Chapter 5
Dynamic Consumption-Labor Framework

We have now studied the consumption-leisure model as a "one-shot" model in which individuals had no regard for the future: they simply worked to earn income, all of which they then spent on consumption right away, socking away none of it for the future. Individuals do, of course, consider their future prospects when making economic decisions about the present. We saw this idea in our study of the two-period consumption-savings model. It should not strike you as unusual, then, that when an individual makes his optimal choice about consumption and leisure in the current period, he recognizes that he will make a similar consumption-leisure choice in the future. In effect, then, it seems there are multiple consumption-leisure choices an individual makes over the course of his lifetime. However, these choices are not independent of each other because consumers can save for the future or borrow against future income.

In this section we will bring the consumption-leisure model together with the consumption-savings model. As we will see, doing so in effect is just "gluing" the two models together. The main benefit is that it allows consideration of a broader range of consequences of macroeconomic policies – in particular it allows us to see that economic policies have their consequences not just in the time period in which they are implemented but also other periods.

Individual's Preferences

With two periods, in each of which the individual makes a consumption-leisure choice, there are four objects which determine the individual's lifetime utility: consumption in period 1, leisure in period 1, consumption in period 2, and leisure in period 2. Denote these, respectively, by c_1, l_1, c_2, and l_2, and let the lifetime utility function be $v(c_1, l_1, c_2, l_2)$. We will assume that this lifetime utility function is **additively-separable** across time in the following way:

$$v(c_1, l_1, c_2, l_2) = u(c_1, l_1) + u(c_2, l_2).$$

The function $v(c_1, l_1, c_2, l_2)$ is the **lifetime utility function** and the function u is the **sub-utility function** which measures utility over consumption and leisure in each of the two periods. Note especially that the function u is the same function in each of the two periods, meaning that the indifference map over c_1 and l_1 is identical to the indifference map over c_2 and l_2. Furthermore, because consumption in two different periods appears in the lifetime utility function, an indifference map over c_1 and c_2 exists, just as in the two-period model we have already consider.

Lifetime Budget Constraint

The more complicated object to describe in this model is the individual's lifetime budget constraint (LBC). Just as in the simpler two-period model, a budget constraint exists for period 1

$$P_1c_1 + A_1 = (1+i)A_0 + (1-t_1)W_1(1-l_1)$$

as well as for period 2

$$P_2c_2 + A_2 = (1+i)A_1 + (1-t_2)W_2(1-l_2),$$

in which W_1 denotes the hourly wage in period 1, W_2 denotes the hourly wage in period 2, t_1 denotes the labor tax rate in period 1, and t_2 denotes the labor tax rate in period 2. All of the other notation is the same as in our simple consumption-savings model and our simple consumption-leisure model. The interpretation of these period-by-period budget constraints is the same as before – in each period the individual has some wealth (which may be negative) and some labor income at his disposal, and he must decide how much to consume and how much to save for the future. The difference here versus the simple consumption-leisure model is that the individual decides how much labor income he earns.[35]

Because the rational individual consider his entire (two-period) lifetime when making his decisions, the relevant budget constraint is a lifetime budget constraint, which we derive using the two period-by-period budget constraints above. First, note that because there is no period 3, it must be that $A_2 = 0$, just as before, because there is no reason to save for after the end of the world. Then, we can solve the period-2 budget constraint to get

$$A_1 = \frac{P_2c_2}{(1+i)} - \frac{(1-t_2)W_2(1-l_2)}{(1+i)},$$

which we can in turn substitute into the period-1 budget constraint. After a few steps of algebra, we have

[35] If this brief description of these budget constraints, as well as the derivation of the LBC to follow, seems unfamiliar, it is a good idea to review the simple consumption-savings model and the simple consumption-leisure model at this point.

$$P_1c_1 + \frac{P_2c_2}{(1+i)} = (1-t_1)W_1(1-l_1) + \frac{(1-t_2)W_2(1-l_2)}{(1+i)} + (1+i)A_0.$$

Finally, as in the consumption-leisure model, we can expand the terms on the right-hand-side and then move the terms involving leisure to the left-hand-side to get

$$P_1c_1 + \frac{P_2c_2}{(1+i)} + (1-t_1)W_1l_1 + \frac{(1-t_2)W_2l_2}{(1+i)} = \left((1-t_1)W_1 + \frac{(1-t_2)W_2}{(1+i)}\right) + (1+i)A_0.$$

As always, it is a good idea for you to verify these algebraic manipulations for yourself.

We will now graph the LBC in expression in three different graphs: in $c_1 - c_2$ space, in $c_1 - l_1$ space, and in $c_2 - l_2$ space. As in the simple consumption-savings model, we will assume for graphical simplicity that $A_0 = 0$, but the results that follow in no way depend on this assumption. Solving the previous expression for c_2 gives

$$c_2 = -\left(\frac{P_1(1+i)}{P_2}\right)c_1 + \frac{(1+i)(1-t_1)W_1}{P_2}(1-l_1) + \frac{(1-t_2)W_2}{P_2}(1-l_2).$$

This equation can be usefully viewed in one of two ways: either c_2 as a function of l_2 (in which case we are thinking of the consumption-leisure decision in period 2) or c_2 as a function of c_1 (in which case we are thinking of the consumption-savings decision that spans period 1 and period 2).

First let's consider graphing the c_2= -…. equation with c_2 on the vertical axis and c_1 on the horizontal axis. The slope of this function is $-(P_1(1+i)/P_2)$. If $c_1 = 0$, then $c_2 = \frac{(1+i)(1-t_1)W_1}{P_2}(1-l_1) + \frac{(1-t_2)W_2}{P_2}(1-l_2)$, while if $c_2 = 0$, $c_1 = \frac{(1-t_1)W_1}{P_1(1+i)}(1-l_1) + \frac{(1-t_2)W_2}{P_1(1+i)}(1-l_2)$ – so we now have the intercepts of this function. Notice that these intercepts depend on the choices of leisure in the two periods, l_1 and l_2.

Alternatively, if we graph c_2= -…. equation with c_2 on the vertical axis and l_2 on the horizontal axis, we see that the slope is $-(1-t_2)W_2/P_2$, just as in our simple consumption-leisure model. If $l_2 = 0$, then $c_2 = -\left(\frac{P_1(1+i)}{P_2}\right)c_1 + \frac{(1+i)(1-t_1)W_1}{P_2}(1-l_1) + \frac{(1-t_2)W_2}{P_2}$, while if $c_2 = 0$, then

$l_2 = 1 - \frac{P_1(1+i)c_1}{(1-t_2)W_2} + \frac{(1+i)(1-t_1)W_1}{(1-t_2)W_2}(1-l_1)$ – so now we have the intercepts of this function. Notice that these intercepts depend on the choice of consumption in period 1 and leisure in period 1.

The main point which emerges from the preceding discussion is that all four choices (of consumption in the two periods as well as leisure in the two periods) are interdependent. Essentially, we need a five-dimensional graph (which obviously is impossible) in order to visualize the solution to this model. So use of graphical tools here is complicated.