Chapter 24
Financial Accelerator

Starting in 2007, and becoming much more pronounced in 2008, macro-financial events took center stage in the macroeconomic landscape. The "financial collapse," as many have termed it, had its proximate cause in the U.S., as several financial-sector institutions experienced severe or catastrophic downturns in the values of their financial assets. Various and large-scale policy efforts were implemented very quickly in the U.S. to try to contain possible consequences.

The motivation behind these policy efforts was not to save the financial sector for its own sake. Instead, the rationale for policy responses was that severe financial downturns often lead to contraction in **real macroeconomic** markets (for example, think in terms of goods markets). Despite a raft of policy measures to try to prevent such effects, the severe financial disruption did cause a sharp contraction of economic activity in real markets: GDP declined by nearly four percent in the third quarter of 2008, the time period during which financial disruptions were at their most severe. This quarterly decline was the largest in the U.S. since the early 1980s, and GDP continued declining for the next three quarters.

But the reason this pullback in GDP was especially worrisome was something history shows is common. When it is triggered by financial turbulence, **a contraction in real economic activity can further exacerbate the financial downturn.** This downstream effect was the real fear of policy makers. If this downstream effect occurred, the now-steeper financial downturn then could *even further* worsen the macro downturn, which in turn could *even further* worsen the financial downturn, which in turn could *even further* worsen the macro downturn, and …. on and on. If this chain of events is set in motion, then it can become extremely difficult for anyone, policy officials or others, to do anything about it.

This type of adverse feedback dynamic between financial activity and macroeconomic activity is referred to by different terms. In media portrayals, terms such as the "financial accelerator," "financial feedback loops," "loan spirals," and others quickly came into use to describe exactly this scenario as both financial and macro conditions deteriorated.

This chapter studies the **financial accelerator** framework, and its broad purpose is to study general properties of events like the one just described. The accelerator model is **not** a new framework, despite its sharp popularity in macroeconomics since 2008. It actually dates back to Irving Fisher and other economists in the 1930s, as they attempted to understand the adverse linkages between macroeconomic activity and financial markets during the Great Depression. In the 1980s and 1990s, Ben Bernanke became one of the world's leading scholars of the Great Depression, and he, first on his own and then later with academic colleague Mark Gertler, built quantitatively richer versions of the accelerator. The framework has been a staple in macroeconomic research since then, but, until 2007 and 2008, had not been used for much practical policy making.

But its appeal as a foundation for macro-finance issues has exploded since 2008, as many policy officials (including Bernanke himself as Chairman of the Federal Reserve at the time!) and researchers have actively developed the model further. The goals have been to both inform policy advice and to simply learn more about the interconnections between macroeconomic markets and financial markets.

To be clear, our study of the accelerator framework is meant as neither a history of the recent financial collapse nor of the "Great Recession" in the U.S. that it precipitated. When scholars such as Fisher, Bernanke, Gertler, and others developed the framework, they of course did not have these very recent events in mind. Rather, they were interested in learning more about the general properties of adverse feedback loops. Recent events have cast a spotlight on thinking more deeply about how financial fluctuations and macroeconomic fluctuations interact with each other through feedback effects when certain shocks affect the economy, and the accelerator framework has once again been viewed as a good starting point.

The accelerator model developed below builds on the multi-period firm analysis of Chapter XX; but it could just as easily be developed in the context of multi-period consumer analysis. To make things as simple as possible, yet rich enough to study the accelerator and related effects, we work with the two-period firm model from Chapter XX, but the ideas extend readily beyond two periods.

There are four building blocks of the accelerator model: i) a multi-period view of firm profit maximization; ii) a financing constraint that captures how financial assets can be important for loans that are used to back physical capital investment purchases; iii) a notion of "government regulation" that operates through financing constraints; and iv) a relationship between firm profits and dividends.

While introducing the building blocks, extended discussion describes fundamentally new ideas that we have thus far not encountered. We then formally work through several results and insights that the framework delivers, including the "accelerator" effect itself. We conclude with some bigger-picture discussion about the framework. But even before describing the building blocks, we have to consider an aspect about the nature of assets that is crucial for the accelerator model.

Risk Properties of Assets

Even before introducing the four building blocks of the accelerator model, we need to describe the natures of the two fundamentally different types of assets that are central for the model. The fundamental difference between assets is in their **risk properties.** At one end of the economic risk spectrum are **riskless assets.** In the model, short-term government bonds are to be thought of as the riskless asset (although we will also consider the marginal product of capital to be a riskless asset when we get into the model's details). At the other end of the economic risk spectrum are **risky assets.** In the model, we will consider stocks (defined exactly as in the infinite-horizon model of Chapter 8) as the risky asset.

For all of our analysis, **risk** is defined to mean the "guarantee" about the value of an asset's payoff at some point in the future. More precisely, at a fixed date in the future, a riskless asset is one whose value is known for sure by market participants, whereas a risky asset is one whose value is not known for sure by market participants.[207] The latter, risky, asset is the one whose value has less guarantee.

No asset is truly riskless. But what matters for the definition is a relative notion of risk. As an example, U.S. aggregate stock returns vary more sharply over time than do U.S. short-term government bond returns.[208] U.S. bond returns do vary, in sometimes unexpected ways – hence one may want to call them "risky." But stock returns are even more risky than U.S. short-term government bond returns. For the purposes of economic analysis, it is thus sufficient to identify stocks as risky and U.S. short-term bonds as riskless,[209] which is helpful taxonomy. Several further aspects about risk and asset returns are worth describing.

First, we should recognize that financial assets that are not bonds (even more precisely, that are not short-term government bonds) by definition **do not guarantee, based on purely economic incentives, any payment(s).** In practice, any "guarantee" provided by risky assets is conferred on them by legal precedents, government decrees, social norms, and so on, which have various degrees of social value – but they are **not** conferred by pure economic incentives. Thus, if stocks carry some "guarantees" of payment(s), they should be thought of as arising for "non-economic" reasons.

Second, there is no reason why "stocks" had to be selected as the model's risky asset. In principle, **any** financial asset that is more risky than U.S. short-term government bonds serves the goals of the model equally well, especially because our analysis, while couched in a formal optimization problem, is ultimately qualitative. A few examples of

[207] In terms of probability and statistics, a riskless asset has a known expected value (the first central moment) and **zero** variance (the second central moment) around that expected value. A risky asset has a known expected value and a **positive** variance around that expected value. "Risk" is implied by the positive variance.

[208] As in Chapter XX, we should think of stocks as something like the S&P 500, which is representative of stock-market aggregates.

[209] Recall the discussion in Chapter XX that U.S. short-term government bonds have long been considered the riskless asset in markets. Of course, it is possible that some (adverse and large) negative shock could prevent the U.S. government from making its next short-term bond repayment. But, in practice, this has never happened in over 200 years of U.S. history.

other financial assets include foreign stock, shares in oil companies, and holdings of financial products based on housing mortgages – the last example in particular is relevant for the recent U.S. financial and economic downturn. But the accelerator framework, developed as it was originally in the 1930s and then re-developed in the 1980s and 1990s, captures much broader ideas than the events of just the past few years. A bit further discussion appears when we describe the first building block of the model, but the broad notion of "stocks" captures the crucial risk idea for the accelerator.[210]

Third, for either riskless or risky assets, one can always define the "interest rate" on that asset. For short-term bonds (which sometimes will be referred to from here on simply as "bonds"), the nominal interest rate is defined by $1+i=\frac{1}{P_1^b}$ (in which we are continuing with our maintained assumption of unit face value of bonds (FV = 1) upon payoff, and the "1" subscript on the price of a bond (P_1^b) is the period in which that price is being paid). For stocks, the nominal "interest rate," or nominal "rate of return," is defined by

$$1+i^{STOCK}=\frac{S_2+D_2}{S_1},$$

in which the notation is exactly as in our earlier study of stock prices: S_t is the nominal price of a share of stock in period t, and D_t is the nominal dividend payment per share of stock in period t. In the accelerator model, any gap between i and i^{STOCK} drives critical results.

Fourth, as a point of terminology, we will refer interchangeably to both i and i^{STOCK} as "interest rates" or "rates of return." For non-bond assets, "interest rates" is unconventional language (rate of return is usually preferred). But from a presentation perspective, using the same terminology for different types of assets emphasizes that there are economic relationships between them and consequences implied by them that matter for some types of transactions. These relationships emerge in detail below.

Fifth, regardless of risk properties, we can measure an asset's rate of return in either nominal terms, as shown above, or in real terms (in which we measure the returns by r and r^{STOCK}), the latter by appropriate application of the Fisher relation. For consistency with earlier analyses, we begin with a nominal view as we now turn to the building blocks of the model.

