# Chapter 23 Optimal Monetary Policy with Sticky Prices

We now reconsider the issue of optimal monetary policy, this time in the Rotemberg sticky-price framework we just finished developing. We will find that the policy advice that arises in a sticky-price view of the economy is qualitatively quite different from the policy advice that arises in a flexible-price view of the economy.

The work we do in this chapter builds on virtually all of the ideas and concepts we have laid out so far. We will rely on the Dixit-Stiglitz-Rotemberg model of price-setting firms subject to menu costs. Our mode of optimal-policy analysis will be identical to the structure by which we analyzed the optimal policy problem in Chapter 17. As there, we must first specify the private-sector equilibrium for any arbitrary policy the government (the central bank) might choose. This in itself requires setting up and solving the optimization problems of the demand and supply sides of the economy; we have already done most of this work, but there are a couple of new elements we Then, as in Chapter 17, in a second step, we determine the policy that introduce. maximizes the representative consumer's utility. The final step is to compute the actual optimal policy, which is done by comparing the solution of the optimal policy problem with the outcome in the private-sector equilibrium; the result is the optimal policy recommendation. We once again – because, we continue to maintain, it seems very natural – adopt the representative consumer's utility as the welfare criterion according to which the central bank ranks various policies.

Thus, just as in our earlier analysis of optimal policy, we can think of the policy-makers as sitting "above" the economy, watching how equilibrium unfolds. We need to make a slight refinement to this view here, however: we will think of the policy-makers as watching how a *symmetric equilibrium* unfolds. Thus, policy-makers understand that for any **given** policy they choose, the private sector (consumers, retail firms, and wholesale firms) will make optimal choices that will result in *some* symmetric equilibrium. All of this by-now quite familiar machinery allows us to continue to think of the optimal policy problem as a problem of choosing the *best* equilibrium, where "best equilibrium" means the one that maximizes the utility of the representative consumer.

## **Retail Firms**

The representative retail firm is again no different from the one we developed in the basic Dixit-Stiglitz and Rotemberg models: a retail firm simply "packages" the continuum [0,1] of differentiated wholesale products and sells the retail consumption basket to

consumers via perfectly-competitive markets. As before, the price of retail goods is determined only through the invisible hand of the market, and the profit-maximizing choice of any arbitrary wholesale good j leads to the demand function for wholesale good j,

$$y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{\frac{\varepsilon}{1-\varepsilon}} y_t, \qquad (1.130)$$

for, as always, any  $j \in [0,1]$ . In terms of setting things up for the description of the full equilibrium below, there are *no* equilibrium conditions stemming directly from profitmaximization by retail firms that we need to keep independent track of. The demand function (1.130) is a sufficient summary of the profit-maximizing choices of retail firms; but, because it will be subsumed inside the analysis of wholesale firms, we inevitably will end up "keeping track" of it.<sup>195</sup>

#### **Wholesale Firms**

Wholesale firms are also no different from the ones we developed in the basic Rotemberg model: a particular wholesale firm, wholesaler j, produces one good, which is imperfectly substitutable with any other wholesale good, and sells it to the retail sector. Because of imperfect substitutability between its good and the good of any other wholesaler, wholesale firm j enjoys some monopoly power, making it explicitly a pricesetter. In (re-)setting its nominal price from one period to the next, however, the

wholesale firm is subject to the *real* quadratic cost of price adjustment,  $\frac{\psi}{2} \left( \frac{P_{jt}}{P_{jt-1}} - 1 \right)^2$ , in

which, as before,  $\psi$  is a parameter that governs the magnitude of menu costs. Even more so than in our analysis so far, the fact that we are adopting the view that menu costs are *real* costs will be critical; it will be the key force shaping the optimal policy prescription at which we eventually arrive.

Finally, profit-maximization by wholesale firm *j*, taking into account the costs of price adjustment, leads *in a symmetric equilibrium* to the New Keynesian Phillips Curve (NKPC),

$$\frac{1}{1-\varepsilon} \left[ 1 - \varepsilon mc_t \right] y_t - \psi \pi_t (1 + \pi_t) + \beta \psi \pi_{t+1} (1 + \pi_{t+1}) = 0, \qquad (1.131)$$

<sup>&</sup>lt;sup>195</sup> In this sense, the distinction between "retail firms" and "wholesale firms" can be thought of as nothing more than a theoretical artifice to keep our thinking straight between consumers and firms.

which links together period-t inflation, period-t+1 inflation, and period-t marginal costs of production. The NKPC (1.131) is one of the (symmetric) equilibrium conditions of our model economy, stemming from optimal price-setting decisions on the part of firms subject to menu costs. Indeed, for our analysis in this chapter, the NKPC is the key equilibrium condition, the one on which we will load essentially all of our intuition.

