Chapter 26
Long-Run Growth

Most of our study focuses on short-run macroeconomic fluctuations, rather than long-run growth. The basic growth framework is important because the heart of modern business-cycle study is tightly connected with growth analysis. The **Solow Growth Model** (and later offshoots) has shaped the way economists approach both long-run growth issues and shorter-run business-cycle fluctuations.

Long-run growth is concerned with how an economy develops over long periods of time. From a utility-maximizing, or welfare-maximizing, perspective, the metric of development in principle should be something akin to "utils." This concept of an economic standard of living is of course a fiction, so it cannot be measured empirically.

Although it can be criticized on several fronts, the main empirical metric used to judge economic standards of living in an economy is real GDP per capita (i.e., GDP per person). Thus, **economic growth** is measured as growth over long stretches of time in real GDP per capita.

For much of history, there was essentially zero economic growth.

Over the past two to three centuries, however, economic growth has been positive. Several reasons (this list is admittedly short) seem to be the Industrial Revolution, the invention of the printing press, refrigerators, the development of personal computers, the widespread use of the Internet, and the concomitant ease of world-wide communication. Nonetheless, growth rates still vary widely from one region of the world to another, as well from one country to another.

There has been much debate about whether growth rates should imply **convergence** of real GDP per capita across countries. The idea of **convergence** is that over long periods of time, per capita GDP should equalize across even widely-differing countries, due to eventual technological diffusion. Evidence on convergence, however, has been mixed. Qualitatively, it seems many industrialized nations have indeed more or less converged to similar "standards of living." However, many developing countries are stuck at far lower standards of living, despite some periods of "catch-up."

A caveat before developing the Solow framework. It is sometimes informally referred to as the **"neoclassical growth model."** However, this is **not true.** In the "neoclassical growth model," savings **decisions** are determined by consumers' (or firms') optimality conditions. In the Solow model, there is no **decision-making** or "optimal choice" of savings; rather, the savings in which the economy engages is strictly a parameter.

We proceed as follows. First, the exogenous sources of growth are described. Next up is the manner by which inputs are transformed into output in the growing economy and how resources are saved over time. To obtain a solution, we have to **detrend** the model, which itself is an inherent connection between growth analysis and business-cycle analysis. After detrending, we compute an analytical result for the long-run capital stock, and how it depends in natural ways on, among other things, aggregate savings propensities and technological innovations of different societies.

Solow Growth Model

A foundational model of economic growth is commonly attributed to Robert Solow in the 1950's. Many other innovators also helped shaped the basic framework – such as Trevor Swan and Nicholas Kaldor, to name two – but Solow's continued development of the framework and future Nobel Prize cast him in the spotlight.

The Solow model takes a production-function approach to explain how **per-capita real GDP increases over time.** Figure 95 displays the exponential growth of per-person real GDP in the U.S. economy since the National Income and Product Accounts (which is the GDP accounting expression, GDP = C + I + G + NX) began during the Great Depression. The population growth rate over the past century has averaged about 1.3%, which emphasizes that Figure 95 shows **per-person** real GDP growth.

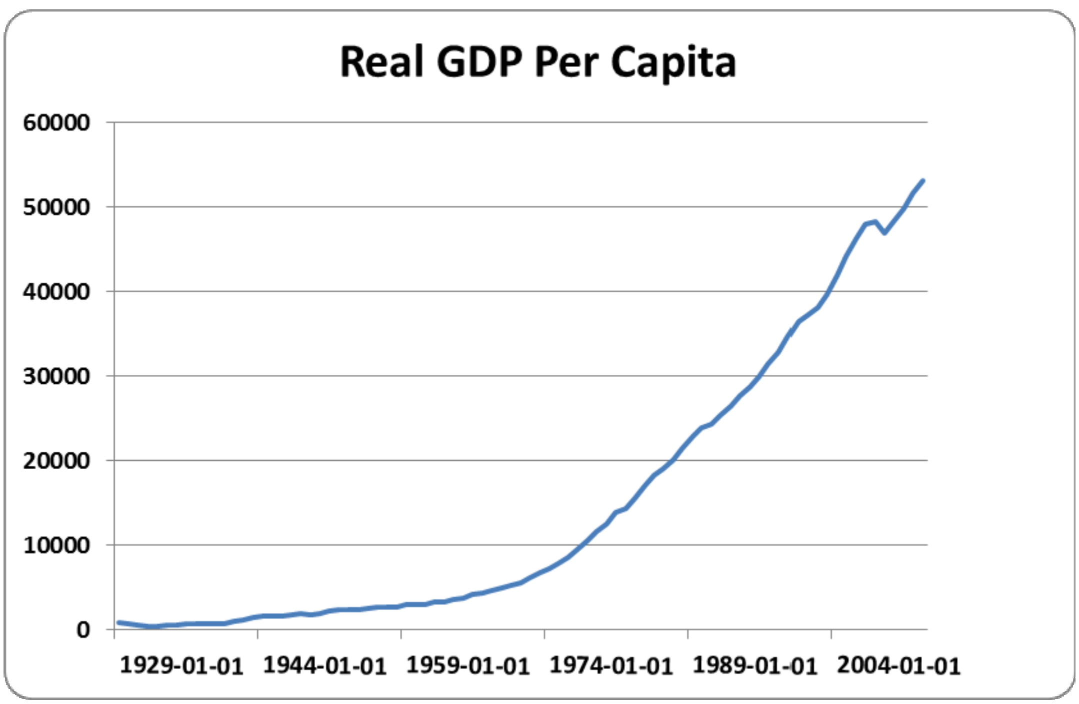


Figure 95. Real GDP per capita in the U.S..

Applied to long-run analysis, the **Solow framework is fundamentally about how, why, and how quickly an economy accumulates physical capital**. There are a number of assumptions the framework makes (as does any framework) in studying the dynamics and implications of this model. Some of these assumptions can be rightly criticized as being too unrealistic. However, the model is extremely valuable in that it provides a useful benchmark for the performance of both other models of long-run growth as well as short-run fluctuations. Moreover, a framework that both performs better and simpler to use has not been fleshed out in the more than half century since Solow's model became prominent.

Exogenous Sources of Growth

The total physical capital stock of an economy K and the total number of people N are the productive factors that in tandem construct total goods and services in the economy. The total quantity of goods and services built in an economy *is* the definition of GDP, which is denoted by Y.[229]

The Solow framework asserts that **long-run growth factors are exogenous to the economy.** That is, they are **inputs** into the analysis, not **outputs** resulting from the analysis. There are various exogenous growth factors we could consider; we will focus on two.