[210] We should also note that assets whose risk properties lie between "purely riskless" and "extremely risky" also exist. For the purposes of this chapter, we do not need to consider such "intermediate" risk levels, the two we have of "riskless" versus "risky" is sufficient.

Building Block 1: Firm Profit Function

The first building block is the firm's dynamic profit function. As stated above, we limit ourselves to a two-period time horizon, with optimization conducted at the start of period one. But note that all of the analysis and results can be readily extended to more than two periods.

Given that stock is the risky financial asset in the model, it appears in the first building block of the framework, the **dynamic profit function,**

$$P_1 f(k_1,n_1) + (S_1 + D_1)a_0 - P_1(k_2 - k_1) - P_1 w_1 n_1 - S_1 a_1 + \frac{P_2 f(k_2,n_2)}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2(k_3 - k_2)}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{S_2 a_2}{1+i},$$

which is an extension of the profit function studied in Chapter XX. The extension is simply that stocks are accumulated by the firm, for the purpose described in the next subsection. As in Chapter XX, because the analysis is being conducted from the perspective of the start of period one, the period-two components of profits are discounted by a (gross) nominal interest rate $1+i$.

Two important points are useful to clarify.

First, a natural question may be: where are the short-term riskless bonds? The answer is that the interest rate i that appears in the discounting is exactly the one on short-term bonds. Thus, even though it superficially appears that bonds are not present in the profit function – they actually do appear. Hence, both the riskless interest rate and the risky interest rate appear in the profit function.

Second, an important distinction to make in reading the profit function is one between the optimization problem faced by **one single (small) firm** versus aggregate market variables. Although we will take the representative-firm approach in analyzing the results of the optimization, at **this stage** of the analysis, the firm is to be viewed as one of the small, atomistic firms in the overall economy. Thus, the terms involving stock in the profit function are **not,** at this stage of the analysis, this particular firm's own stock. If they were this particular firm's own shares, it would be hard to understand why (in the ensuing analysis) stock prices and dividends would be taken as given. Stock prices and stock dividends are to be thought of in their usual aggregate terms, and they are taken as given until we get to the first-order conditions. This distinction is exactly the one between partial-partial equilibrium, partial equilibrium, and general equilibrium that we have drawn several times.

From an analytical perspective (and as always in considering the two-period framework) the firm needs neither physical capital at the start of the non-existent "period three" –

hence, $k_3 = 0$ – nor financial assets at the start of the non-existent "period three" – hence, $a_2 = 0$.[211]

Building Block 2: Financing Constraint

The second building block of the accelerator framework is its critical conceptual idea. All of the analysis ultimately revolves around it, so it is important that we clearly understand it, both technically and conceptually.

An important practical issue for many firms is that in order to purchase **physical assets** (think large-scale expenditures for investment in machinery, equipment, computers, and so on), they require **a sufficient (market) value of financial assets,** which facilitates the **borrowing** that is needed in order to finance their purchase. The "market value" nature of financial assets is important: it indicates that **both the price and the quantity of financial assets** held by a firm matter for its ability to borrow.

This raises a question: why does a firm need to borrow at all? In Chapter XX, firms simply demanded as much labor and as much capital as was profit-maximizing: there was nothing formal within the framework that concerned borrowing. In certain situations, however, a firm may need to borrow for large-scale investment purchases. In these cases, a particular type of market imperfection, which is viewed as central in financial theory, necessitates that a firm "back" a loan, or "pledge collateral against" procurement of a loan. The proceeds of the loan are then used for physical capital expenditures. By inherent properties of assets, it is "risky assets" that must be used to back the loans obtained for capital expenditures (which are to be interpreted as "riskless" assets).[212] These points are expanded below. But a critical connection with the basic firm analysis of Chapter XX is that all of the ideas to be presented below **could** indeed have been present there, as well; for a reason to be made very precise, though, they can all be thought of as zeroing out in Chapter XX.

To make progress with the mathematics of the model, the expression that forms the centerpiece of the second building block is $P_1 \cdot inv_1 = \quad S_1 \cdot a_1$ (in which $inv_1 = k_2 - k_1$ is physical capital investment). Or, to instead express things in terms of only k, simply substitute the definition of investment. This allows the **financing constraint** to be written a bit more explicitly as

[211] Nor any other assets, if there were other assets in the formal framework.

[212] We are using "stock" as the risky asset, but an appropriate interpretation is that any financial asset(s) could be held by firms to back borrowing that will be used for physical capital purchases. The details of exactly which financial assets are collateralizable are country-specific and/or market-specific and/or industry-specific, and they are governed by various private-market arrangements and government regulations. The details of such institutional setups are beyond the scope of our study.

$$P_1(k_2 - k_1) = \quad S_1 \cdot a_1 .$$

This financing constraint will be modified in a slight, but important, way in the next subsection. So this will not be the exact way we use it in the analysis later. Nonetheless, if this constraint did not exist, **none** of the rest of the framework matters, a point that will be established rigorously when we study the model's insights.

Regarding the formal expression of the financing constraint as written so far, the left-hand side is the nominal value of physical investment expenditures, $P_1(k_2 - k_1)$, the firm plans to undertake in period one. On the right-hand side, the term $S_1 a_1$ is the firm's **market value of collateral** for the loan. The market value of collateral is the backing pledged by the firm in order to obtain the loan, whose proceeds in turn will be used to purchase physical assets. Even though the financing constraint is not yet in its technical final form, it is close enough to its final form that three important points are worth discussing (the first two of which are highly related, but we disentangle them to make them conceptually easier to understand).

In financial theory, one of the most important market imperfections is **informational asymmetries.** We distinguish two aspects of informational asymmetries: those arising between potential borrowers and potential lenders, and those arising between the pair and the overall economic environment as time evolves. For both aspects, a simple illustrative example is the case of an individual seeking a mortgage loan in order to purchase a house.

First, no matter how many credit references, income verifications, and other means-testing a potential lender conducts, a potential borrower fundamentally knows more about his own personal circumstances when the time for (long-lived) loan repayments begins. This informational asymmetry provides the potential lender an incentive to not make a loan in the first place, even if the loan would be beneficial to both the lender and the borrower. The incentive driving the lender is fear that he will not be repaid.

Private markets have developed a way to manage some of the consequences of this aspect of informational asymmetries: lenders require borrowers to "put some skin in the game" at the time a loan is originated. If the potential borrower puts down, say, 20% of the total value of the house, this affects the lender's incentives to loan the remaining 80%.[213] The lender now effectively knows that, if the borrower does "walk away" from the loan repayments very quickly, the borrower would at the very least have lost 20% of the value of the house. And that cost may be large enough that it would induce the potential borrower to not approach the lender in the first place, unless he was serious about making a steady stream of repayments.

[213] 20% down payments for home mortgages was a long-standing norm in the U.S., until the several years before the events of 2007 and 2008, when down payment requirements sharply declined.

Such "down payment" requirements affect not only consumers, but also firms when they are making large-scale purchases. The intuitive way to think about the market value of assets on the right-hand side of the financing constraint is thus as a down payment that is being used to back a loan for use on purchases of capital goods. The firm then makes a steady stream of repayments that slowly repays the loan.

The "steady stream of repayments" raises the second of the two aspects of informational asymmetries: there is inherently a **maturity mismatch** between the financial asset being used as a down payment, and the physical asset for which the loan is being made. This aspect does not involve any "malicious" informational asymmetry between borrower and lender. Instead, the asymmetry is between the perfectly-aligned goals of the borrower-lender pair and the overall economic environment – the latter naturally changes over time even if the aligned borrower-lender goals do not.

The maturity mismatch is captured in the model in a simple way by including **a_1, not a_0, on the right-hand side of the financing constraint.** The reason for a_1 appearing on the collateral side can be described in purely technical terms: a_0 is pre-determined at the start of period one, implying there is no choice by the firm about its value. In order for the framework to make testable predictions, the firm should have some choice about the right-hand side of the collateral constraint, hence the inclusion of a_1.

