

Chapter 6

Firms: Labor Demand, Investment Demand, and Aggregate Supply

We have studied in depth the consumers' side of the macroeconomy. We now turn to a study of the firms' side of the macroeconomy. Continuing with our representative agent paradigm, we suppose there is a single "representative firm" in the economy. This firm must use labor and capital (machines) in order to produce the output good ("all stuff") that consumers demand. Condensing all of the inputs that a firm uses into the two categories "labor" and "capital" is a useful simplification.³⁵ In studying firms, we again adopt the two-period idea from the consumption-savings model.

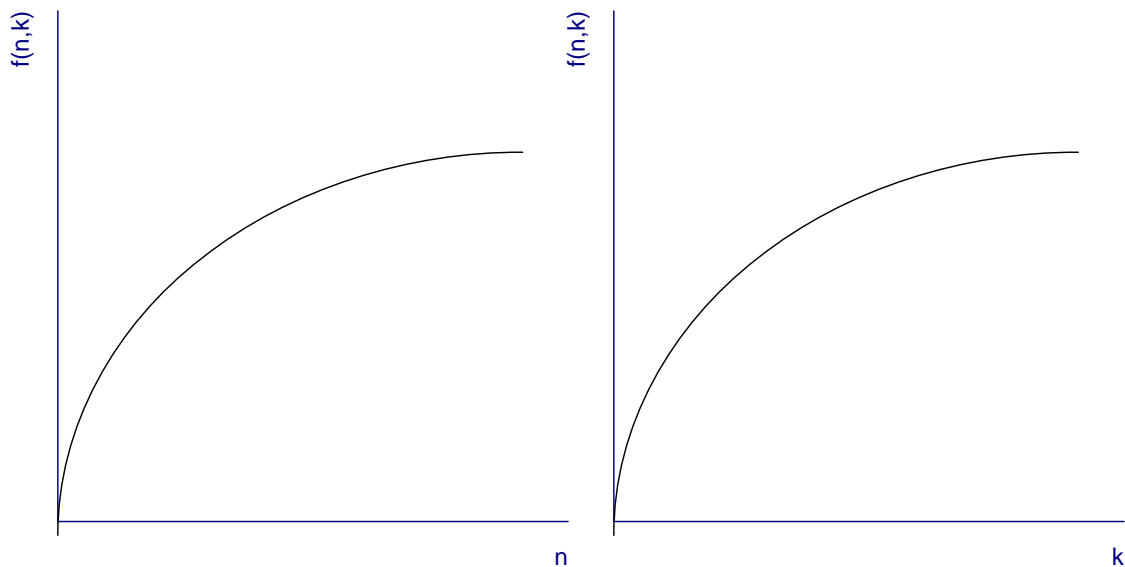


Figure 24. The production function $f(k, n)$ displays diminishing marginal returns to both labor and capital. The left panel shows that as capital is held constant, increases in labor increase output at an ever-decreasing rate. The right panel shows that as labor is held constant, increases in capital increase output at an ever-decreasing rate.

³⁵ Indeed, if you think about it, there really is only one thing that is not readily categorized as either labor or capital – land. Thus, we abstract from land.

Aggregate Production Function

We assume a standard production function $f(k, n)$, where n denotes the number of hours of labor the firm hires (from consumers) and k denotes the number of machines (capital) the firm uses. As should be familiar from basic microeconomics, we assume this production function displays diminishing marginal returns to both labor and capital. Specifically, holding the amount of labor fixed, increases in capital increase total output at an ever-decreasing rate, and holding the amount of capital fixed, increases in labor increase total output at an ever-decreasing rate, as shown in Figure 24.

Profit Maximization – The Firm’s Labor Hiring Decision

Again as familiar from basic microeconomics, the goal of a firm is to maximize profit, which equals total revenue minus total cost. Let P_1 be the period-1 price of the output good (again, the same “all stuff”) in terms of dollars and W_1 the period-1 hourly wage rate in terms of dollars. Total revenue for the firm in period 1 is then simply $P_1 \cdot f(k_1, n_1)$, and the total labor cost of the firm is $W_1 \cdot n_1$. The “1” subscript in all of the preceding indicates the time-period we are considering, namely period 1 of the two-period model. Similarly, in period 2 total revenue for the firm is $P_2 \cdot f(k_2, n_2)$ and the total labor cost is $W_2 \cdot n_2$. Because the firm exists for both periods of the two-period world, **the goal of the firm is to maximize lifetime profits, which is the sum of profits in each of the two periods.**