An issue that we have so far left unanswered in our analysis of wholesale firms is the underlying determinants of the marginal cost of production. We simply asserted that wholesale firms operate a production technology that exhibits a marginal cost of production independent of the quantity that it produces. Because specifying an optimal policy problem requires fully specifying the nature of private-sector equilibrium outcomes, which in turn requires specifying how production actually occurs and hence its underlying costs, we no longer can be silent about the underlying determinants of the marginal costs of production.

We will assume here, as we did in our first look at optimal monetary policy with flexible prices, the simplest possible physical production technology for wholesale firms, linear in labor:  $y_{jt} = f(n_{jt}) = n_{jt}$ . In principle, each wholesale firm could hire a quantity of labor different from other wholesalers, which is why in general we might need the subscript *j* on labor. However, in keeping with the symmetric equilibrium analysis we wish to pursue, we drop this potential asymmetry and from here on simply assert that every wholesale firm will actually purchase the same quantity of labor as every other wholesale firm, allowing us to write wholesale firm *j*'s production technology as  $y_{jt} = f(n_t) = n_t$ .

This labor is hired in a perfectly-competitive labor market. Thus, in the labor market, wholesale firms take as given the real wage rate  $w_t$ . Some simple logic will now allow us to conclude that, given the quite simple production structure we have set up, the marginal cost of production for wholesale firm *j* in period *t* is simply the market real-wage rate  $w_t$ . To understand this, note that by definition (real) marginal cost is the resources a firm must spend in order to produce one more (the marginal) unit of output. Given our linear-in-labor production technology, production of one more (the marginal) unit of output requires one more (a marginal) unit of labor. For the wholesale firm, the cost of hiring one more (the marginal) unit of labor is simply the market real wage. In turn, due to perfect competition, the market real wage is independent of any input or output decisions made by wholesale firm *j* – this is nothing more than the assertion that wholesale firms are price-takers in the labor market.

In order to expand output by one unit, then, the firm must spend  $w_t$ , meaning the real wage *is* the firm's marginal cost of production.<sup>196</sup> Thus, the simple relation

<sup>&</sup>lt;sup>196</sup> This is not a completely general conclusion, but rather one that follows from the specific production function we have adopted here. A more general, say Cobb-Douglas, production function involving both labor and capital would render the link between marginal costs and wages, while still close, less one-for-one.

is an equilibrium condition of the environment we are considering.

## **Consumers in a Cashless Economy**

In the New Keynesian view of the economy and monetary policy, the object called "money" actually plays no *physical* role whatsoever. This may seem surprising because the New Keynesian framework has become the dominant theoretical framework for the analysis of *monetary* policy. This de-emphasis of the physical role of money is also a stark departure from the MIU and CIA frameworks, in which we spent considerable effort trying to articulate the medium-of-exchange role – the physical role – of money.

Recall that the benchmark policy prescription we arrived at in those flexible-price frameworks was that, in the long-run, the optimal monetary policy entails implementing the Friedman Rule of deflation at the rate of consumer impatience – or, in terms of nominal interest rates, setting i = 0. This policy recommendation followed from the desire of the optimal-policy-maker to avoid distorting the consumer's consumption-leisure optimality condition away from the economically-efficient one; an i > 0 (equivalently, recall, a steady-state money growth rate  $\mu > \beta$ ) is precisely what caused a distortion in this optimality condition. In turn, this distortion stemmed fundamentally from the cash-in-advance constraint – a *physical*, medium-of-exchange, role for money.

The New Keynesian framework, in contrast, asserts that the medium-of-exchange role is *not* the most important role played by money in a developed economy. It thus simply ignores – completely – money's role as a medium of exchange. Thus, in our analysis here we will have no CIA or MIU aspects whatsoever.

Instead, the New Keynesian framework emphasizes only the unit-of-account role of money – the simple fact that society, however it does so, generally agrees upon an accepted "language" or "standard" in which all (most?) prices will be quoted. *How* a society "agrees upon" a common unit of account is an open question in economics; the New Keynesian view has nothing novel to say about why intrinsically-useless pieces of paper printed by a country's central bank are, in modern times, almost universally an economy's unit of account.