But to understand it conceptually, it is helpful to continue the example of an individual person pursuing a mortgage loan in order to purchase a house. Obtaining a mortgage loan typically requires a down payment (as above, say 20% of the market value of the house). In the process of completing the loan, the individual has to make several decisions about his own personal finances. These decisions are intended to obtain the 20% down payment in a liquid form to pass on to the lender.[214] Regardless of the precise decisions, the key aspect is that there are some **decisions** that the potential borrower had to make in the process of going to the bank, withdrawing funds from certain accounts,

[214] For example, the borrower may have to withdraw funds from a protected savings account against which checks cannot be written and then transfer it to his own checking account. Regarding passing on the down payment to the lender, it is convenient to think of a "down payment" on a home-mortgage application as being "in cash." But it is technically not cash. Technically, the down payment is a type of "short-term bond." The bond nature of a down payment arises because (given the magnitude of resources involved) a potential lender inevitably asks for a "certified check" from an individual's bank. The certified check is a verification provided by the individual's personal bank that the funds are actually in his bank account, and that the funds are being held for the explicit purpose of payment to the lender. These details are unlike the case of an individual handing over literally cash, or of providing an uncertified check. The uncertified check provides no verification of the availability of funds when the lender tries to redeem it (which again raises the consequences of the first, "malicious," aspect of informational asymmetries); whereas hard cash is generally not accepted (partly for legal reasons) for such large transactions. The individual's own financial institution essentially has issued a "short-term bond," which will be repaid (out of the borrower's funds) when the lender redeems the certified check. From an operational standpoint, these "bank-issued short-term bonds" are equivalent to a reliable government's short-term bonds – the key aspect is that they are both short-term.

depositing extra funds in other accounts if necessary, obtaining a certified check, and so on.

In contrast, the very nature of a house, which is the ultimate reason for borrowing in this example, makes it a much longer-term asset: the average house can last for decades. So the "maturity mismatch" is that the financial asset used to collateralize the loan on which the house is actually purchased is much shorter in time horizon than the long-lived physical asset being bought. The long-lived nature of the physical asset is the source of the long-lived "steady stream of repayments" by the borrower back to the lender.

The same maturity mismatch idea applies to firms' financing of physical capital purchases collateralized by loans secured by stock. New machines, new factories, new delivery vehicles, and so on last for much longer periods of time than the financial assets being used to collateralize loans for their purchase. Their long-lived nature provides part of the source of profits for many periods, which in turn is the source of the long-lived steady stream of repayments by the firm back to the lender.

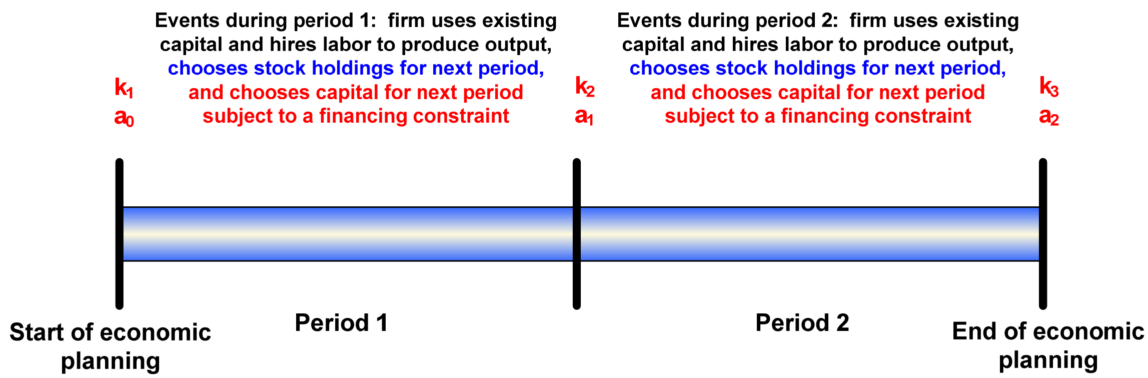
In the two-period setup, the end of the second period makes things rather stark because there is no more need for either physical capital or financial assets. Extending the analysis beyond two periods, and, importantly, allowing capital to be productive and therefore profit-generating for many periods, brings the maturity mismatch idea squarely into view in the model.[215] While the starkness of the two-period framework mutes the maturity mismatch idea a bit, it does capture it in a simplified form and it does not obscure the economic insights provided by the accelerator framework.

The third point is more technical. The financing constraint should properly be considered an **inequality** constraint: $P_1(k_2 - k_1) \leq \quad S_1 \cdot a_1$. The fully correct analysis of inequality-constrained optimization problems requires the use of Kuhn-Tucker optimization tools, which is a generalization of the Lagrange optimization tools we have been using (recall that the Lagrange method formally applies only to **equality** constrained problems). To keep things in line with our Lagrange-based methodology, we will formally assume that the financing constraint always holds with strict equality – or, in more technical language, that the financing constraint "always binds."

However, when we briefly discuss the consequences of ever-**increasing** financial market returns (which will not be our main analytical experiment), we will have to move away from the Lagrange-based predictions. The richer Kuhn-Tucker analysis would allow us to rigorously establish what happens in this particular case; but we will simply approach it qualitatively. More details are provided when we get to that point, but the important idea is that **a borrower cannot be compelled to borrow more than he wants to**

[215] To see this, suppose we take a weekly view of time periods. If new physical capital takes 52 weeks to build and be ready for use by a firm, and it takes only one week to arrange financing-related decisions, then the financial constraint would intuitively read $P_1(k_{52} - k_1) = \quad S_1 \cdot a_1$.

borrow, even though he can be compelled to borrow less than he wants to borrow. This **asymmetry** is important, one that will be reflected in the permissible values of the Lagrange optimization, which we discuss when we begin considering first-order conditions.



To recap, the financing constraint is the central building block of the accelerator framework. It arises due to fundamental informational asymmetries that affect the borrowing/lending transaction. Although there are other crucial elements of the model, if the financing friction were not present, the entire analysis below would collapse, and the predictions would literally return to those of the baseline Chapter XX, as we will point out in key places. The constraint is a summary way of portraying markets' mechanisms for trying to mitigate the consequences of informational asymmetries that are impossible to avoid in any interesting financial transaction. From a more analytical perspective, the constraint also captures the idea that a firm (more generally, any potential borrower) has to make purposeful decisions about both the value of collateralizable financial assets and about the quantity of physical investment it wants to purchase using loans backed by those financial assets.

Building Block 3: Government Regulation

The financing constraint is to be viewed as a primitive feature of private-market transactions, ones plagued by important informational asymmetries, that arises directly from private parties' incentives. Given the existence of this constraint, it permits the government a channel by which it can possibly regulate market transactions in which informational asymmetries are present.

Specifically, let's layer into the financing constraint above a catch-all "government regulatory measure" $R > 0$, so that the financing constraint with which we will **actually** work is

$$P_1(k_2 - k_1) = R \cdot S_1 \cdot a_1 .$$

Although this form of the financing constraint looks nearly identical to the one introduced above (it would be exactly identical if $R = 1$), it is useful to think of this expression as distinct from the "basic" financing constraint that arose directly in the private sector. Thus, despite their formal near-similarity, it is very useful to keep the second and third building blocks conceptually separate.

The measure $R > 0$ (more precisely, its inclusion in the financing constraint as written above – we will sometimes simply say "$R > 0$" as a shorthand way of describing the entire idea) is the third building block of the accelerator framework. Except for brief discussion immediately below regarding the nature of R, we will stick with the very general interpretation that it is controlled by the government. Extra precision about R is not critical for analysis of the accelerator.

If we do interpret R as reflecting only government regulation, or government oversight, it is easy to imagine that it consists of various components. For example, suppose the U.S. Securities and Exchange Commission (SEC) and the U.S. Treasury are the only two government agencies that have any role in the process of setting R. For certain applied questions, it may be useful to think of R as being decomposed into $R = R^{TRES} \cdot R^{SEC}$, which is the multiplicative product of each agency's own regulatory scheme. If the Federal Reserve System is also involved in providing such regulation, then it may also be helpful to think in terms of $R = R^{TRES} \cdot R^{SEC} \cdot R^{FED}$. Decompositions of this type may be useful in considering the details of government regulatory policy and its implementation.

One could instead think of R as being set by both government regulation (by one or many underlying institutions) and by private-sector "norms" regarding borrowing and lending. In this case, it can be useful to decompose R into $R = R^{GOV} \cdot R^{PRIV}$, which emphasizes the private-sector / government spectrum. Then, just as above, one could decompose R^{GOV} into finer sub-categories if needed; by analogy, one could also decompose R^{PRIV} into finer sub-categories if needed.

However, in the basic analysis, we will simply consider $R > 0$, because R is taken as given from the perspective of private-market participants in a financial transaction. Our analysis has nothing concrete to say about how different groups might organize to "lobby" various government agencies and/or private-market organizations to change (components of) R. While interesting as talking points, this more advanced analysis requires bringing in additional constraints that describe the organizing process, the lobbying process, and so on. For the general analysis of the accelerator framework, though, it is overkill. In the interest of keeping things as simple as possible, and to fix some language for the rest of the analysis and discussion, let's return to describing $R > 0$ as a catch-all government regulation measure that affects private-market financial transactions.

