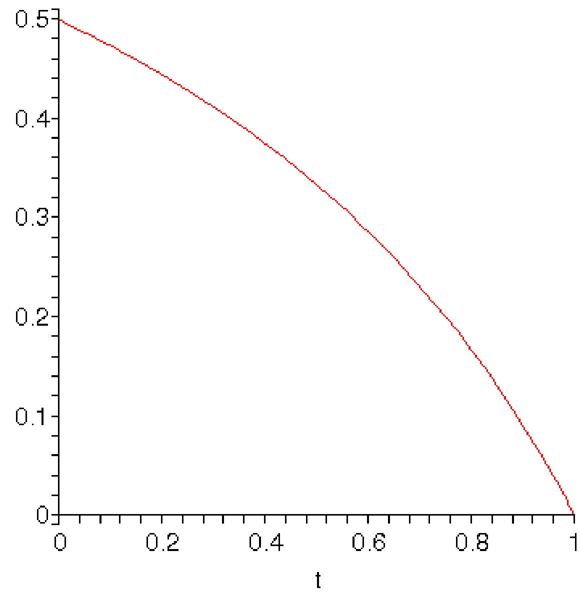

Figure 85. For the given utility function, equilibrium labor supply (plotted on vertical axis) as a function of the labor tax rate (plotted on the horizontal axis).

Thus, for our assumed utility function, the equilibrium quantity of labor does indeed fall as the labor tax rises. That is, $n'(t) < 0$ in this example, meaning that a Laffer Curve could potentially arise in this example.

With expression (1.83), the government budget constraint in turn can be written as

$$\frac{t(1-t)}{1-t+(1-t)^2} = govt, \tag{1.84}$$

which of course is simply $t \cdot n(t) = govt$ with (1.83) substituted in. The left-hand-side of (1.84) is the function $TR(t)$, that is, it shows total tax revenue as a function of the tax rate. If we plot the $TR(t)$, we have exactly the function shown in Figure 84. Thus, a Laffer Curve does indeed arise for this particular utility function.[161]

[161] A Laffer Curve does **not** arise for every utility function; thus, one needs to begin from the basics in order to determine whether a Laffer Curve arises for any particular utility function under consideration.

Now, let's conclude our analysis – that is, let's finally determine the optimal tax rate that finances a given amount of government spending. Suppose that $govt = 0.1$ is the amount of spending that needs to be financed. Figure 84 shows us that there are *two* tax rates that raise this amount of total revenue: $t = 0.22984$ and $t = 0.87015$ both raise 0.1 units of revenue. So is the government indifferent about which one it chooses? Or is there still a meaningful decision for the *optimal* policy-maker to make?

To answer the question, we (the policy-maker, that is) must evaluate the representative consumer's utility under the two possible choices of tax rates – after all, the policy-maker is trying to maximize the consumer's utility, albeit subject to the need to raise a given amount of revenue through taxation. If the government chooses the higher tax rate, $t = 0.87015$, then expression (1.83) tells us that $n = 0.1149$, meaning $1 - n = 0.8851$. The consumer budget constraint (1.72) then reveals that $c = (1 - 0.87015)\cdot 1 \cdot 0.1149 = 0.01492$. In turn, this means that the representative consumer's utility is $u(0.01492, 1 - 0.1149) = 2\sqrt{0.01492} + 2\sqrt{1 - 0.1149} = 2.12589$. If the government instead chooses the lower tax rate, $t = 0.22984$, the same analysis reveals that $n = 0.43508$, $1 - n = 0.56492$, $c = (1 - 0.22984)\cdot 1 \cdot 0.43508 = 0.33806$, thus $u(0.33806, 1 - 0.43508) = 2.66608$. The objective of maximizing utility then dictates that the government should choose the lower tax rate; it raises the same amount of total revenue and allows the representative consumer to achieve higher total utility.

It may seem, from the latter part of our analysis, that we used "just" the government budget constraint to obtain the optimal tax rate. This is not true. In the government budget constraint (1.78), we needed to know the *private-sector function* $n(t)$. We could not obtain this equilibrium "reaction function" that the government needs to know without *first* solving for the private-sector equilibrium; once we solved for the private-sector equilibrium, then we were able to proceed to the formulation and solution of the policy problem. The first part of our analysis – solving for the private-sector equilibrium – was exactly the same (with one caveat that we return to below) as in our study of optimal monetary policy. The second part of the analysis differed a bit because of the presence of the government budget constraint. In particular, we did not need to formulate a Lagrangian because in essence the policy problem boiled down to solving one equation (the government budget constraint) in one unknown. The twist was that two solutions arose, so that we had to compare the two solutions to see which delivered higher utility. But in both our earlier study of optimal monetary policy and our study here of optimal fiscal policy, the idea and approach are similar: first solve for the private-sector equilibrium, from which we need to glean a private-sector equilibrium reaction function, then formulate and solve the policy problem.

The only caveat in our formulation of the private-sector equilibrium here, recall, was that we used the household budget constraint (1.72) rather than the economy's resource constraint (1.75). The reason using either turns out to be equivalent in our analysis here is that we knew that we were ultimately going to use (had to use…) the government budget constraint in the optimal policy problem. Any two of the three – the household budget constraint, the government budget constraint, and the resource constraint – imply the third holds as well. For example, if we sum the household budget constraint and the government budget constraint (and use the equilibrium condition $w=1$) we immediately have the resource constraint. Thus as long two of the three conditions are satisfied, the third is satisfied as well. It turns out to be more convenient in the formulation of an optimal fiscal policy problem to use the household budget constraint rather than the resource constraint in the specification of the private-sector equilibrium.