One main exogenous source for economic growth is population growth gr_N, defined as

$$gr_N \equiv \frac{N_{t+1} - N_t}{N_t} \left(= \frac{N_{t+1}}{N_t} - 1 \right),$$

in which N_t represents aggregate employment in period t.

Another input to economic growth is growth in the "productivity" of the economy. Broadly stated, "productivity" is how well or how easily the various factors of production mesh with each other in producing output. One example is that the knowledge and skills of employees are getting better over time. Another is that the quality of the physical machinery or computers or smartphones that one employee is working with to produce output is improving over time.

Based on these examples, productivity is fundamentally considered to be **"labor-augmenting"** or that the **"effectiveness of labor"** drives long-run growth. Because of

[229] Two notes. First, K and Y are intentionally meant to be upper-case letters. Second, N can be interchangeably be interpreted as total number of people or total number of hours.

long-lasting improvements in technology or skills, one employee can produce more output as time marches on.

Whatever the various and many innovations over the decade have been, **denote by X_t a worker's labor-augmenting productivity during time period *t*, and its growth rate gr_X as**

$$gr_X \equiv \frac{X_{t+1}-X_t}{X_t}\left(=\frac{X_{t+1}}{X_t}-1\right).$$

Both gr_N and gr_X are asserted to be constant for every time period, hence the lack of time subscripts. As mentioned above, in the U.S. population growth rate has been roughly $gr_N = 0.013$ per year since 1900. In principle we could allow these percentage growth rates gr_N and gr_X to vary over time, but that would start to bring us into the realm of business-cycle analysis, which is not necessary to study growth. Here, our focus is on long-run growth, so we can purposely omit shorter-run economic ups and downs.

Aggregate Production Function

The canonical production functional form for aggregate GDP is the **Cobb-Douglas production function**

$$Y_t = K_t^{\alpha}\left(X_t N_t\right)^{1-\alpha}$$

in which ***α*, which is a number between 0 and 1, measures the "importance" of *K*** in the production of GDP.[230] Consequently, **the number 1-*α*, which also lies between 0 and 1, measures the "importance" of *XN*** in the production of GDP.[231] The second argument *XN* is commonly known as **"effective labor."** The Cobb-Douglas production function is a workhorse in both the study of long-run growth and business-cycle fluctuations.

To understand the economic importance of α, consider the extreme case of $\alpha = 0$. In this case, it is only worker's labor efforts that create GDP – an economy's production process is very **labor intensive** if α is close to zero. On the opposite extreme, $\alpha = 1$, labor efforts have virtually nothing to do with producing goods and services – this economy's GDP is essentially entirely produced with machines and factories and robots, thus the economy is very **capital intensive** because α is close to one.

[230] Note that this is **not at all** the same concept of α as in the Keynesian macroeconometrics frameworks.

[231] "*XN*" should not be confused with "*NX*" – the latter is the typical acronym for "net exports" (or, equivalently, the "trade balance").

Naturally, different economies display varying degrees of capital intensity. A widely-accepted view is that the capital intensity of GDP production in advanced economies is $\alpha = 1/3$, which arises from econometric estimation for the U.S. and other developed economies. For the sake of generality, we will continue using the more general notation α.

This $Y_t = K_t^{\alpha}\left(X_t N_t\right)^{1-\alpha}$ form of writing the aggregate production function emphasizes a long-standing view that decades-long and centuries-long growth fundamentally occurs due to **labor-augmenting productivity,** which is the term X_t. Alternative terminology for X is **total factor productivity (TFP)** or the **Solow residual.**[232] We will use these three terms interchangeably.

Regardless of terminology, this constantly growing X_t is the centerpiece of Solow growth analysis. Difficult-to-measure changes in X_t over time could be thought of as a "measure of ignorance" about how various economies' **inputs** – which are K and N – yield **output** Y.[233]

Aggregate Savings and Aggregate Investment

The Solow model is a closed economy structure, which implies that, in the basic GDP accounting expression, we have (GDP_t =) $Y_t = C_t + I_t + G_t$. **The GDP accounting equation is the resource frontier of the overall economy,** which simply means that all goods produced – which is aggregate supply, the left-hand side of the accounting equation – are absorbed by one of the several expenditure components of aggregate demand (the right-hand side).

Government spending is not crucial for the analysis of the Solow framework, so let's simplify the resource constraint further to $Y_t = C_t + I_t$. Then, for ease of use below, let's rearrange it as

$$Y_t - C_t = I_t.$$

Next, using our definition of **savings** from the basic consumption-savings framework (recall the definition, $S_t = Y_t - C_t$), we have that economy-wide savings is the source of funding for economy-wide investment,

[232] An algebraic transformation allows us to equivalently express the production function as $Y_t = A_t \cdot K_t^{\alpha} N_t^{1-\alpha}$. The transformation to get from the X version to the A version is as follows: $Y_t = K_t^{\alpha} X_t^{1-\alpha} N_t^{1-\alpha}$ → $Y_t = X_t^{1-\alpha} \cdot K_t^{\alpha} N_t^{1-\alpha}$. Then define $A_t = X_t^{1-\alpha}$, which gets us to $Y_t = A_t \cdot K_t^{\alpha} N_t^{1-\alpha}$.

[233] In *business-cycle analysis,* this X term also arises and plays an important role, but Solow never intended for it to be a critical component of the study of short-run ups and downs.

$$S_t = I_t,$$

which is simply due to accounting identities.

The Solow framework does **not** feature "optimizing" consumers or firms.[234] Thus, there is no "consumption-savings optimality condition" that pins down S_t.

The Solow framework asserts that aggregate savings is a constant fraction *s* (be careful between uppercase S and lowercase *s*!) of GDP in each period. The relationship between the savings rate $s \in (0,1)$ (the percentage of savings) and aggregate savings S_t is

$$S_t = s \cdot Y_t.$$

Aggregate gross investment is defined as

$$I_t = K_{t+1} - (1-\delta)K_t,$$

in which $\delta \in (0,1)$ **stands for the rate of depreciation of capital K_t used in the period-*t* production process.** It is natural that capital **depreciates,** or wears out, over time. Physical equipment, such as factories, engines, printers, computer monitors, and so on, naturally wear down, or simply become obsolete, over time. This is the notion that is captured in the depreciation concept. The assumption of **constant** depreciation over time is a decent approximation to what empirical studies suggest is the annual rate of depreciation of the U.S. capital stock, which is approximately 8% per year, hence δ = 0.08 for the U.S. economy at an annual frequency. The quantity of capital goods that depreciates **during the production process of period *t* and thus can no longer be used in period *t*+1 is δK_t.**

Note the use of the word "gross" in the definition. If capital never depreciated – that is, if δ = 0 – then the definition above boils down to $I_t^{net} = K_{t+1} - K_t$, which is known as **aggregate net investment** (hence the superscript "net"). Thus, δ = 0 implies that $I_t = I_t^{net}$. Because capital gradually wears out during usage in the production process, $I_t > I_t^{net}$. Thus, some portion of I_t is **replacement investment,** and the rest is directed towards accumulating the physical capital stock. Empirically, it is gross investment I_t that is measured in the GDP accounts.