Given this interpretation, what exactly is R? Some examples include institutions such as rules regarding filing of proper documentation, full disclosure ("truth in lending") laws, regulations that provide for direct lending in some markets and/or geographic regions, or regulators looking favorably at some sub-markets. But these are all talking points, because, once again, the model makes no statements about the sources of R.

Regardless of the interpretation of R, it plays an important role in markets. To see this directly, examine again the financing constraint that contains R. Now including $R > 0$, its literal statement is that for a given market value of collateralizable assets S_1a_1, the amount that can be used as the backing for a loan to be used for physical investment is ***R*** **times that market value.**

In financial analysis, R is referred to as the **leverage ratio,** which measures the **multiple** of the market value of collateralizable assets up to which borrowing can occur for the purchase of physical assets. Intuitively, a very high value of R indicates "fragility" (which need not be, but could be, excessive) on the part of a borrower, a point that has been brought up frequently in discussing the U.S. financial and economic situation starting in 2007. Finally, if R is set solely by government regulation, purposeful changes in R, holding S_1a_1 constant, imply that the amount that can be borrowed scales directly with R.

To recap, $R > 0$, as embedded in the financing constraint, is the third building block of the accelerator framework. In the analysis below, **this** version of the financing constraint will appear in the formal problem, **not** the primitive form $P_1(k_2 - k_1) = \quad S_1 \cdot a_1$ described earlier. The primitive form should be thought of as a mechanism originated solely by private-market participants in order to manage the consequences of informational asymmetries. Given the existence of the basic constraint, the government can impose some "regulation" on financial transactions through it. The financing constraint in its final form is thus written as $P_1(k_2 - k_1) = R \cdot S_1 \cdot a_1$ (a special case of which is obviously R = 1). But, to be clear, the regulatory aspect could not even manifest itself if not for the existence of the constraint in the first place. It is important to keep these two ideas distinct, even though the third building block builds directly on the second building block.

Building Block 4: Profits and Dividends

Dividends are the payments made by publicly-traded companies to their shareholders, who are ultimately the owners of public companies. Corporate dividend policies naturally differ amongst countries, amongst industries within a country, amongst sub-industries within industries, and so on. Differences reflect different economic structures, different governing institutions, and various degrees of social and cultural norms regarding acceptability.

Adopting a U.S.-centered, representative (publicly-traded) firm approach, it is instructive to examine the share of total corporate profits paid out as dividends. **[Figure [XX] plots the S&P 500 dividend payout rate, which is simply the fraction of total corporate profits of the S&P 500 firms that are paid out as dividends]**

[GRAPH OF S&P 500 DIVIDEND RATE]

While the dividend rate has clearly declined over time, a stark point to note is that there was **not** much of a change in the rate as the financial turmoil and ensuing U.S. Great Recession of 2007 – 2009 occurred.

The fourth building block closes the macro-finance link with the statement that the percentage ρ (the Greek letter "rho") of profits paid out as dividends is constant over time. Formally, the nominal quantity of per-share dividends relates to ρ, P_2, and real profits according to

$$D_2 = \rho \cdot P_2 \cdot profit_2 ,$$

in which $profit_2$ is real per-share profits, and in which attention is limited to just period two because of the two-period framework being studied. If we extend the framework beyond two periods, this relationship simply generalizes to $D_t = \rho \cdot P_t \cdot profit_t$.

Analysis Part I: Basics

Having established the four building blocks of the model, we can now begin studying its predictions. In terms of formal optimization, the Lagrangian for the problem is

$$P_1 f(k_1, n_1) + (S_1 + D_1)a_0 - P_1(k_2 - k_1) - P_1 w_1 n_1 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2(k_3 - k_2)}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{S_2 a_2}{1+i}$$
$$+ \lambda \left[R \cdot S_1 \cdot a_1 - P_1 \cdot (k_2 - k_1) \right]$$

in which λ is the Lagrange multiplier on the financing constraint. The next step would be to compute first-order conditions. But it is very helpful to discuss important economic intuition about the multiplier λ.

When presenting the second building block, we asserted that the financing constraint will be assumed to always hold with equality in the formal analysis. However, it need not hold with equality in practice. A consequence of the **possibility** that it holds with equality is that the Lagrange multiplier must be (weakly) positive at the optimal choice **– that is, $\lambda \geq 0$ must hold at the optimal choice.** The multiplier λ cannot be a strictly negative value at the optimal choice. The non-negativity of λ is a condition we have not at all encountered before, and its meaning is important.

The non-negativity of λ may seem a somewhat technical point, but it is actually easy to describe in terms of economic insight. Note that the non-negativity of λ is simply an **asymmetry regarding λ.** The reason this mathematical asymmetry about λ arises as part of the optimal solution of the accelerator framework is that it reflects the conceptual asymmetry about borrowing that is an input into the framework. If the firm optimally chooses not to borrow at all for the purpose of physical capital purchases, then it optimally decides that there is no need for it to rely on the financing restriction. There is nothing that **compels** a firm to find a lender to procure a loan if it optimally chooses to not do so. If the firm optimally chooses to not borrow, then $\lambda = 0$.[216] Important here is the repeated use of the term **optimal.**

This intuition is a very general and powerful one in optimization analysis, regardless of whether it is an economics application, an engineering application, a physics application, or any application: **if asymmetries inherently exist in the inputs to the optimization problem, then multipliers must themselves display asymmetries at the optimal solution.** Once again, important here is the use of the term **optimal.**

The housing example is again useful for further illustrative intuition; for simplicity, let $R = 2$ throughout the rest of this example. Suppose the optimal level of spending for (the total value of) a house is \$100,000, and an individual has, say, \$200,000 in funds in his personal bank account that have been optimally chosen for the **sole purpose of backing a loan for the house. Then there is optimally no reason for the individual to obtain a loan at all!** He can simply pay \$100,000 directly for the purchase of the house with his own available funds, without any need for borrowing. Optimally choosing to not rely on the financing constraint exactly implies $\lambda = 0$ at the optimal choice.[217]

The Kuhn-Tucker analysis (which, to reiterate, we are not employing) properly rules out strictly negative values of λ and, intuitively, inserts $\lambda = 0$ when this situation arises. In the context of the examples above, Kuhn-Tucker thus mathematically delivers the correct result.

[216] This result can be stated more powerfully, given the structures built into the accelerator model, as will become clear through the first-order conditions and ensuing analysis below: even if a firm does have to borrow, if borrowing interest rates are identical to our standard notion of interest rates – that is, if $i = i^{STOCK}$ -- then $\lambda = 0$.

[217] A more everyday example is a personal favorite. I enjoy driving fast. What if speed limits everywhere were 300 miles per hour? You are allowed to drive as fast as you want, but your speed cannot exceed 300 miles per hour. In principle, this is a constraint imposed on my optimal speed. But in practice, it is one that is irrelevant for my optimal choice because I cannot purchase a car that drives that fast. The constraint exists, but it does not affect my behavior, hence the multiplier on it at the solution of my optimization problem about how fast to drive is zero.

Instead, the straightforward Lagrangian analysis leads to the conclusion that the value of λ, in the same examples, is strictly negative. This is despite the logical conclusions above that λ cannot be strictly negative due to the optimal lack of reliance on the financing constraint. The incorrect conclusion that $\lambda < 0$ in turn leads to other downstream conclusions that, unfortunately, are also incorrect. The bottom line is that, in cases where the financing constraint simply "does not bind," (i.e., is optimally ignored) the formal Lagrange analysis leads to incorrect results.

The opposite case, however – that is, when $\lambda \geq 0$ turns out as part of the result – works just fine from the perspective of both Lagrange and Kuhn-Tucker analyses. The opposite case is that of a borrower being compelled to borrow because his pledgeable funds are insufficient to pay for the optimally-chosen spending. In the housing example (and with $R = 2$) from above, suppose the individual has only \$50,000 in his personal bank account to optimally pledge against a loan for the purpose of buying the \$100,000 house. In this case, he must collateralize his funds in order to borrow enough to pay for the house. In this simple example, he can take out a loan of \$50,000 with his own personal \$50,000 pledged as collateral, and then use the resulting \$100,000 to purchase the house. This result is reflected technically in a (weakly) positive value of λ, which both the Kuhn-Tucker and Lagrange analyses correctly deliver.

This discussion regarding numerical values of multipliers at the optimal choice should strike you as intuitive – read it over several times, though, to allow it to sink in. Also think of similar personal situations, like purchasing a car or paying to attend college, which are also events that may or may not have required obtaining a loan.