With this modified view of the role of money in the economy, the representative consumer's problem is a bit simpler to state and characterize than in our flexible-price consideration of optimal monetary policy in Chapter 17. The representative consumer begins period t with nominal bond holdings  $B_{t-1}$  and stock (a real asset) holdings  $a_{t-1}$ . The period-t budget constraint of the consumer is

$$P_t c_t + P_t^b B_t + S_t a_t = W_t n_t + B_{t-1} + (S_t + D_t) a_{t-1}, \qquad (1.133)$$

where the notation again is as in the MIU model of Chapter 14 and the CIA model of Chapter 17:  $S_t$  is the nominal price of a unit of stock,  $D_t$  is the nominal dividend paid by each unit of stock, and  $P_t^b$  is the nominal price of a one-period, zero coupon nominal bond with face-value \$1. Compared to the budget constraints analyzed there, however, note the *absence* of nominal money. In the New Keynesian "cashless" view of the economy, because the physical medium-of-exchange function of money is deemphasized, we simply completely ignore it in the consumer's optimization problem.

Notice that because we have dropped nominal money from the representative consumer's budget constraint, we also have dropped, compared to Chapter 17, the term  $\tau_t$ , which was the lump-sum means by which the monetary authority achieved changes in the (physical) money supply.

Denoting by  $\lambda_t$  the Lagrange multiplier on the period-*t* budget constraint, the sequential Lagrangian for this problem is

$$u(c_{t}, 1-n_{t}) + \beta u(c_{t+1}, 1-n_{t+1}) + \beta^{2} u(c_{t+2}, 1-n_{t+2}) + \dots + \lambda_{t} \Big[ W_{t}n_{t} + B_{t-1} + (S_{t} + D_{t})a_{t-1} - P_{t}c_{t} - P_{t}^{b}B_{t} - S_{t}a_{t} \Big] + \beta \lambda_{t+1} \Big[ W_{t+1}n_{t+1} + B_{t} + (S_{t+1} + D_{t+1})a_{t} - P_{t+1}c_{t+1} - P_{t+1}^{b}B_{t+1} - S_{t+1}a_{t+1} \Big] + \dots$$

$$(1.134)$$

In terms of analyzing just the consumer optimization problem, the absence of the CIA constraint makes the "cashless" economy quite a bit simpler than the CIA model.

In period t, the consumer chooses  $(c_t, n_t, B_t, a_t)$  -- note the absence of  $M_t$  from the list of objects over which to optimize. The first-order-conditions with respect to each of these four choice variables, respectively, are:

$$u_1(c_t, 1 - n_t) - \lambda_t P_t = 0 \tag{1.135}$$

$$-u_2(c_t, 1-n_t) + \lambda_t W_t = 0 \tag{1.136}$$

$$-\lambda_t P_t^b + \beta \lambda_{t+1} = 0 \tag{1.137}$$

$$-\lambda_t S_t + \beta \lambda_{t+1} (S_{t+1} + D_{t+1}) = 0$$
(1.138)

Analysis of the rest of this now-cashless structure, from the consumer optimization point of view, proceeds just as in Chapter 17.

Combining conditions (1.135) and (1.136) immediately yields the familiar (from Chapter 2) consumption-leisure optimality condition,

$$\frac{u_2(c_t, 1-n_t)}{u_1(c_t, 1-n_t)} = w_t.$$
(1.139)

In contrast to the consumption-leisure optimality condition derived in Chapter 17, monetary policy does *not* potentially drive a wedge into consumers' optimal choices. Thus, in stark contrast to the forces driving optimal-policy recommendations in a flexible-price view of the economy, in the cashless sticky-price view the forces driving optimal-policy recommendations are, as we will see, something quite different.

Finally, just as in Chapter 17, we can express a consumption-savings optimality condition. Due to the *lack* of a CIA constraint here, it is simple to express this in terms of the marginal utility of consumption, rather than in terms of the marginal utility of leisure as we did in Chapter 17; here, we simply have

$$\frac{u_1(c_t, 1-n_t)}{u_1(c_{t+1}, 1-n_{t+1})} = \beta(1+r_t), \qquad (1.140)$$

which results from condensing (1.135) and (1.138) and defining the gross real interest rate 1+r as the real return on stock holdings.<sup>197</sup>

Conditions (1.139) and (1.140) are thus equilibrium conditions stemming from the consumer (demand) side of the economy.