[234] Recall from our study of the history of macroeconomics, microeconomic-level optimization structures came into use for macroeconomic analysis after the Lucas Critique of the late 1970's.

Equilibrium

To develop an **equilibrium** for this model requires both using the relationships described in the previous section and **normalizing, or de-trending,** the framework in an appropriate manner.

First, using the relationship $S_t = s \cdot Y_t$, equating aggregate savings with aggregate gross investment gives

$$I_t = s \cdot Y_t .$$

Next, using the definition of aggregate gross investment to substitute for I_t, we have

$$K_{t+1} - (1-\delta)K_t = s \cdot Y_t ,$$

an important expression to which we will soon return.

If the economy being analyzed displays **non-negative long-run growth** (which is the perspective we adopt), $gr_N \geq 0$ and $gr_X \geq 0$ must both be true. In turn, Y_t would grow explosively over the decades and centuries, as Figure 95 suggests.

The Solow framework (indeed, all of our macro frameworks) cannot handle "infinite" levels of GDP, or, for that matter, infinite quantities of consumption or savings or investment. The framework thus must be **normalized, or scaled,** in an appropriate manner. **The appropriate scaling factor(s) is (are) the exogenous source(s) of growth.** Referring to business-cycle analysis, this process is exactly the **detrending** procedure!

In our setup, the two sources of growth are population and labor-augmenting productivity. Hence, the appropriate scale factor in any given time period is $X \cdot N$. To perform the detrending, let lowercase letters denote **per unit of effective labor** variables. Define

$$y_t \equiv \frac{Y_t}{X_t \cdot N_t} \quad \text{and} \quad k_t \equiv \frac{K_t}{X_t \cdot N_t}$$

as **GDP per unit of effective labor** and **capital per unit of effective labor**, respectively. For ease of language from here on, we will refer interchangeably refer to "per unit of effective labor" as **per capita.**

Using the definitions of "per-capita" y_t and "per-capita" k_t, the GDP production function $Y_t = K_t^{\alpha} \left(X_t N_t \right)^{1-\alpha}$ can be rewritten in per-capita terms with a few algebraic steps. First, detrend these variables, which gives us

$$\frac{Y_t}{X_t \cdot N_t} = \left(\frac{K_t}{X_t \cdot N_t}\right)^{\alpha} \left(\frac{X_t \cdot N_t}{X_t \cdot N_t}\right)^{1-\alpha}.$$

Next, use the definitions to rewrite as

$$y_t = k_t^{\alpha} \cdot 1^{1-\alpha}.$$

As per basic mathematics, $1^{1-\alpha} = 1$, thus the aggregate production function is simply

$$y_t = k_t^{\alpha}.$$

This function expresses per-capita output as a function of the per-capita capital stock; it is illustrated in Figure 96. A crucial point to notice is that **because $\boldsymbol{\alpha} < 1$, the production function displays diminishing marginal product in capital.** Also illustrated in Figure 96 is the per-capita savings function, which, in the Solow framework, is per-capita output times the savings rate $s \in (0,1)$.

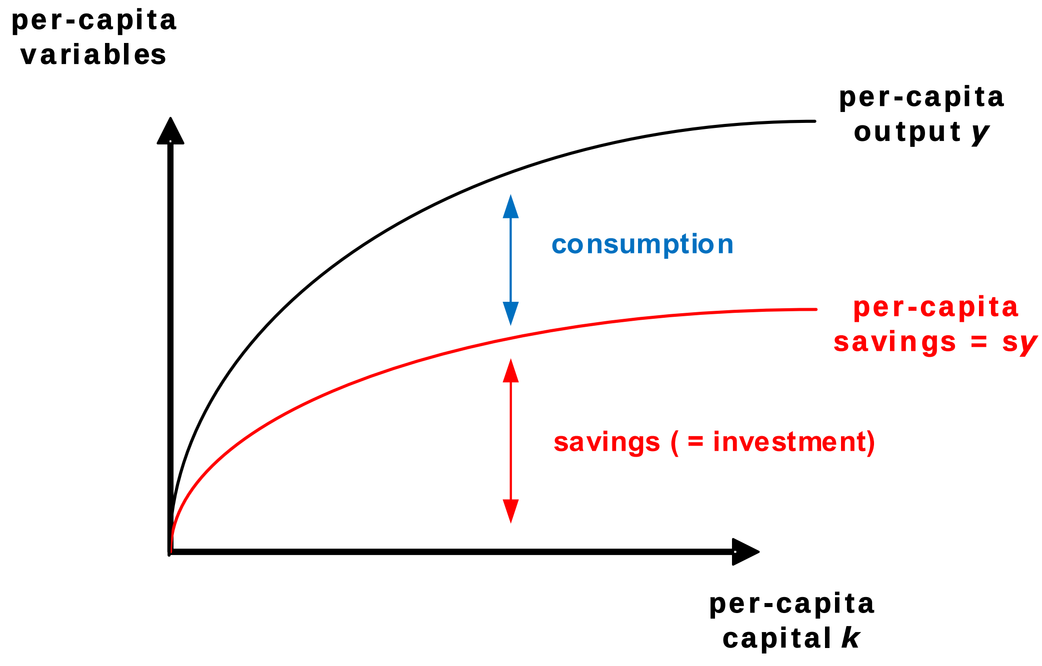


Figure 96. The decomposition of output y between savings for the future (in red) and immediate consumption.

To analyze k in the Solow model, begin with $K_{t+1} - (1-\delta)K_t = s \cdot Y_t$, which first has to be stated in detrended terms. Proceeding step by step with the algebra, first divide by $X_t N_t$, which gives

$$\frac{K_{t+1}}{X_t \cdot N_t} - (1-\delta)\frac{K_t}{X_t \cdot N_t} = s \cdot \frac{Y_t}{X_t \cdot N_t}.$$

Next, substitute the y_t and k_t definitions into the right-hand side and the second term on the left-hand side, which leads to

$$\frac{K_{t+1}}{X_t \cdot N_t} - (1-\delta)k_t = s \cdot y_t,$$

or, rewritten slightly using the per-capita production function $y_t = k_t^{\alpha}$,

$$\frac{K_{t+1}}{X_t \cdot N_t} - (1-\delta)k_t = s \cdot k_t^{\alpha}.$$

All of these terms are now normalized, with the important exception of the first expression on the left.