In none of the models studied thus far have non-negativity issues regarding multipliers arisen. This is because asymmetry has fundamentally not appeared in models thus far. In such cases, the value of multipliers at the optimal choice could be positive or negative – there is no asymmetry conditions regarding values of the multiplier. But asymmetries do (easily) arise in the accelerator framework, due to the basic economic asymmetry in the need for borrowing.

In all of the formal analysis of the accelerator, we will limit attention to cases in which $\lambda \geq 0$ turns out to be part of the optimal solution. The discussion regarding asymmetries is nonetheless raised here because one may wonder why situations like the financial collapse of 2007-2008 and associated downstream events do not occur "all the time." The basic reason is simply the asymmetry regarding borrowing, which is reflected in the asymmetry regarding λ.

Given the setup of the accelerator framework and limiting attention in the formal analysis to cases in which $\lambda \geq 0$ is part of the optimal solution, we could in fact re-interpret the analysis of Chapter XX as being the special case of $\lambda = 0$ (which is intuitively the knife-edge case between the Kuhn-Tucker and Lagrange cases). What $\lambda = 0$ at the optimal choice means is that despite the existence of informational asymmetries that require a financing constraint, they turn out to not at all matter for the results of Chapter XX. More precisely, the financing constraint ends up not at all affecting either the capital demand or the labor demand functions if $\lambda = 0$ is in place. We will see these points formally below.

However, by allowing $\lambda \geq 0$ and not assuming $\lambda = 0$ (note this distinction), the richer accelerator analysis allows us to study a crucial issue (besides that of the accelerator effect itself): the market and/or regulatory settings that allow for $\lambda = 0$ to arise as an **outcome,** rather than being imposed as an **assumption.** Stated more technically, the accelerator allows us to consider how or why $\lambda = 0$ can emerge **endogenously,** rather than simply being assumed **exogenously.** We will revisit this important economic question after obtaining first-order conditions and doing some other preliminary analysis.

First-Order Conditions

Based on the Lagrangian above, the first-order conditions with respect to n_1 and n_2 are

$$P_1 f_n(k_1, n_1) - P_1 w_1 = 0$$

and

$$\frac{P_2 f_n(k_2, n_2)}{1+i} - \frac{P_2 w_2}{1+i} = 0.$$

Cancelling terms appropriately in each of these expressions gives $w_1 = f_n(k_1, n_1)$ and $w_2 = f_n(k_2, n_2)$. If the analysis is extended beyond two periods, the first-order condition on labor is $w_t = f_n(k_t, n_t)$, for every time period t. Irrespective of whether the time horizon is two periods or longer, these labor demand conditions are identical in functional form to those from Chapter XX. Thus, up to first-order, there is no shift of the labor demand functions; but further discussion about this appears below.

Given the particular setup of the framework, in which it is only physical capital purchases that are subject to financing constraint (which is the most common form of the accelerator model), it is only physical capital investment in period one that is (potentially!) directly affected by financing issues. The first-order conditions with respect to k_2 and a_1 are thus the heart of the analysis. These conditions are, respectively,

$$-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} - \lambda P_1 = 0$$

and

$$-S_1 + \frac{S_2 + D_2}{1+i} + \lambda \cdot R \cdot S_1 = 0.$$

Zooming in again on the multiplier $\lambda \geq 0$, its appearance is what differentiates the first-order condition on k_2 from the simpler one that appears in basic firm analysis. The basic firm analysis is to be thought of as the special case of $\lambda = 0$. In order to work out the implications of the more general case in which $\lambda \geq 0$ is part of the optimal solution, a joint analysis of both of the immediately preceding first-order conditions is required.

The ensuing analysis takes up two distinct, but related, questions: first, what economic and/or regulatory conditions cause $\lambda = 0$ to emerge as an **outcome** (rather than being assumed); and second, how, in the case of $\lambda \geq 0$, the capital demand function modifies. The first issue requires analysis of only the first-order condition on stock; the second issue requires joint analysis of both the first-order condition on stock and the first-order condition on physical capital. Once we have the modified capital demand function in place, we can then directly study the accelerator effect itself.

When Does λ = 0?

An important question is the conditions (if any exist) under which $\lambda = 0$ emerges as an outcome as part of the optimal choice. Studying this question requires only the first-order condition on stock. Because doing so spotlights the insights, isolating the λ term from this first-order condition is helpful. Simultaneously emphasizing the **real** (as opposed to nominal) nature of the accelerator, although not required, is also helpful.

The full set of algebraic rearrangements (which is simply several steps of algebra) appears in the Appendix; proceeding here directly to the resulting expression, the multiplier λ that emerges is

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}.$$

Based on earlier discussion, we know that $\lambda < 0$ cannot occur. The fact that it seems that $\lambda < 0$ can occur reflects the use of purely Lagrange tools. Thus, we can formally ignore the case of $\lambda < 0$ because the Kuhn-Tucker analysis would properly insert $\lambda = 0$ in its place. In terms of rates of return, we can thus ignore the case of $r - r^{STOCK} < 0$.

Discarding the case of $\lambda < 0$, the expression states that two basic conditions determine whether or not $\lambda = 0$ (or, more precisely, whether or not λ is such a small positive number that it is tantamount to zero).

First, if $r - r^{STOCK} = 0$, then $\lambda = 0$ emerges as an outcome of the analysis. This result is irrespective of the precise numerical value of $R > 0$. Intuitively, if the real returns on riskless assets are aligned with the real returns on risky assets, then, despite the presence of informational asymmetries and the attendant financing constraint, they turn out to simply not matter at the optimal choice. This is all captured by $\lambda = 0$. An exactly analogous result ($i - i^{STOCK} = 0$) emerges if we prefer thinking in terms of nominal rates of return.

If instead $r - r^{STOCK} > 0$, then the expression states (again given the maintained assumption $R > 0$) that $\lambda > 0$ strictly. We have already discussed the interpretation of the strict positivity of the multiplier: the firm must actually rely on the constraint to obtain a loan, due to financial assets that are insufficiently large to purchase the physical capital outright without a loan.

But consider also the case of $r - r^{STOCK} > 0$ simultaneously with a regulatory measure R so large that it can be effectively interpreted as $R = \infty$ (more properly, think in terms of mathematical limits, $\lim_{R \to \infty}$). If R is extremely large, then a very small market value of financial assets can be leveraged up to a very large loan for the purpose of physical capital purchases. An extreme example again using housing markets illustrates this: suppose that \$1 of financial assets could be leveraged up to obtain a loan that can pay for a \$1,000,000 house. In this case, R = 1,000,000. If R is only government regulation, it is quite lax regulation! To also use other language introduced earlier, the marginal borrower in this case has a lot of "fragility" in the sense that if some shock (either personal or aggregate) affects his ability to repay the remainder of his loan, he may be more hard-pressed to do so than if he were compelled, at his optimal choice, to put up more collateral to obtain the loan (which is tantamount to a lower, finite, value of $R > 0$). The example makes the point that, whether it is controlled just by the government or by some combination of government and private-market conditions, an extremely high R effectively renders moot the financing constraint even if $r - r^{STOCK} > 0$.

Stated more formally, the mathematical limit

$$\lim_{R \to \infty} \left[\frac{r - r^{STOCK}}{1 + r} \right] \cdot \frac{1}{R} = 0$$

shows the result that $\lambda \to 0$ as $R \to \infty$. Financial markets or sub-markets sometimes seem to be characterized by very lax regulation for one reason or another; some economists and policy officials interpret the events in housing and housing-mortgage markets in the years leading up to 2007 and 2008 as being excessively lax.

Capital Demand Function

With the condition $\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}$ (which, recall, is nothing but the first-order condition on stock), the next step we can take is to jointly analyze it simultaneously with the first-order condition on k_2. The resulting condition characterizes the capital demand function. To obtain the capital demand function, we do not have to proceed this way. We could instead directly analyze the first-order condition on k_2 above because it does contain the critical object λ, and it is shifts in the capital demand function induced by changes in λ that is the economic issue of interest. But, combining the first-order condition on k_2 with the first-order condition on a_1 in order to eliminate λ allows us, in this case, to think more directly in terms of economics.