#### Government

We can still say that the government "prints money," and thus we can still speak of the growth rate of money. However, because we are being much less explicit about the physical medium of exchange used in the economy, we can get away without actually articulating a government budget constraint that describes the printing of money. So, in New Keynesian tradition, we will simply leave this aspect of the economy in the background.

#### **Resource Constraint**

As usual, the resource constraint of the economy describes the transformation of inputs into total output (GDP) as well as all of the possible different uses of total output. In our Rotemberg sticky-price economy, there are *two* uses for final output: consumption (of "goods" – remember that in symmetric equilibrium, we blur the distinction between

<sup>&</sup>lt;sup>197</sup> See the analysis in Chapter 17 for a reminder of these details.

"retail goods" and "wholesale goods," even though the distinction is crucial for the derivation of the NKPC) and the real, physical, costs associated with price adjustment. After all, firms must expend resources – whatever exactly the "menu costs" are – in order to change their prices. Thus, the resource constraint, in a symmetric equilibrium (which allows us to drop the distinction between  $P_{it}$  and  $P_t$ ) is

$$c_t + \frac{\psi}{2} \left(\frac{P_t}{P_{t-1}} - 1\right)^2 = y_t.$$
 (1.141)

Using the definition of inflation,  $1+\pi_t = P_t/P_{t-1}$ , we can instead express the economy-wide resource constraint as

$$c_t + \frac{\psi}{2} (\pi_t)^2 = y_t.$$
 (1.142)

Finally, because we are limiting ourselves to a symmetric equilibrium, *in equilibrium* we can speak interchangeably of "retail goods," "wholesale goods," and "consumption baskets." Properly speaking, what we care about for the resource constraint is how the consumption baskets of the economy are produced. Because of symmetry, though, this is equivalent to caring about how wholesale goods are produced. We have assumed that wholesale goods are produced according to a linear-in-labor production technology. We can thus substitute the simple relation  $y_t = n_t$  into (1.142) and express the welfare-relevant resource constraint as

$$c_t + \frac{\psi}{2} (\pi_t)^2 = n_t.$$
 (1.143)

#### **Equilibrium and Steady-State Equilibrium**

Before proceeding to consideration of the optimal policy problem, we must be clear about the precise nature of the private-sector equilibrium. As always, equilibrium is a collection of prices and quantities that in concert make all markets clear, given that both demand (consumer choices) and supply (firm choices) decisions in the economy are made optimally. In our model economy here, equilibrium is described by the NKPC (1.131); the relation (1.132) linking marginal costs with real wages; the consumer optimality conditions (1.139) and (1.140); and the resource constraint (1.143).

Because condition (1.132) is so simple, let's simply substitute it into the NKPC. Doing so leaves us with a description of equilibrium which is condition (1.139), condition (1.140), condition (1.143), and the NKPC

$$\frac{1}{1-\varepsilon} \left[ 1-\varepsilon w_t \right] n_t - \psi \pi_t (1+\pi_t) + \beta \psi \pi_{t+1} (1+\pi_{t+1}) = 0.$$
(1.144)

Next, impose steady-state on these four equilibrium conditions.<sup>198</sup> Doing so leaves us with, respectively,

$$\frac{u_2(c,1-n)}{u_1(c,1-n)} = w, \qquad (1.145)$$

$$\frac{1}{\beta} = 1 + r \,, \tag{1.146}$$

$$c + \frac{\psi}{2}\pi^2 = n, \qquad (1.147)$$

and

$$\frac{1}{1-\varepsilon} \left[ 1-\varepsilon w \right] n - (1-\beta)\psi \pi (1+\pi) = 0.$$
(1.148)

Our task in what follows is to continue to condense these expressions as far as possible.