To detrend $\frac{K_{t+1}}{X_t \cdot N_t}$, multiply and divide it by $X_{t+1} \cdot N_{t+1}$, so that the previous expression can be stated as

$$\left(\frac{K_{t+1}}{X_{t+1} \cdot N_{t+1}}\right) \cdot \left(\frac{X_{t+1} \cdot N_{t+1}}{X_t \cdot N_t}\right) - (1-\delta)k_t = s \cdot k_t^{\alpha}.$$

To continue proceeding, a couple of observations are required. First, the bracketed term $\frac{K_{t+1}}{X_{t+1} \cdot N_{t+1}}$ **is simply k_{t+1} by definition!** Second, notice that $\frac{N_{t+1}}{N_t} = 1 + gr_N$ reflects the population growth rate; similarly, $\frac{X_{t+1}}{X_t} = 1 + gr_X$ reflects the growth rate of technology.

Inserting these definitions leads to

$$k_{t+1} \cdot (1+gr_X) \cdot (1+gr_N) - (1-\delta)k_t = s \cdot k_t^{\alpha}.$$

After two more steps of algebra, a final rewriting is

$$k_{t+1} = \frac{s \cdot k_t^{\alpha}}{(1+gr_X)\cdot(1+gr_N)} + \frac{(1-\delta)k_t}{(1+gr_X)\cdot(1+gr_N)},$$

which emphasizes k_{t+1} on the left-hand side. This expression is the **equilibrium law of motion for the (per-capita) capital stock *k*** in the Solow model.

The equilibrium law of motion is the heart of the framework. The law of motion states that for given values of s, α, gr_X, gr_N, and δ, the beginning-of-period t+1 capital stock k_{t+1} is **completely** determined by k_t. Figure 99 below, which we will consider at more length soon, plots the law of motion.

Long-Run Capital Stock

Figure 97 describes the steady-state equilibrium for the capital stock k. We can solve for steady-state k^* analytically, which is a powerfully sharp result of the Solow framework.[235] Several steps of algebra will get us there.

The most important step is to begin by imposing $k_{t+1} = k_t = k^*$ in the law of motion. If the economy arrives at k^*, then the equilibrium law of motion tells us that it will remain at k^* forever. **Mathematically, a steady-state is the condition at which an object stops evolving over time; dropping time subscripts** is the way to operationalize a steady-state.

Imposing steady state in the law of motion gives us

$$k^* = \frac{s \cdot \left(k^*\right)^\alpha}{(1+gr_X)\cdot(1+gr_N)} + \frac{(1-\delta)k^*}{(1+gr_X)\cdot(1+gr_N)}.$$

The remainder of the algebra is to isolate the k^* term.

From this previous expression, subtract the second term on the right-hand side, which yields

$$k^* \cdot \left[1 - \frac{1-\delta}{(1+gr_X)\cdot(1+gr_N)}\right] = \frac{s \cdot \left(k^*\right)^\alpha}{(1+gr_X)\cdot(1+gr_N)}.$$

The additive terms in square brackets can be combined so we have

$$k^* \cdot \left[\frac{(1+gr_X)\cdot(1+gr_N) - (1-\delta)}{(1+gr_X)\cdot(1+gr_N)}\right] = \frac{s \cdot \left(k^*\right)^\alpha}{(1+gr_X)\cdot(1+gr_N)}.$$

Next, cancelling the $(1+gr_X)\cdot(1+gr_N)$ terms allows us to simplify a bit to arrive at

$$k^* \cdot \left[(1+gr_X)\cdot(1+gr_N) - (1-\delta)\right] = s \cdot \left(k^*\right)^\alpha.$$

Collecting the k^* terms on both the left-hand side and the right-hand side, and shifting the square-bracketed term to the right-hand side gives

[235] Impressive both in the 1950's and still today, and continues to be the foundation of modern business-cycle analysis.

$$\left(k^*\right)^{1-\alpha} = \frac{s}{(1+gr_X)\cdot(1+gr_N)-(1-\delta)}.$$

Finally, raise both sides of the expression to the power $\frac{1}{1-\alpha}$ to obtain **the analytic steady-state *k**, which is the main result of the Solow analysis.**

Solow Framework Steady-State *k**

$$k^* = \left[\frac{s}{(1+gr_X)\cdot(1+gr_N)-(1-\delta)}\right]^{\frac{1}{1-\alpha}}.$$

This Solow steady-state expression for k^* contains several parts. To consider the basic economics, suppose there is neither population growth nor technological growth, which means $gr_X = gr_N = 0$. Steady-state k^* in this case is

$$k^* = \left[\frac{s}{\delta}\right]^{\frac{1}{1-\alpha}},$$

which depends on the per-period savings rate s, the per-period depreciation rate δ, and the capital share α. It is clear from this expression that for a given capital share α, either an increase in s or a decrease in δ leads to a higher steady-state capital stock k^*.[236] In turn, a larger value for steady-state GDP, $y^* = \left(k^*\right)^{\alpha}$, is achieved.

Stated in everyday terms, an increase in savings leads to an increase in wealth (wealth in this framework is the capital stock k).

Left for you as exercises are further comparative statics of k^* (and hence of y^*, which, by definition of steady state, is also constant) with respect to the set of other parameters: $\{\alpha, gr_X, gr_N\}$.

[236] Stated mathematically, the steady-state elasticity of k^* with respect to s is strictly positive, and the steady-state elasticity of k^* with respect to δ is strictly negative.

Regardless of precise parameter values for $\{\alpha, gr_X, gr_N, s, \delta\}$, the phase diagram in Figure 97 displays the steady-state equilibrium k^* as the intersection of the savings supply function and the **replacement investment demand function (or, equivalently stated, the "break-even" investment demand function).** In steady-state equilibrium, the quantity of replacement ("break-even") investment demanded is simply that required to replace depreciated capital (δk), **plus the additional per-capita resources for population growth ($gr_N k$), plus the additional per-capita resources for technological growth ($gr_X k$).**

If the economy achieves equilibrium k^*, there will be no further change in k.[237]