As in Chapter XX, let's specialize attention to the Cobb-Douglas production function, $f(k,n) = k^{\alpha} n^{1-\alpha}$, which has associated marginal product of capital $f_k(k_2, n_2) = \alpha k^{\alpha-1} n^{1-\alpha}$.[218] Some algebra is again involved in obtaining an analytic form for the capital demand function, and the derivations appear in the Appendix. Proceeding here directly to the resulting expression, the capital demand function is characterized by

$$r = \left[\frac{R}{R+1}\right] \alpha k_2^{\alpha-1} n_2^{1-\alpha} + \frac{r^{STOCK}}{R+1},$$

which is a generalization of the simpler capital demand function that appeared in Chapter XX. The formal way to see this is to again consider the mathematical limit as regulation R becomes very lax,

$$\lim_{R \to \infty} \left\{ \left[\frac{R}{R+1}\right] \alpha k_2^{\alpha-1} n_2^{1-\alpha} + \frac{r^{STOCK}}{R+1} \right\} = \alpha k_2^{\alpha-1} n_2^{1-\alpha}.$$

The right-hand side is simply the marginal product of capital for the Cobb-Douglas production function, in which case the standard condition $r = f_k(k_2, n_2)$ emerges.

For the accelerator analysis below, we will consider the case in which R is strictly positive, but is not so large that it can be considered to be infinite. That is, our benchmark for the rest of the analysis will be a finite positive R, $(0 < R < \infty)$. The interest is then on how changes in the $r - r^{STOCK}$ term affect the capital investment demand function.

[218] Note that we are abstracting from total factor productivity (TFP), for the sake of some parsimony in the notation. But TFP could easily be introduced in exactly the same way as in the basic firm analysis.

All of the analysis up to now can be thought of as partial equilibrium (that is – one small, atomistic firm) in nature. For the rest of the analysis, we switch to a general-equilibrium viewpoint (the representative firm) because we will be describing the stages through which, among other effects, the equilibrium quantity of investment is affected. Because it allows intuition to be described in a clear way, a good starting point is exactly the capital demand function from Chapter XX. The downward-sloping capital demand function qualitatively plotted in black in Figure 86 represents the basic capital demand function (that is, under the case $\lambda = 0$, which results if $r - r^{STOCK} = 0$).[219] Also indicated is the profit-maximizing quantity of physical capital investment, which is simply the equilibrium in Figure 86; this is a key point for the subsequent analysis.

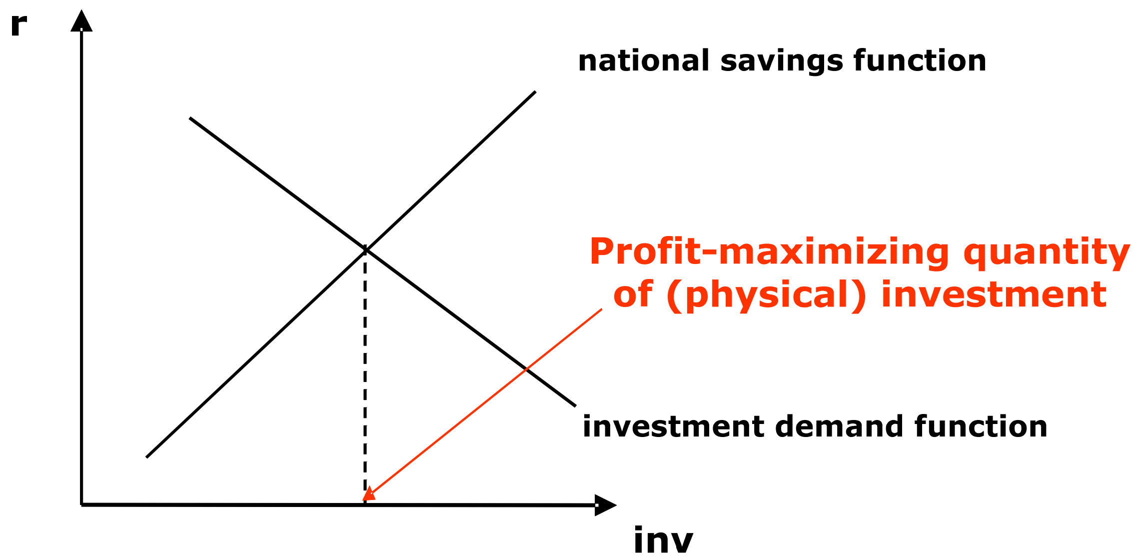


Figure 86. Equilibrium in the physical investment market, in the case of financing constraints not affecting capital demand at all, due to $r - r^{STOCK} = 0$.

Labor Demand Function

Before proceeding to the accelerator effect itself, let's briefly consider labor demand. The first-order conditions on n (regardless of time period) do not directly contain λ. The financing constraint thus apparently does not directly shift the labor demand conditions at all.

The result is more nuanced, however, and it depends on the depth of analysis we are considering. If we take functional forms and the values of k (in particular, it is k_2 that is important for the accelerator effect in the formal model) as given, then we arrive at exactly the conclusion reached above: the financing constraint does not affect the labor

[219] Just as in our basic firm analysis, the diagram is qualitative because it uses linear functions, even though the Cobb-Douglas function implies strictly convex functions. For our qualitative purposes, this is sufficient.

demand functions. Intuitively, this (non-)effect arises from the fact that nothing regarding n appears in the financing constraint.

However, in doing a complete joint analysis of the firm's optimal decisions for both labor **and** capital, the optimal value of k_2 in principle can be different from the one that is optimal in the basic firm analysis of Chapter XX. Inserting this possibly new value of the optimal k_2 into the first-order condition on n shows that the labor demand function would in principle be "shifted" after all.

This complete joint solution is not difficult to obtain, but it requires a little more algebra than just examining whether, conditional on functional form and a particular value of k (whether or not it is the optimal k), the labor demand function shifts. As we have encountered a few times in earlier models, the issue is one of **partial-partial equilibrium** (analyzing the implications of only the first-order condition on n and no other expressions) versus **partial equilibrium** (analyzing the joint implications of both the first-order conditions on labor and capital). In terms of vocabulary that we have also identically used earlier, the former corresponds to the case of **zero first-order effects** on the labor demand function; the latter corresponds to the case of examining **higher-order effects** on the labor demand function. Zero first-order effects are simple to analyze graphically; the presence of higher-order effects are harder to analyze graphically, and they are instead more amenable to solving the model jointly for both the optimal k and the optimal n.

Moving away from the details of the particular way in which we have constructed the accelerator framework, a broader reason that labor demand **can be affected directly** by financing constraints is if some aspect about labor expenditures **directly** appeared in the financing constraint. Such a setup is also admissible. In this case, the first-order conditions on n **would** directly contain terms arising from the constraint (in particular, would contain terms that involve λ). The labor demand functions would then directly – that is, to first-order – be affected by the financing constraint. But our baseline accelerator model is not set up this way.

Analysis Part II: The Accelerator Effect

We now proceed to the accelerator effect itself. The starting point is Figure 86, which is drawn for the case of $r - r^{STOCK} = 0$. Figure 86 displays equilibrium in the investment market when the physical investment demand function is exactly the one studied in Chapter XX. While we do not have to begin exactly here, this point of departure makes it simple to describe the ultimate economic insights; but the economics is the same if we start at some other equilibrium.

Several points are worth clarifying before conducting the main analytical experiment.

First, given $(0 < R < \infty)$, the main interest is in how changes in the interest rate gap $r - r^{STOCK}$ affect the investment demand function. An important observation is that the accelerator model does **not** explain why the interest rate gap itself might change. Changes in the interest gap are thus viewed from the perspective of the model as shocks, in the way we have studied earlier. The accelerator instead primarily focuses on understanding macro-finance dynamics following such shocks.

Second, regarding directionality of shocks, the relevant experiment in the formal analysis is any shock that causes returns on risky financial assets to **decline** compared to returns on riskless assets. That is, the relevant experiment is any shock that causes the gap $r - r^{STOCK}$ to become **larger.** Consideration of the other direction for shocks is briefly discussed later, but is only qualitative due to the asymmetry of borrowing at the heart of the model.

Third, the analysis is mostly graphical. This is partly because the basic results are fairly intuitive, given the effort introducing the building blocks and the analysis already conducted. A truly complete analysis would require much more numerical precision through computer simulations, which is beyond the scope of our analysis. Rather, the goal here is to describe the insight of the accelerator effect, which is easy. The analysis is also graphical because we will adopt an equilibrium-centered view, which requires both the demand and supply sides of the market, as shown in Figure 86.

Fourth, to simplify the analysis even further, suppose that r is constant as r^{STOCK} declines. As we will see as we work through the stages of the experiment, r itself will also decline (in equilibrium). But the quantitative decline in r will not be as large as the (possibly very sharp) declines in r^{STOCK} (which again raises the issue of numerical solutions). This simplification thus also does not obscure the economic insight.