As we did in Chapter 17, let's use the resource constraint (1.147) to eliminate the *n* terms in the other equilibrium conditions; this leaves us with

$$\frac{u_2\left(c,1-c-\frac{\psi}{2}\pi^2\right)}{u_1\left(c,1-c-\frac{\psi}{2}\pi^2\right)} = w, \qquad (1.149)$$

$$\frac{1}{\beta} = 1 + r$$
, (1.150)

and

$$\frac{1}{1-\varepsilon} \left[1-\varepsilon w\right] \left(c+\frac{\psi}{2}\pi^2\right) - (1-\beta)\psi\pi(1+\pi) = 0.$$
(1.151)

Next, let's substitute for w in the NKPC using condition (1.149), which yields as the pair of equilibrium conditions

$$\frac{1}{\beta} = 1 + r \tag{1.152}$$

and the rather unfriendly-looking condition

<sup>&</sup>lt;sup>198</sup> Thus, just as in Chapter 17, we limit our analysis to steady-state optimal policy.

$$\frac{1}{1-\varepsilon} \left[ 1 - \frac{\varepsilon u_2 \left( c, 1 - c - \frac{\psi}{2} \pi^2 \right)}{u_1 \left( c, 1 - c - \frac{\psi}{2} \pi^2 \right)} \right] \left( c + \frac{\psi}{2} \pi^2 \right) - (1 - \beta) \psi \pi (1 + \pi) = 0.$$
(1.153)

Despite its "cashless" nature, the New Keynesian view does affirm that in the long-run (i.e., in the steady-state), a simple monetarist link between the rate of inflation,  $\pi$ , and the rate of money growth, g, exists.<sup>199</sup> Thus, imposing  $g = \pi$  on the previous two expressions leaves us with

$$\frac{1}{\beta} = 1 + r \tag{1.154}$$

and

$$\frac{1}{1-\varepsilon} \left[ 1 - \frac{\varepsilon u_2 \left( c, 1 - c - \frac{\psi}{2} g^2 \right)}{u_1 \left( c, 1 - c - \frac{\psi}{2} g^2 \right)} \right] \left( c + \frac{\psi}{2} g^2 \right) - (1-\beta) \psi g (1+g) = 0 \quad (1.155)$$

as the set of (now only two) equilibrium conditions.

For the optimal-policy analysis to follow, condition (1.154) can essentially be ignored. In the cashless view, condition (1.154) is only useful insofar as it pins down, in steady-state, a nominal interest rate *once* a money growth rate (and hence inflation rate) has been decided upon by the central bank. That is, a completely-standard Fisher relation does exist in the New Keynesian view, and it is essentially nothing more than condition (1.154). In steady-state, as usual, we can open up the real interest rate as

$$1 + \pi = \beta(1+i);$$
 (1.156)

or, invoking the long-run monetarist link between money growth and inflation,

$$1+g = \beta(1+i).$$
 (1.157)

<sup>&</sup>lt;sup>199</sup> However, the fundamental source of such a long-run monetarist link between money growth and inflation is left unspecified in the New Keynesian view. In contrast, recall, the CIA and MIU frameworks clearly articulated the source of the long-run link – the steady-state of a money demand condition is the source of the monetarist link in both the MIU and CIA frameworks.

Thus, in the ensuing optimal-policy analysis, once the central bank has chosen the welfare-maximizing g, condition (1.157) (equivalently, condition (1.154)) simply tells it what i to set to achieve the chosen g; condition (1.154) (equivalently, condition (1.157)) does not play any direct role in the policy problem.<sup>200</sup> Rather, it is condition (1.155) – which, despite its now cumbersome form, is simply the NKPC – that is the essential equilibrium condition for the optimal policy problem.

As we did in Chapter 17, then, let's take stock of where we've arrived before proceeding to the optimal policy problem. After setting up and solving both retail firms' and wholesale firms' profit-maximization problems, as well as consumers' (cashless) utility-maximization problem, we defined the full equilibrium. We then imposed steady-state on these conditions and proceeded to condense them into a single (albeit not very compact) expression, condition (1.155). Condition (1.155) – which, we emphasize again, is nothing more than the NKPC – describes the steady-state equilibrium of the entire private sector of the economy. What condition (1.155) describes is how the steady-state equilibrium level of consumption depends on the steady-state rate of growth of the nominal money supply. As in Chapter 17, we've compacted the entire model economy (i.e., its setup and solution) into a single expression. There is no (reliable) shortcut for all the analysis we have done; one must go through the entire solution of the demand and supply sides, description of the equilibrium, and then (and only then) can one impose steady-state.