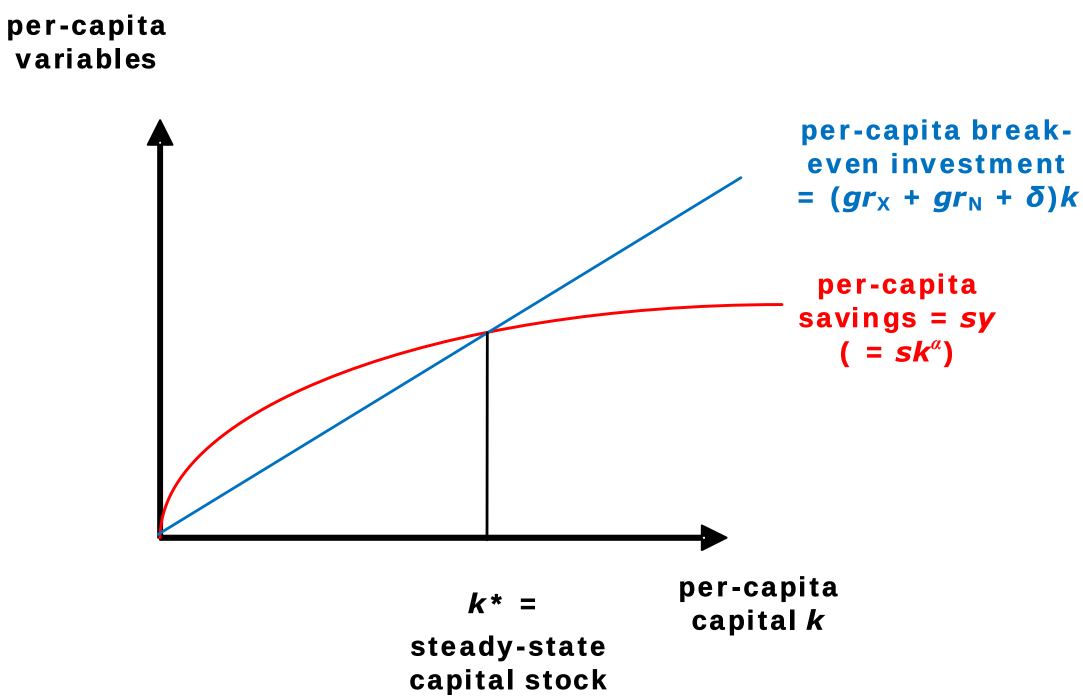


Figure 97. Intersection of savings function and break-even (alternatively, replacement) investment function determines the long-run (aka steady-state) level of physical capital.

[237] In terms of differential equations (or the discrete time analog we are considering here, which are difference equations), k^* is a **stable steady state** of the economy – once it reaches k^*, it will never depart from it. Unless, that is, some "shocks" cause it to, which is the point at which modern macroeconomic business-cycle analysis begins.

Transitional Dynamics of Growth

That leads to a next natural question: if the economy is away from steady-state k^*, how, if at all, does it **converge** to k^*?

To consider convergence, we return to the fully-fledged dynamic law of motion for the capital stock

$$k_{t+1} = \frac{s \cdot k_t^{\alpha}}{(1+gr_X)\cdot(1+gr_N)} + \frac{(1-\delta)k_t}{(1+gr_X)\cdot(1+gr_N)},$$

with time indices explicitly included. The timeline in Figure 98 is helpful for the analysis.

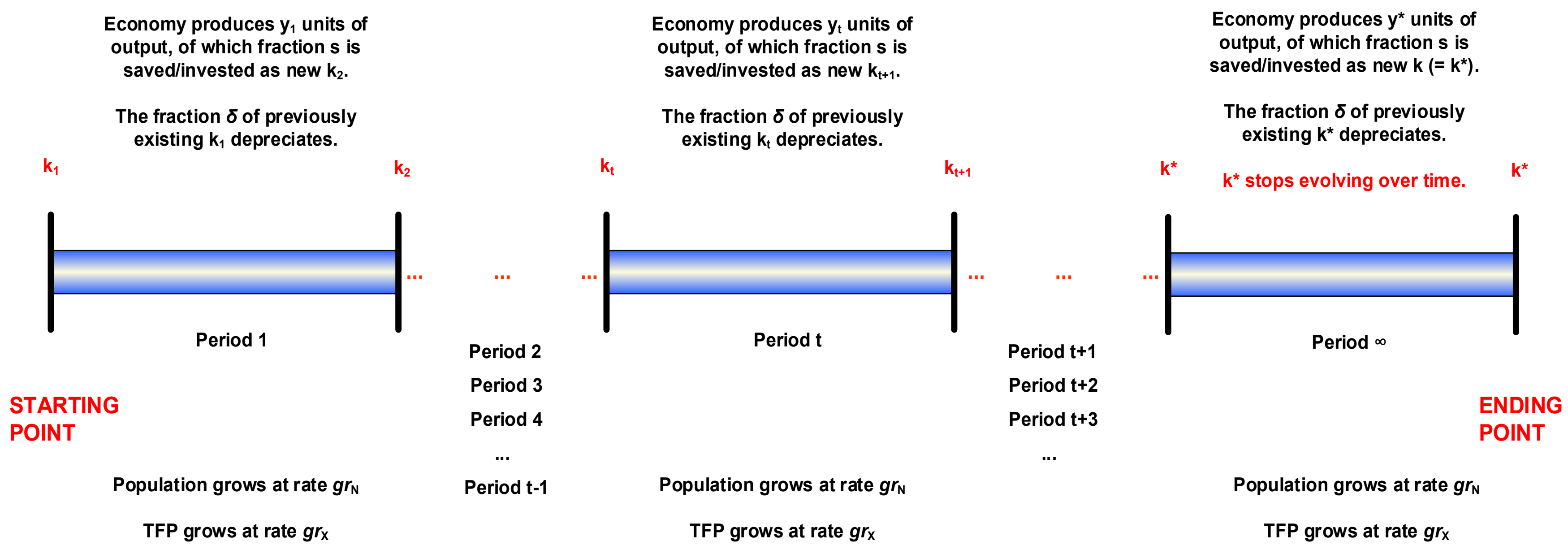
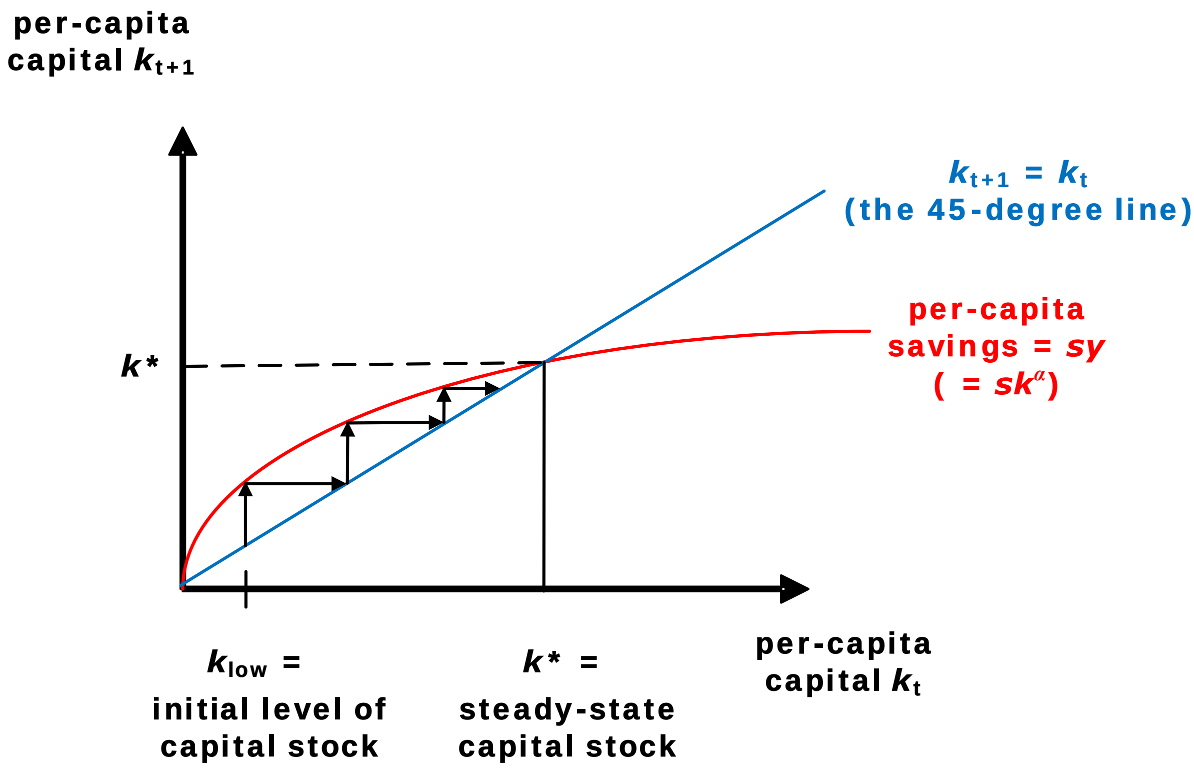