With these several points in mind, start from Figure 86 and consider a negative shock to r^{STOCK} that causes $r - r^{STOCK} > 0$. Our analysis of the capital demand function above shows that the widening of the interest gap $r - r^{STOCK}$ shifts it leftward. This is plotted in Figure 87. The adverse development in the rate of return on financial assets means that it is now more expensive for a firm to use a given quantity of financial assets to back a loan to use for physical capital investment. Starting from any point on the investment demand function, fewer capital goods can thus be purchased.

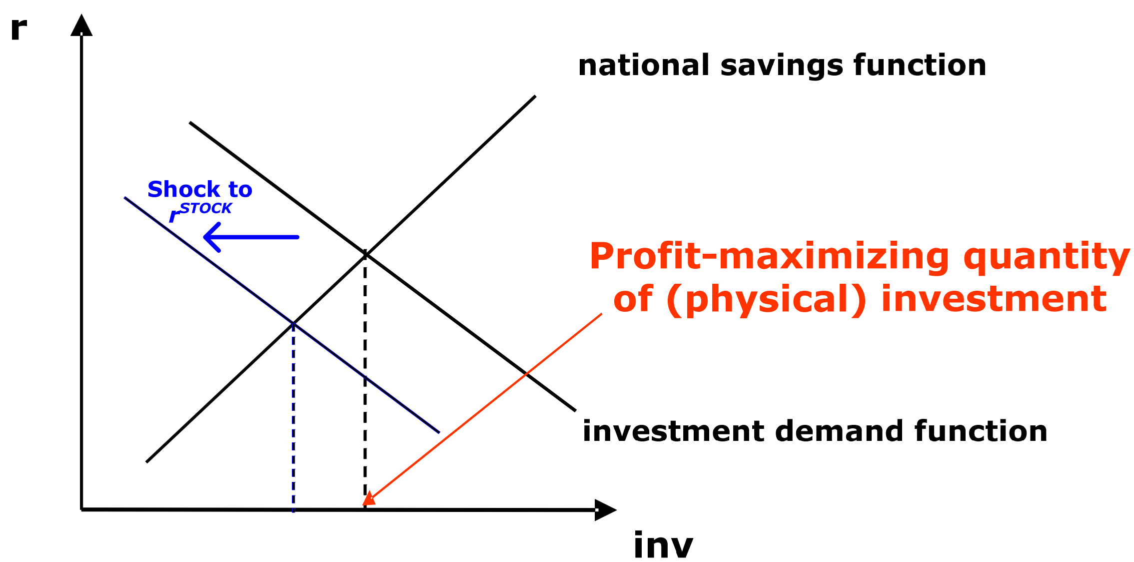


Figure 87. After a decline in r^{STOCK} relative to r, $r - r^{STOCK} > 0$, which shifts the physical capital investment function inwards.

Next, focusing on an equilibrium-centric view, the equilibrium quantity of investment declines. The pullback in investment in turn means that profits decline. This effect on profits occurs because the starting point in Figure 86 was one in which profits were at a maximum. Any other point thus necessarily implies smaller profits, including the new equilibrium level of profits in Figure 87.

It is this stage at which the "accelerator" part of the framework kicks in. The fourth building block of the model, which describes the fairly stable relationship between profits and dividends, means that dividends decline. The return on risky assets is $1+i^{STOCK} = \dfrac{S_2 + D_2}{S_1}$ (or, to express it in real terms, $1+r^{STOCK}$, divide $1+i^{STOCK}$ by the gross goods inflation rate $1+\pi_2$). The decline in dividends thus means that the **return on risky assets declines even further.**

Note the very stark nature of this conclusion.

The analysis began with an adverse **shock** to r^{STOCK}. The background reason is unknown, due to its very nature as a shock, but it sets into motion some events. One of the conclusions that the model then predicts is that $\boldsymbol{r^{STOCK}}$ **declines even further.** Stated very bluntly, the **input** to the analysis is a negative shock to r^{STOCK} – and one of the **outputs** of the analysis is that r^{STOCK} declines even further. In more technical terms, an **exogenous** negative shock to r^{STOCK} leads to the **endogenous** result that r^{STOCK} declines even further.

You should stop and re-read the last few paragraphs again. This is a very dramatic conclusion. Its nature is not something we have seen before.

This **now predicted (endogenous) decline in r^{STOCK}** causes the investment demand function to shift in even further than the shift illustrated in Figure 87.
Figure 88 illustrates this further shift.

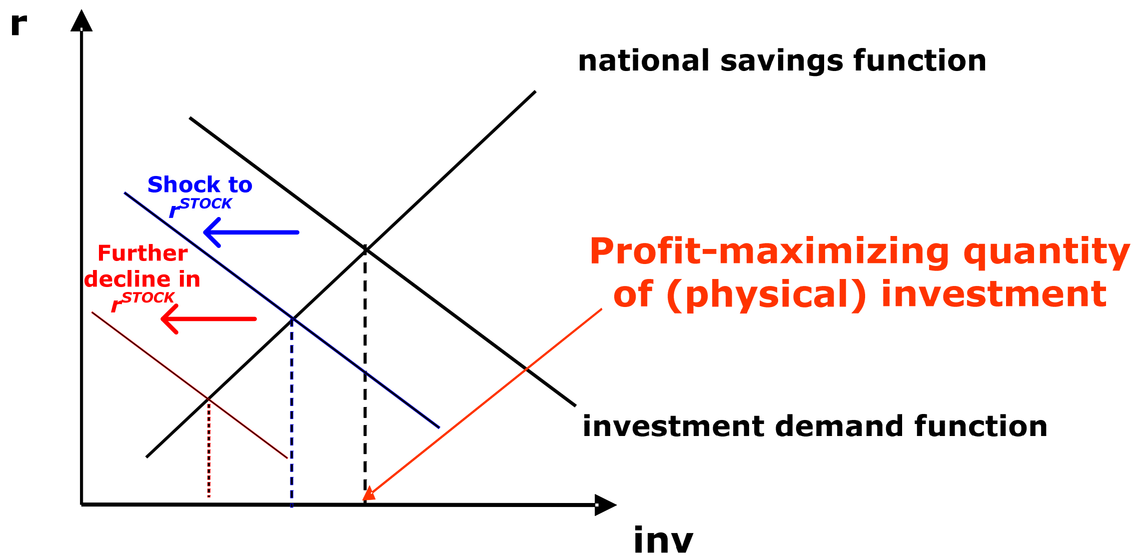


Figure 88. A further, endogenous, decline in r^{STOCK} relative to r makes $r - r^{STOCK}$ even more strictly positive, which shifts the physical capital investment function even further inwards.

Again taking an equilibrium-centric view, the equilibrium quantity of investment falls even further. The decline in investment thus means that profits fall even further, because the market quantity has moved even further away from its starting point in Figure 86. The stable relationship between profits and dividends (the fourth building block) then predicts that dividends fall even further. In turn, the return on risky assets falls even further – that is, by even more than illustrated in
Figure 88.

But this **now predicted *EVEN* sharper decline** in r^{STOCK} causes the investment demand function to shift inwards even further than the shift illustrated in

Figure 88. But this means that equilibrium investment falls even further, which in turn means that profits decline even further, which in turn means that dividends fall even further. But this means that the returns on risky assets fall ***EVEN*** further. And the effects continue on and on.

For parsimony, we will not sketch any further diagrams. But it should be clear where things are heading from the point of view of the framework.

They are heading towards a very severe and jointly-connected downward spiral in macroeconomic outcomes and financial outcomes. This is exactly the accelerator effect: once a financial downturn (captured in this analysis by a decline in r^{STOCK}) begins, if it is sufficiently widespread, then the adverse feedback loop kicks in. Intuitively, the adverse feedback loop arises due to the linkage between profits, which reflects fundamentally macro outcomes, and dividends, which are a fundamental aspect of finance.

Discussion

This chain of logic returns us to the very beginning of the chapter: what are policy authorities to do in the face of such events? In a short and almost, but not quite, facetious sense – who knows.

In a slightly less short, and slightly more serious, sense – increase R.

If we are thinking specifically about the events that occurred in the U.S. in 2007 and 2008, the Federal Reserve, the U.S. Treasury, and many other regulatory agencies were trying to do exactly this – increase R. What exactly was the nature by which R was increased, if it was successfully increased at all? Thinking back to the events of that period, some assorted slices of policy were "quantitative easing," "high-quality financial injections," and some changes in the literal regulatory structure, which, along with other things, required some firms to hold on to larger quantities of financial assets on their balance sheets. But these are all talking points, because the accelerator model does not take a stand on any of this. As discussed when introducing the third building block, the model makes no prediction regarding R; rather, R is simply taken as given by the framework.