## **Formulation and Solution of Optimal Policy Problem**

From the point of view of the central bank, condition (1.155) describes how the private sector of the economy responds (in steady-state) to its chosen monetary policy. Although generally not amenable to an analytic solution, condition (1.155) indeed *defines the steady-state equilibrium c as a function of g*. To emphasize this *functional* dependence, let's from here on write c(g). As in Chapter 17, then, the optimal-policy maker takes this "private-sector equilibrium reaction function" as given when maximizing the (steady-state) utility of the representative consumer.

Referring back to Chapter 17 for details, the representative consumer's lifetime (steadystate) utility is given by

$$\sum_{s=0}^{\infty} \beta^{s} u \left( c(g), 1 - c(g) - \frac{\psi}{2} g^{2} \right) = \frac{u \left( c(g), 1 - c(g) - \frac{\psi}{2} g^{2} \right)}{1 - \beta}.$$
 (1.158)

<sup>&</sup>lt;sup>200</sup> Or, if we framed things in terms of the central bank choosing *i* directly, then condition (1.157) would pin down the appropriate *g* to set to hit the target *i*. In our steady-state analysis, choosing one or the other instrument, *g* or *i*, are completely equivalent.

The optimal policy problem thus boils down to choosing a (steady-state) growth rate of money that maximizes (1.158). Mathematically, no constraints are required on this optimization problem *because we have already built all constraints imposed by equilibrium into* (1.158). Observe that, except for the fact that the resource constraint and hence the substitution for c in the utility function is not as simple as in Chapter 17, the policy-maker's objective function (1.158) is identical to that in Chapter 17.

Being careful to apply the chain rule, the first-order condition of (1.158) with respect to g is

$$u_{1}\left(c(g), 1-c(g)-\frac{\psi}{2}g^{2}\right) \cdot (c'(g)-\psi g) - u_{2}\left(c(g), 1-c(g)-\frac{\psi}{2}g^{2}\right) \cdot (c'(g)+\psi g) = 0, (1.159)$$
$$u_{1}\left(c(g), 1-c(g)-\frac{\psi}{2}g^{2}\right) \cdot c'(g) - u_{2}\left(c(g), 1-c(g)-\frac{\psi}{2}g^{2}\right) \cdot (c'(g)+\psi g) = 0 (1.160)$$

where, as in Chapter 17, c'(g) is how steady-state equilibrium consumption responds to a marginal change in the money growth rate (i.e., it is the derivative of the function c(g) with respect to g).<sup>201</sup>

Rearranging terms, we have

$$u_{1}\left(c(g),1-c(g)-\frac{\psi}{2}g^{2}\right)\cdot c'(g)-u_{2}\left(c(g),1-c(g)-\frac{\psi}{2}g^{2}\right)\cdot c'(g)-u_{2}\left(c(g),1-c(g)-\frac{\psi}{2}g^{2}\right)\cdot \psi g=0.(1.161)$$

If it were the case that  $\psi = 0$ , this first-order condition would simplify exactly as in Chapter 17. In the case of  $\psi = 0$ , the *c*'(*g*) terms would cancel, and we would be left with the conclusion that, if policy were being conducted optimally,

$$\frac{u_2(c(g), 1-c(g))}{u_1(c(g), 1-c(g))} = 1, \qquad (1.162)$$

exactly as we found in Chapter 17.

However, with  $\psi > 0$ , things are a bit more complicated. If we divide the condition (1.161) by  $u_1\left(c(g), 1-c(g)-\frac{\psi}{2}g^2\right) \cdot c'(g)$ , we have

<sup>&</sup>lt;sup>201</sup> Note that we've dropped the 1- $\beta$  term from this first-order condition because it does not affect the solution of the policy problem – i.e., we're just dropping a constant.

$$\frac{u_2\left(c(g), 1-c(g) - \frac{\psi}{2}g^2\right)}{u_1\left(c(g), 1-c(g) - \frac{\psi}{2}g^2\right)} \left[1 + \frac{\psi g}{c'(g)}\right] = 1, \qquad (1.163)$$

or, putting the terms in square brackets over a common denominator,

$$\frac{u_2\left(c(g), 1 - c(g) - \frac{\psi}{2}g^2\right)}{u_1\left(c(g), 1 - c(g) - \frac{\psi}{2}g^2\right)} \left[\frac{c'(g) + \psi g}{c'(g)}\right] = 1.$$
(1.164)