Figure 98. Timeline for dynamic long-run growth analysis. Eventually, the economy reaches the long-run (or steady-state) per-capita capital stock k^*.

Consider an economy in period t – let's call it an "emerging economy" – that has $k_t < k^*$. **The question to be answered is: is k_{t+1} closer to k^* than k_t is, or is k_{t+1} further away from k^* than k_t is?** In other words, is the economy **towards** its steady state or **away from** its steady state?

The mathematical details of the solution are left to a more advanced course.[238] In a nutshell, though, the law of motion answers the question: $k_{t+1} > k_t$ **, thus k_{t+1} is closer to k^* than was k_t.** Repeating this logic forward, the law of motion informs us that $k_{t+2} > k_{t+1}$ **, thus k_{t+2} is closer to k^* than was k_{t+1}.** Iterating the logic forward yet again, the law of motion tells us that $k_{t+3} > k_{t+2}$ **, thus k_{t+3} is closer to k^* than was k_{t+2}.** And so on, **until the capital stock converges to k^*.**

The economics of convergence is due to the concavity of the production function (given that $\alpha \in (0,1)$). The phase diagram in Figure 99 conveys this point. Note carefully the axes in Figure 99: k_t appears on the horizontal axis, and k_{t+1} appears on the vertical axis. Figure 99 is thus fundamentally about the **dynamic growth path** of the economy.



[238] The mathematics involves various stability theorems regarding difference equations.

Figure 99. Dependence of k_{t+1} (vertical axis) on k_t (horizontal axis), as embodied in the equilibrium law of motion for capital k.

Figure 100 illustrates the same convergence idea in different coordinates. Figure 100 shows that if $k < k^*$, per-capita savings **is larger than** per-capita break-even investment. Hence, the quantity of new capital produced is higher than that required for the break-even condition. The total stock of capital therefore rises.

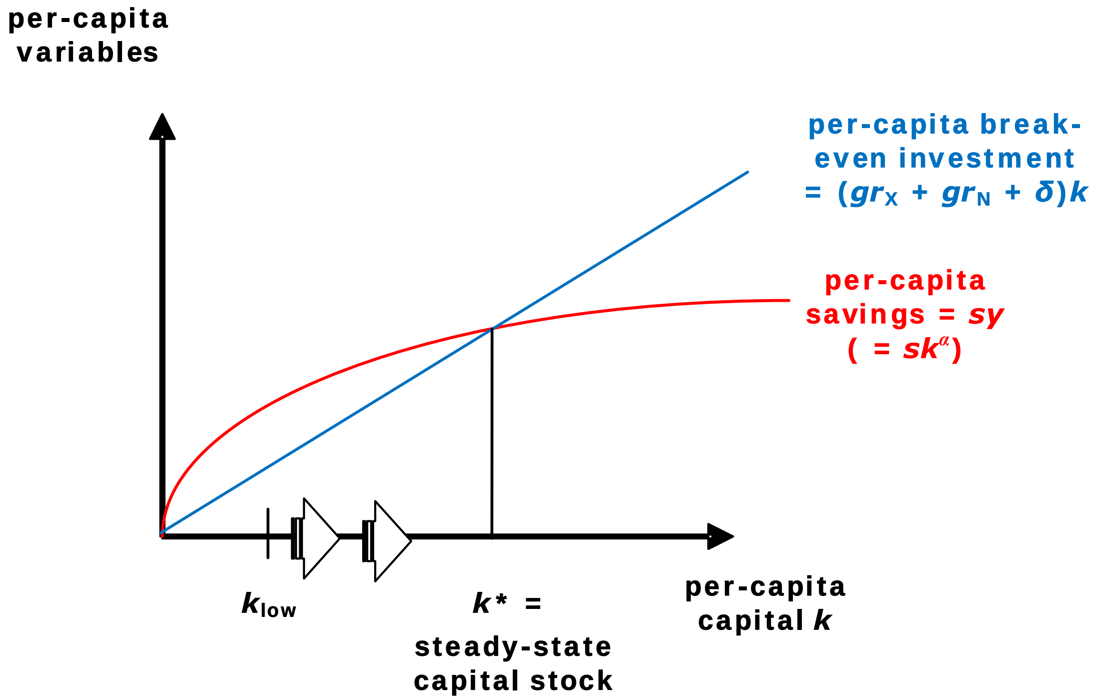


Figure 100. If k is below k^*, savings exceeds break-even investment, causing k to increase and converge towards k^*.

The mechanism by which the capital stock *decreases* until it hits steady state if it starts *above* k^* is analogous – the above logic simply operates in reverse.[239]

Shortcomings of the Solow Growth Model

One major shortcoming of the Solow model is that it predicts that each country eventually converges to steady state. If one believes that "mature" economies (such as the U.S., the western European economies, Japan, and so on) have all converged to their

[239] You should work through this logic yourself.

steady states,[240] then we should observe identical per-capita capital stocks in all mature economies. But this is not what evidence shows.