The timing of events for the firm in the two-period model is depicted in Figure 25. It begins period 1 with some initial amount of capital k_1 , which it cannot alter during period 1. That is, the initial amount of capital is what it must use in production in period 1 – in terms familiar from basic microeconomics, it is a fixed cost because it cannot be altered during period 1.³⁶ During period 1, the firm then decides how much labor n_1 it wants to hire (a variable cost) as well as how much capital k_2 it would like to have next period. When period 2 begins, the firm has k_2 units of capital to use in production during period 2, and it then decides how much labor n_2 to hire and how much capital k_3 it would like to have in period 3. But because the firm knows there is no period 3, it makes no sense for it to choose anything other than $k_3 = 0$!³⁷

Thus, the decisions that the firm makes are labor hirings in period 1 and period 2, as well as by how much to change its capital between period 1 and period 2. This latter decision, by how much to change its capital stock, is the firm’s **investment decision**.

³⁶ This is a useful way of thinking about how firms operate: at any point in time, it is much harder to change the number of machines a firm has (because it takes time to build new machines, etc.) than it is to change the number of hours of work a firm hires (i.e., simply hire overtime labor).

³⁷ Just as in the two-period consumption model we studied, the rational individual (here, the rational firm) will always choose to have zero wealth at the end of the economy. The capital (machines, equipment, etc.) is the wealth (i.e., the assets) that a firm owns.

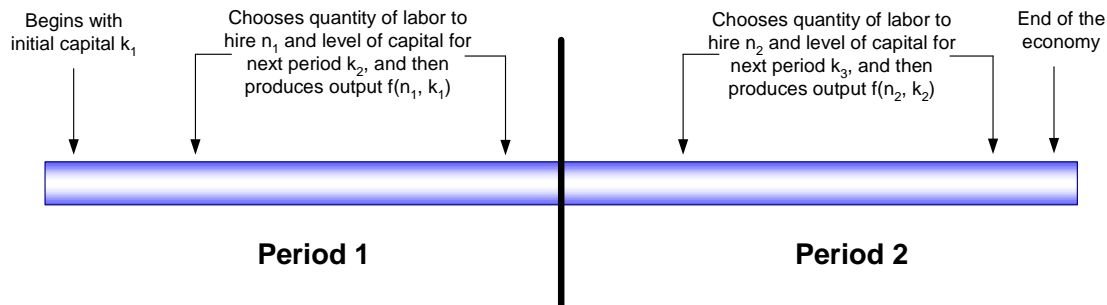


Figure 25. Timing of events for the firm in the two-period model.

Before we study the firm's investment decision, let's briefly consider its decision about how many workers to hire. In fact, this is something with which you are already familiar from basic microeconomics. The firm's demand for labor is a **derived demand**. To briefly review: in period 1, the capital k_1 is fixed, so that hiring additional workers increases total output at an ever-decreasing rate according to the law of diminishing marginal product. The price P_1 is determined by the market (think perfect competition here), so that marginal revenue product (defined as price of the output good times marginal product) declines the more labor the firm hires. The derived demand curve for labor is simply the marginal revenue product curve, as shown in Figure 26.