Solving for the MRS  $u_2(.)/u_1(.)$ , we have

$$\frac{u_2\left(c(g), 1 - c(g) - \frac{\psi}{2}g^2\right)}{u_2\left(c(g), 1 - c(g) - \frac{\psi}{2}g^2\right)} = \frac{c'(g)}{c'(g) + \psi g}.$$
(1.165)

The  $c'(g) - \psi g$  terms cancel. Rearranging, we thus are left with the conclusion that

$$\frac{u_2\left(c,1-c-\frac{\psi}{2}g^2\right)}{u_1\left(c,1-c-\frac{\psi}{2}g^2\right)} = 1,$$
(1.166)

which states that *if policy is chosen optimally*, the representative consumer's MRS between consumption and leisure equals one. Note that this conclusion is completely identical to the one that emerged in Chapter  $17!^{202}$ 

 $<sup>^{202}</sup>$  The reason that this conclusion is identical in the two seemingly very different frameworks is that – aside from the precise forms of the arguments inside the marginal utility function – is that the marginal rates of *transformation* between consumption and leisure are simply *one* in both our model here and the model considered of only optimal monetary policy – examine the resource constraints in the respective environments to see this. At the end of the day, optimal policy is about trying to achieve economic efficiency, and the condition that essentially *defines* economic efficiency (as discussed in the topic of economic efficiency) is that marginal rates of substitution be equated to marginal rates of transformation. This basic underlying force behind optimal policy-setting has nothing to do with whether or not price adjustment is costly.

What will be quite different, though, is the next step we take, which is to translate condition (1.166) into an actual policy *recommendation* for g. Exactly as we proceeded in Chapter 17, this final step requires comparing the condition that describes the implications of optimal policy – which is condition (1.166) – with the condition that describes the mapping between any given (whether optimal or not) policy and the private-sector equilibrium outcome – which in our model here is condition (1.155).

This is an extremely daunting task; the precise setting for g that would make condition (1.155) exactly coincide with condition (1.166) is an extremely complicated expression. Unless we make one small modification to the analysis, that is – a modification that New Keynesian analysis typically makes. Let's first make this modification, draw the policy implications that stem from it, and defer until below what the economic content or meaning of this modification might be.

In the NKPC, condition (1.155), suppose that an  $\varepsilon$  term were present in the denominator of the second term inside the large square brackets. That is, suppose the NKPC were

$$\frac{1}{1-\varepsilon} \left[ 1 - \frac{\varepsilon u_2 \left( c, 1 - c - \frac{\psi}{2} g^2 \right)}{\varepsilon u_1 \left( c, 1 - c - \frac{\psi}{2} g^2 \right)} \right] \left( c + \frac{\psi}{2} g^2 \right) - (1 - \beta) \psi g (1 + g) = 0. \quad (1.167)$$

If this (somehow) were the form of the NKPC, the  $\varepsilon$  terms in the second term inside the large square brackets would obviously cancel out, leaving

$$\frac{1}{1-\varepsilon} \left[ 1 - \frac{u_2 \left( c, 1 - c - \frac{\psi}{2} g^2 \right)}{u_1 \left( c, 1 - c - \frac{\psi}{2} g^2 \right)} \right] \left( c + \frac{\psi}{2} g^2 \right) - (1-\beta) \psi g (1+g) = 0$$
(1.168)

as the NKPC.

Based on this now-modified NKPC, it actually is quite simple to determine what money growth rate g would make the (modified) NKPC coincide with condition (1.166).

Simple observation tells us that setting g = 0 makes condition (1.168) coincide with condition (1.166). Moreover, g = 0 is the unique steady-state money growth rate that does so. Thus, taking as given for a moment the "modification" we just made to the NKPC, the benchmark New Keynesian policy prescription is that a central bank ought to implement (in the steady-state, technically – i.e., on average) a zero growth rate of money, which in turn achieves a zero inflation rate.

Alternatively, if we wish to think about the optimal policy in terms of a prescription for the nominal interest rate, condition (1.157) (recall we stated above that condition (1.157) would only be necessary to use if we wanted to map from g to i) tells us that  $i = \frac{1}{\beta} - 1 > 0$ . The optimal policy recommendation in the sticky-price framework can

thus equivalently be thought of in terms of this precise *strictly positive* nominal interest rate, or in terms of a zero money growth rate/zero inflation rate.<sup>203</sup> It is usually the latter feature of New Keynesian policy recommendations that anchors our thinking.