One highly plausible modification to the theoretical model is to allow different countries to have different savings rates. That is, even if economies have the same production processes, perhaps one country has aggregate savings = $s_1 y$ and another has aggregate savings = $s_2 y$. As long as they both have the same depreciation rates (and growth rates of population and exogenous technological change), the steady-state level of per-capita capital in the two economies will be different, as seen in Figure 101. Referring back to **Figure 96**, this implies different **"propensities to consume"** across economy 1 and economy 2. The economy with the **lower propensity to consume – and hence the higher propensity to save –** eventually reaches a higher value of steady-state per-capita capital and hence a higher value of per-capita GDP.

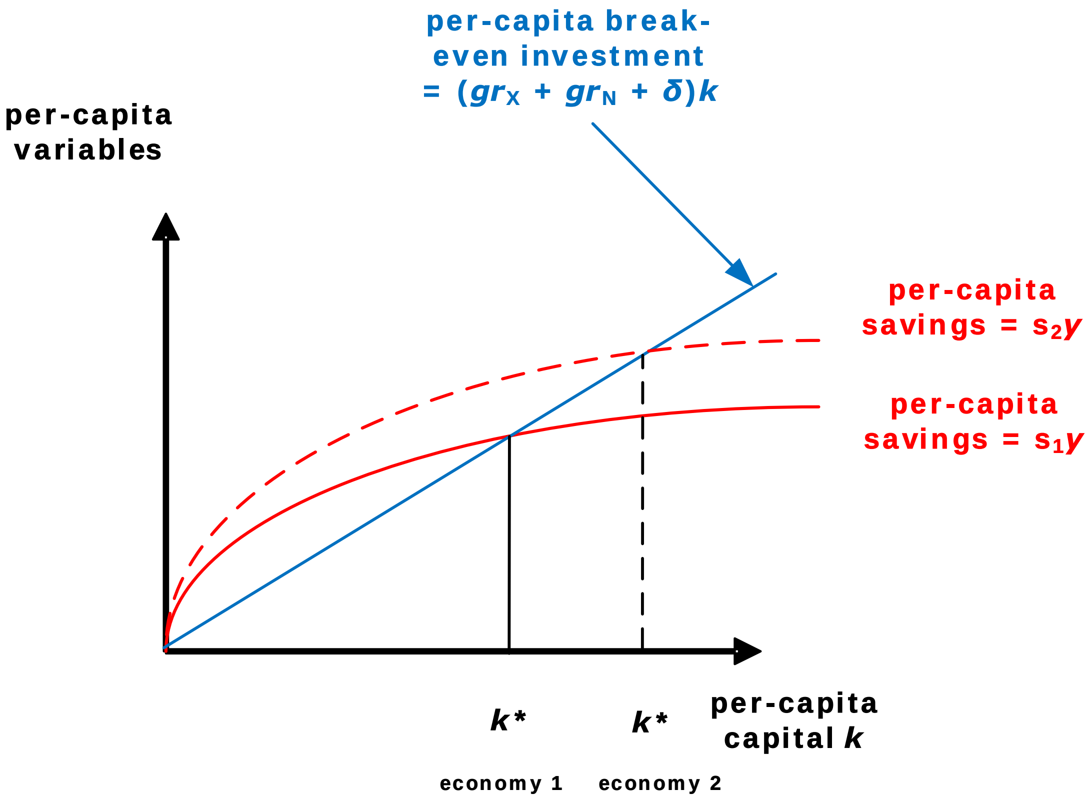


Figure 101. Different rates of savings. The economy with savings rate $s_2 > s_1$ converges to higher steady-state per-capita k^*.

[240] Which at first blush is not a terrible hypothesis – these economies have had a lot of time to converge to their supposed steady states.

Another feature of the Solow model that could be extended is to allow for different depreciation rates of capital in different economies. It is left to you as an exercise to show that different values of δ across countries would imply different long-run capital stocks.[241]

A **seeming** shortcoming of the model's predictions is the implication that economies eventually reach a state of zero growth. That is, once steady state is k^* achieved, regardless of the precise savings rate s or other parameters $\left\{\alpha, gr_X, gr_N, \delta\right\}$, it seems there is no further economic growth: actual investment always equals break-even investment forever after convergence is achieved. Casual inspection of the diagrams in Figure 99 or Figure 100.

The message, however, is more subtle.

Diagrams such as Figure 99 or Figure 100 seem to show that **growth eventually shrinks to zero.** But recall that **"per-capita"** is a shorthand way of stating **per-effective units of labor**. In order to conduct our analysis, we **detrended the model by effective units of labor *XN*.**

If we "reverse" the detrending procedure that allowed for a **finite value for *k**,** we see that **the overall, actual, economy being analyzed is experiencing positive growth** as long as the population is growing or some relevant notion of productivity is growing.

Which of these two concepts of "growth" is more important – "per-capita growth," or "overall" growth – is left for the reader to decide. But the universal consensus amongst economists is that **per-capita growth** is the notion that is of utmost importance.

Empirical consensus thus far indicates that most economists do not believe the U.S. has permanently stopped growing, despite the recent downturn. The population growth rate has been declining for several decades (that is, gr_N , though positive, seems to be declining over the past several decades, which is not be confused with $gr_N < 0$).

Which then implies that sustained long-run per-capita growth must be driven by the exogenous technological growth rate gr_X. The idea proposed by Solow was that there are some components of production, and hence economic growth, which are essentially impossible to quantify – a "measure of ignorance" of economists, if you will. One can in principle measure the quantity of machines, computers, trucks, airplanes, and so on that are being put to productive purposes. Similarly, one can measure the number of people employed.

[241] Also left to you as exercises is to show that different values across countries of either exogenous population growth gr_N or exogenous technological progress gr_X would imply different long-run per-capita capital stocks.

Given these measures, if it turns out that k^{α} does not equal the measured quantity of goods and services *y,* then there is something else out there, some other valuable and productive "knowledge-based" input, that matters for sustainable growth.

What does this unknown productive input represent? Is it clean water supplies that are delivered unnoticed to your faucet? Is it the sudden emergence of refrigerators that allowed families to save food for the coming weeks? Is it the widespread use of automobiles that sprang up in the first half of the 20th century? Is it the rapid adoption during the 1980's of the Microsoft Windows operating system? Or Apple's sheeny, have-to-have-it tech products? Or smartphones and smart-tablets? Perhaps it's the social networking that the smart-stuff enabled?

Probably all of these.