What about the opposite of the situation analyzed above, in which r^{STOCK} increases relative to r? It is again helpful to start this analysis with the case drawn in Figure 86, in which $r - r^{STOCK} = 0$. If r^{STOCK} rises relative to r, then it is clear that $r - r^{STOCK} < 0$. If we follow the chain of logic and the exact analytical expressions of the arguments laid out above, then we would claim that $\lambda < 0$.

But we know from our earlier discussion about the Lagrange multiplier that $\lambda < 0$ cannot occur at the optimal choice! The smallest possible value of λ is $\lambda = 0$, at which point there is no need for the firm to use loans backed by financial assets to help pay for physical capital purchases. The accelerator effect by its very nature does not work in the opposite direction, and this follows from the asymmetry regarding borrowing already studied. Or, stated in terms of Figure 86, there is an optimal, profit-maximizing, quantity of physical capital investment. If the firm is given the chance to invest more in physical assets for a given market value of financial assets – that is, for the physical capital demand function to shift outwards at every r – it would optimally choose to not invest any further. There is no "acceleration" on the upside.[220]

The financial accelerator model has been in existence for decades. In the U.S., events described by it do not occur very often – in the past roughly 100 years, the Great Depression and the "Great Recession" of 2007-2009 are really the only events that can be classified as accelerator periods. But when they do occur, the adverse effects, or possible adverse effects, can be very pronounced. The interpretation of many policy authorities and academic researchers is that the after-effects of the events of 2007-2009 are not yet over.

[220] Stated more subtly, there could be acceleration for a while, but it would eventually choke off. The point at which it would choke off is as soon equilibrium investment reaches the point at which it is truly profit-maximizing, which was the highlighted point in Figure 86.

Appendix A: Isolating λ from first-order condition on financial assets

The following presents the algebra that isolates the Lagrange multiplier λ from the first-order condition on a_1. Repeated here for convenience is the first-order condition on a_1,

$$-S_1 + \frac{S_2 + D_2}{1+i} + \lambda \cdot R \cdot S_1 = 0.$$

To isolate the λ term, first rearrange this expression to get

$$\lambda = \left[S_1 - \frac{S_2 + D_2}{1+i} \right] \cdot \frac{1}{R \cdot S_1}.$$

Next, pull the S_1 term outside the square brackets inside the square brackets, which gives

$$\lambda = \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{1}{1+i} \right] \cdot \frac{1}{R}.$$

Then, multiply and divide the second term in parentheses by P_1 and P_2, which gives

$$\lambda = \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{P_1}{P_2} \cdot \frac{P_2}{P_1} \cdot \frac{1}{1+i} \right] \cdot \frac{1}{R}.$$

Using the definition of goods-price inflation between period 1 and period 2, $1+\pi_2 = \frac{P_2}{P_1}$, in both the numerator in denominator of the previous expression gives

$$\lambda = \left[1 - \frac{S_2 + D_2}{S_1} \cdot \frac{1}{1+\pi_2} \cdot \frac{1+\pi_2}{1+i} \right] \cdot \frac{1}{R}.$$

Next, insert the definition of the interest rate (or rate of return) on stock, $1+i^{STOCK} \equiv \frac{S_2 + D_2}{S_1}$, to replace the term involving nominal prices and dividends of stock, which gives

$$\lambda = \left[1 - \frac{1+i^{STOCK}}{1+\pi_2} \cdot \frac{1+\pi_2}{1+i} \right] \cdot \frac{1}{R}.$$

Use of the usual Fisher relation allows us to express the rates of return in real units (rather than in nominal units),

$$\lambda = \left[1 - \frac{1+r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}.$$

A final algebraic step inside the square brackets yields

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R}.$$

This final expression is what is used in the analysis in the main text. Despite several steps of rearrangement, note that it fundamentally is the same first-order condition on a_1 based on the Lagrangian.

Appendix B: Construction of capital demand function

The following shows how to combine the first-order conditions on a_1 and on k_2 to obtain predictions about the capital demand function. The reason both conditions are required is that the multiplier λ appears in each. Start with the expression for λ obtained above (which is simply a re-expressed version of the first-order condition on a_1)

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R},$$

and insert it in the first-order condition on k_2 (which, repeated for convenience, is

$$-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} - \lambda P_1 = 0).$$

This gives the single expression

$$-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} - \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R} P_1 = 0,$$

which will now be rearranged in several steps. While there are several steps, keep in mind this is just algebra.

First, divide the entire expression by P_1, which gives

$$\frac{P_2 f_k(k_2, n_2)}{P_1(1+i)} + \frac{P_2}{P_1(1+i)} - \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R} = 1$$

(in which we have also moved the $-P_1$ term over to the other side of the expression in the same step). Then, using the definition of inflation between period one and period two, $1+\pi_2 = \frac{P_2}{P_1}$, this expression becomes

$$\left(\frac{1+\pi_2}{1+i}\right) f_k(k_2,n_2) + \frac{1+\pi_2}{1+i} - \left[\frac{r-r^{STOCK}}{1+r}\right]\cdot\frac{1}{R} = 1.$$

Next, apply the Fisher relation, which gives

$$\frac{f_k(k_2,n_2)}{1+r} + \frac{1}{1+r} - \left[\frac{r-r^{STOCK}}{1+r}\right]\cdot\frac{1}{R} = 1.$$

This expression is helpful because it shows that if $r = r^{STOCK}$ or if $R \to \infty$, then the capital demand function obtained in basic firm analysis, characterized by $f_k(k_2,n_2) = r$, emerges.

Going further is useful, though, if $r - r^{STOCK} > 0$ (which in turn implies $\lambda > 0$). Multiplying the previous expression by $1+r$ gives

$$f_k(k_2,n_2) + 1 - \left[\frac{r-r^{STOCK}}{R}\right] = 1+r,$$

or, canceling the 1's on each side,

$$f_k(k_2,n_2) - \left[\frac{r-r^{STOCK}}{R}\right] = r.$$

As in basic firm analysis, let's focus on the Cobb-Douglas production function, $f(k,n) = k^{\alpha} n^{1-\alpha}$, for the rest of the analysis. This is simply because virtually all practical studies in macroeconomics use this particular form because it is empirically relevant; however, any production function that displays a constant elasticity of substitution between k and n (the Cobb-Douglas case is just one example) works.

The Cobb-Douglas function implies that the marginal product of capital (in period two, in particular, which is the period of interest regarding the capital stock in the formal model, due to the fact that k_1 is fixed at the beginning of period one) is $f_k(k_2,n_2) = \alpha k_2^{\alpha-1} n_2^{1-\alpha}$. Substituting this in the previous displayed expression,

$$\alpha k_2^{\alpha-1} n_2^{1-\alpha} - \left[\frac{r - r^{STOCK}}{R}\right] = r\,.$$

As in basic firm analysis, this expression defines the capital demand function, and we want to represent it in r-k_2 space. To get there, there is a bit more algebra to do, because we need to isolate the r terms on one side of the expression.

Proceeding with this part of the algebra, first open up the term in square brackets, which gives

$$r = \alpha k_2^{\alpha-1} n_2^{1-\alpha} - \frac{r}{R} + \frac{r^{STOCK}}{R}$$

(which also interchanges the left-hand and right-hand sides of the expression for clarity). Then, combine terms involving the riskless rate r on the left-hand side, which gives

$$\left[1 + \frac{1}{R}\right] r = \alpha k_2^{\alpha-1} n_2^{1-\alpha} + \frac{r^{STOCK}}{R},$$

or, combining terms in the square brackets on the left-hand side,

$$\left[\frac{R+1}{R}\right] r = \alpha k_2^{\alpha-1} n_2^{1-\alpha} + \frac{r^{STOCK}}{R}.$$

Finally, multiply both sides by $\dfrac{R}{R+1}$ to get the final form

$$r = \left[\frac{R}{R+1}\right] \alpha k_2^{\alpha-1} n_2^{1-\alpha} + \frac{r^{STOCK}}{R+1},$$

which is the modified version of the capital demand function that appears in the main text. As noted in the main discussion,

$$\lim_{R\to\infty} \left\{ \left[\frac{R}{R+1}\right] \alpha k_2^{\alpha-1} n_2^{1-\alpha} + \frac{r^{STOCK}}{R+1} \right\} = \alpha k_2^{\alpha-1} n_2^{1-\alpha},$$

The right-hand side of which is simply the marginal product of capital from the Cobb-Douglas production specification. Thus, as $R \to \infty$, the relation above converges to $r = \alpha k_2^{\alpha-1} n_2^{1-\alpha}$, which is exactly the condition that characterizes the capital demand function in basic firm analysis.