Zero inflation is the cornerstone New Keynesian policy prescription. The first observation to make is that this prescription is in marked contrast to the optimal *deflation* prescribed by the Friedman Rule that we obtained in Chapter 17. The economics behind the zero-inflation prescription are quite simple, which is part of the reason why it has nearly-universally captured the imagination of policy-makers.

Thinking about the details of the Rotemberg model, the menu *costs* of price *adjustment* are, as the term obviously implies, are *costs* of price changes. These costs are social costs, which we know because they enter they appear in the resource constraint of the economy. On the other hand, from the point of view of the economy as a whole (rather than from the point of view of a single firm) there are absolutely no benefits whatsoever of price adjustment. We can conclude this because, again, the resource constraint only contains *costs* of price adjustment.

Standard economic decision-making principles dictate that choices should balance marginal costs and marginal benefits. From the point of view of social planner, however, there are no marginal benefits to price adjustment; there are only marginal costs. Hence, simple logic would lead us to conclude that an activity – in our case, nominal price adjustment – that only entails costs and no benefits whatsoever ought to be completely eliminated.

Zero inflation achieves exactly this. If, in symmetric equilibrium, there is zero inflation, by definition no firm is ever changing its prices. Zero nominal price adjustment means there are zero menu costs of price adjustment being borne by the economy. In other language that should be familiar, there are zero *deadweight losses* being incurred by the economy if there is zero inflation. In the Rotemberg formulation, the menu costs of price adjustment are purely deadweight losses; optimal policy – indeed, economic efficiency – requires eliminating deadweight losses.<sup>204</sup>

<sup>&</sup>lt;sup>203</sup> For example, if  $\beta = 0.95$ , a commonly-accepted value at an annual frequency, then we have that the optimal nominal interest rate associated with a zero inflation rate is roughly i = 0.05.

<sup>&</sup>lt;sup>204</sup> We should point out that these statements and ideas are not particular to the Rotemberg sticky-price model; they also are true of the now more-popular Calvo sticky-price model, which you might encounter in an even more advanced course in macroeconomics.

#### A Helping Hand from Fiscal Policy

To arrive at the zero-inflation policy prescription, we introduced an  $\varepsilon$  into the NKPC; clearly, the economic fundamentals we have laid out did not warrant this. Introduction of the  $\varepsilon$  is a reflection of some (in our analysis, unmodeled) **fiscal policy intervention**.

From the point of view of economic efficiency, there are two distinct distortions in our Rotemberg view of the economy.<sup>205</sup> First, monopolistic competition *in and of itself* causes a deadweight loss, even if all price adjustment is costless. On top of the deadweight loss stemming from monopolistic competition, the menu costs of price adjustment impose a deadweight loss, as we discussed above.

In principle, there is no way for one policy tool – monetary policy's setting of a money growth rate – to simultaneously correct two inefficiencies. Correcting two independent (separate) deadweight losses in general requires two independent (separate) policy instruments. The predominant view in the modern New Keynesian tradition is that fiscal policy "should" be used to offset deadweight losses arising from monopolistic competition, which frees up monetary policy to deal with "just" the deadweight losses arising from menu costs of price adjustment.

The  $\varepsilon$  term we introduced in moving from equation (1.155) to equation (1.168) effectively inserts the required corrective fiscal policy.<sup>206</sup> Our subsequent analysis then led us to conclude that, *given the presence of this corrective fiscal policy*, the goal of monetary policy in helping deliver an economically efficient outcome is to target zero inflation, which, in the steady state, requires setting a zero money growth rate.

A broader lesson here is that achieving the mantra, often invoked by policy-makers, of the desirability of "low and stable inflation" requires some fiscal preconditions. That is, it is difficult, if not impossible, for monetary policy to do its job without some appropriately complementary conduct of fiscal policy. With a supportive fiscal framework, a given monetary policy can often be quite ineffective or even do the opposite of what it was originally intended to do. Indeed, this latter idea was the underlying theme of our analyses of the joint effects of monetary policy and fiscal policy in Chapter 15 and Chapter 16. Here, we've seen that complementarity between fiscal and monetary policy also is important for the determination of optimal policy-setting.

<sup>&</sup>lt;sup>205</sup> Which is also true of the now-more-popular Calvo model.

<sup>&</sup>lt;sup>206</sup> Although here do not go into the details of how and why this modification can be thought of as an appropriate setting for fiscal policy.