But these leaps in **technology** are hard to measure before they occur. Who could have predicted that Apple would make a big splash in the 1980's ….and then would near bankruptcy in the 1990's…and then would a decade later magically take over a huge segment of the music industry and communications?

Other than maybe Steve Jobs himself, probably very few.

These leaps in **technology** and **innovation** may be the prime "measure of ignorance" in the Solow model.

But inputting this in the Solow model, increases in productivity would rotate the production function up (in a non-parallel manner, because zero capital input would still yield zero output). **An upwards shift in the production function would then cause an upwards shift in the savings function,** ***even if the savings RATE s does not change.*** This can be seen in Figure 102.

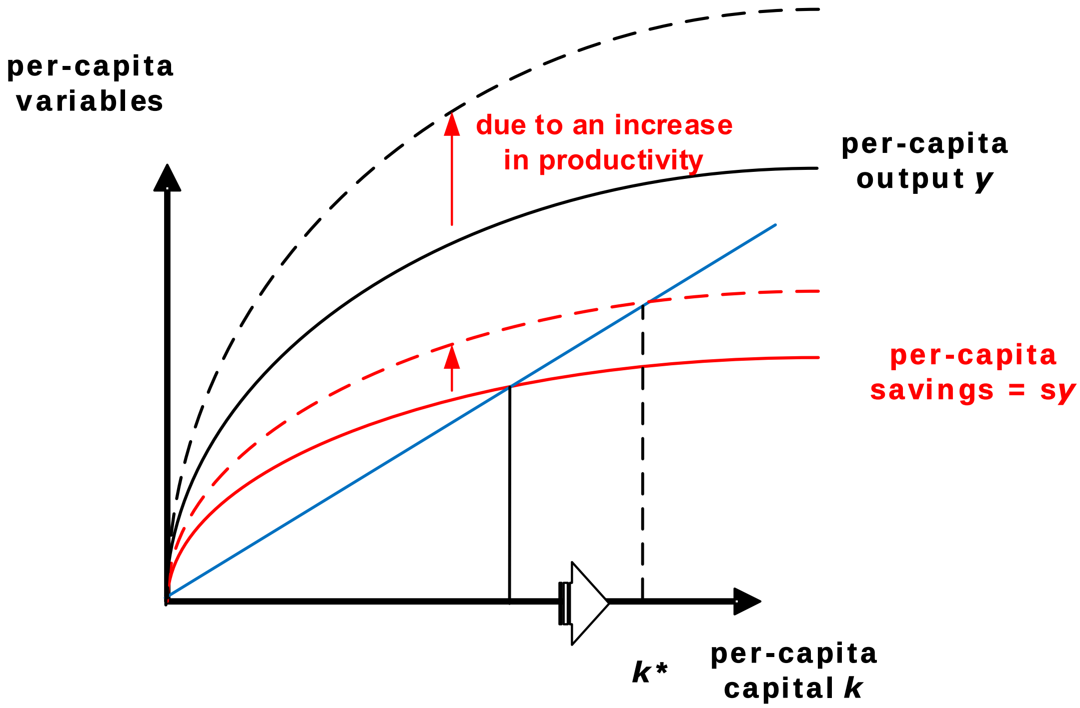


Figure 102. An increase in **total factor productivity** increases the total quantity of goods and services produced, holding fixed the break-even rate ($gr_X + gr_N + \delta$) and the savings rate s. In turn, steady-state k^* increases.

From the perspective of the Solow growth model, the primary reason for inexorable long-term economic growth, at least in mature economies, is **continuing technological innovation.** However, the Solow model does not have anything to say about *why* this ever-advancing technology occurs.

Endogenous growth theory attempts to address the **"why"** question. An overview of endogenous growth appears below. It is very brief, though, because the main thread of "macroeconomics" – which is business-cycle analysis – is not based on endogenous growth.[242]

Endogenous Growth Theory (aka New Growth Theory)

The main focus of endogenous growth theory is to offer explanations to address the shortcomings of the Solow model, namely that in the long run economic growth ceases – or, if growth continues, it occurs because of some unexplained change in the state of technology.

[242] Indeed, it has proven difficult so far for "mainstream" macroeconomic theory (in particular, when **shocks** are included in the framework) to be built on top of endogenous growth models. The profession still awaits an innovation on this.

The main concept that endogenous growth theory applies to amend the Solow model is that there exist positive externalities[243] to innovation and research and development (R&D) activities. For example, when a firm develops a new method for writing software, it will benefit that firm directly because of increased sales and the customers of the firm will benefit because of the new product. However, other firms in the economy, simply by being exposed to the new ideas generated by the innovating firm, will also benefit. The exposure to new ideas will presumably enhance their design and manufacture, etc. of new products – which in turn will help yet other consumers and lead to more ideas available for yet other firms to use.

The positive externalities stemming from knowledge accumulation will occur even if innovating firms are granted patents or copyrights for their inventions and development. A patent or copyright indeed grants certain rights to its holder – but it cannot prevent the dissemination of **ideas** through an economy, and it is **ideas** that fundamentally drive technological progress. Thus, in the language of the Solow growth model, the technology parameter X increases over time – implying positive economic growth even in the long run.

However, a firm, when deciding how much input to use in its R&D activities with the goal of create new products or services, will not take into account the positive externalities of its innovations. In the language of the theory of externalities, the private (i.e., to the firm) marginal benefit from innovation is smaller than the social marginal benefit. Or, another way of stating this is that the private marginal cost of innovation is larger than the social marginal cost of innovation. Thus, the amount of resources that a firm will use for research and development purposes will be smaller than the amount of resources that it *should* use *if* it cared about maximizing the welfare of the entire economy.[244]

The above discussion leads to a very important point: there is clearly a role for government intervention in promoting innovation. Assuming that governments *do* care about maximizing the welfare of its citizens (even when private firms seek only to maximize their own profits), various policies can be implemented which encourage the socially optimal amount of innovation to occur. The most obvious in the context of the above example is for government to use public funds to subsidize research and development. Such a policy would have the effect of lowering the private marginal cost of research – which then induces the firm to engage in the socially optimal amount of research and development.

Because ideas can disseminate through the economy in the manner described, the original innovating firm cannot rest on its laurels. It will know that other firms will soon try to

[243] You should be familiar with the notion of externalities from basic microeconomics.
[244] Note that this does not imply that corporations are "evil" – they simply act to maximize their own private gain, which is what economics usually considers as the most rational goal.

copy its products and enhance them – which will spur the original firm to continue developing new ideas. Thus, in this manner, the state of knowledge continually evolves.

Other ways that governments can encourage technical progress are through encouraging international trade and improving the quality and quantity of education. Again, both of these policies would expose domestic economic agents to more ideas (in the externality manner), which is the ultimate engine of economic growth.