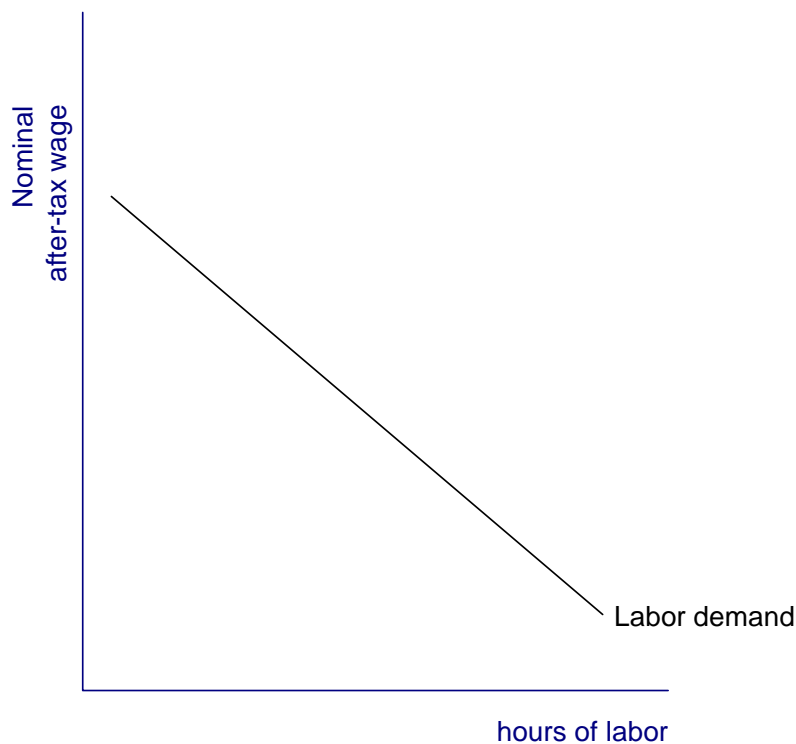


Figure 26. The downward-sloping labor demand curve, which is simply the marginal revenue product curve of labor.

Notice that now that we have a labor demand curve in our model of the macroeconomy, if we superimpose our backward-bending labor supply curve in Figure 26 then the after-tax wage is determined. Again as familiar from basic microeconomics, the labor demand curve shifts with changes in either the marginal product of labor or the price P of the goods the firm sells, which in this case is “all stuff.”

Profit Maximization – The Firm’s Investment Decision

Consideration of the firm’s investment decision is a bit more involved. As mentioned above, a useful simple way of thinking about what is meant when a macroeconomist uses the term “investment” is as the change in the amount of capital used by firms between one period and the next.³⁸ In the context of our two-period model, this change only has interesting meaning between period 1 and period 2. Mathematically, define the variable inv to denote the change in capital between period 1 and period 2 – specifically,

$$inv = k_2 - k_1. \quad (37)$$

To be even more precise, this notion of investment is termed **net investment** – net investment in any given period is defined as the change in the capital stock between that period and the next.³⁹ To analyze the net investment decision, we will make an assumption which at first glance seems to be a (too) gross simplification. We will assume that “consumer goods” (the “all stuff” that individuals buy) are completely identical to “capital goods” (the machines, etc. that firms use to produce consumer goods). Of course, this cannot literally be true – it’s doubtful that any individual consumer has ever purchased an auto assembly line for use in his own home, for example. But there are quite a number of goods that have uses both in production of other goods and services as well as end-user value in themselves. For example, over 60% of households in the U.S. own a personal computer. If you walk into almost any place of business, you are almost certain to see many desktop computers populating employees’

³⁸ It is crucial to distinguish the macroeconomic notion of investment from one that may be more familiar from colloquial usage. Macroeconomic **investment** is the sum of business purchases of **capital goods** (goods that businesses use in the production of other goods and services), new homes built, and addition to firms’ inventories. In everyday language, we often use the term “investment” to refer to someone’s collection of stocks and bonds, as in “I invested in 100 shares of Microsoft stock last week.” In formal economics, this latter type of activity is **not** investment. In fact, this latter type of activity is termed savings, a topic we will encounter later. It is very important, however, to keep this terminology straight, as it is often a source of confusion when discussing matters of savings and investment. As we discuss soon, there is in fact a deep connection between macroeconomic savings and macroeconomic investment. For now, however, we are only considering the topic of investment. Because it is consumers (for the most part) that purchase homes, we see from the above definition of investment that investment encompasses activities of both consumers and firms. However, for convenience of exposition, we will simply speak of investment as being undertaken by firms only.

³⁹ The Appendix considers **gross investment**, which is the more-common notion of investment in macroeconomic discussion. As the derivations in the Appendix show, however, qualitatively net investment and gross investment behave similarly.

workspaces. Firms use PCs in production, just as households use PCs for their everyday “consumption” purposes. There are other examples, too, of goods that are usefully thought of as both consumer goods as well as capital goods. There is also a host of examples of goods that are either one type or the other, but not both. It is not useful to get entangled in such a discussion at this point, however. Rather, we will simply make the assumption in our theoretical model that consumer goods and capital goods are identical in order to see how far such an assumption can carry us.

Now because capital goods and consumer goods are the same, the dollar price of a unit of the capital good must be the same as the dollar price of a unit of the consumer good. In terms of the notation already developed, the price of one unit of capital is P_1 in period 1 and P_2 in period 2. To determine inv , then, we essentially need to determine the firm’s “demand for capital” k_2 (when it is still period 1) as a function of some relevant price.

Because the demand for capital is a derived demand just as is the demand for labor, the marginal revenue product of capital is simply the price of what the firm produces times the marginal product of capital. The marginal product of capital decreases as capital increases (see Figure 24), while the price of the good the firm produces (“all stuff”) is constant from the perspective of the firm if it is in perfect competition. Thus, the marginal revenue product curve describes the benefit to the firm of having capital, and it is a negative function of its price. The harder question is determining what the relevant intertemporal “price” (i.e., not simply P) of capital is. This is the task to which we now turn.

The simplest way of determining the relevant intertemporal price of capital is to consider the economic cost (which includes opportunity costs!) the firm faces when it purchases capital. If the firm owns its machines, then we may be tempted to say that there is zero economic cost for the firm (ignoring, say, costs of electricity or water, etc.). On the other hand, there are many firms that do not own their own machines, but rather rent them from some other organization, most often a bank. If a firm is renting its machines, then the bank is essentially charging it some rate of interest for the privilege of doing so. **This interest rate must, under well-functioning markets, reflect the opportunity cost to a firm that does own its own machines because such a firm could rent its machines to another firm.** As such, whether or not a firm owns its own machines, the cost of capital to the firm is positive, and in particular, the same for either type of firm. So we may as well adopt the point of view of a firm that owns its own capital in both periods.

In period 1, then, the dollar value of the capital the firm will use next period is $P_1 \cdot k_2$ (notice carefully the subscripts here) dollars. That is, whatever amount of capital the firm chooses to have in period 2, its dollar value in period 1 is $P_1 \cdot k_2$. In period 2, this capital is worth a total of $P_2 \cdot k_2$ dollars. The reason the firm must choose its amount of capital for period 2 in period 1 is because it generally takes some time to “build new machines” – here we are supposing that it takes one period. This “time to build” represents an opportunity cost, because the firm could have instead put the $P_1 \cdot k_2$ dollars in its bank account, where it would earn the nominal interest rate i , thus leaving it with $(1+i)P_1 \cdot k_2$

dollars at the beginning of period 2 (i.e., after the interest has been paid on the account). If time-to-build were not an issue, the firm could then use these proceeds to purchase at the beginning of period 2 the amount of capital it wished to use in production during period 2. But in period 2 the price of each unit of capital is P_2 . Because the algebraic units of $(1+i)P_1 \cdot k_2$ is dollars and the algebraic units of P_2 is (dollars / unit of good), we must divide the former by the latter in order to return to units of goods.⁴⁰ But time-to-build is a real issue. Thus, the opportunity cost to the firm of carrying k_2 machines from period 1 into period 2 is

$$\frac{P_1(1+i)}{P_2} k_2. \quad (38)$$

Notice carefully the terminology here – the reason the opportunity cost arises is because hold onto **in period 1 the capital it will use in period 2**, the firm could have instead placed that money in a bank and earned the market nominal interest rate i . The term multiplying k_2 should look familiar to you from our study of the Fisher Equation. Specifically, letting π_2 denote the inflation rate between period 1 and period 2, you should recall that $P_1(1+i)/P_2 = (1+i)/(1+\pi_2)$, from which it follows that the cost of each unit of capital to the firm is

$$\frac{(1+i)}{(1+\pi_2)}. \quad (39)$$

Again recall from our discussion of the (exact) Fisher Equation that expression (39) is nothing more than $(1+r)$, where r denotes the real interest rate. As in consumer theory, both of these ways of expressing the cost of each unit of capital, either $(1+r)$ or $(1+i)/(1+\pi_2)$, will be useful depending on which issues we are considering. The conclusion from the preceding analysis: **the relevant price (which is really an opportunity cost) of capital is the real interest rate.**

We already concluded above (before our discussion of what the appropriate intertemporal price of capital is) that the demand for capital must be a negative function of its price because it is a derived demand. But because investment and capital are not quite the same thing (recall our definition $inv = k_2 - k_1$ above), we have one more small step to make before we can describe the investment demand function, which is fortunately a simple one. Capital in period 1 is fixed – the firm has no control over it. Thus, it is simply a constant from the perspective of the firm. As such, the qualitative shape of the capital demand function is the same as the investment function. The investment function is shown in Figure 27 with investment on the horizontal axis and the real interest rate on the vertical axis.

⁴⁰ Write out these expressions with the associated algebraic units to convince yourself of this.

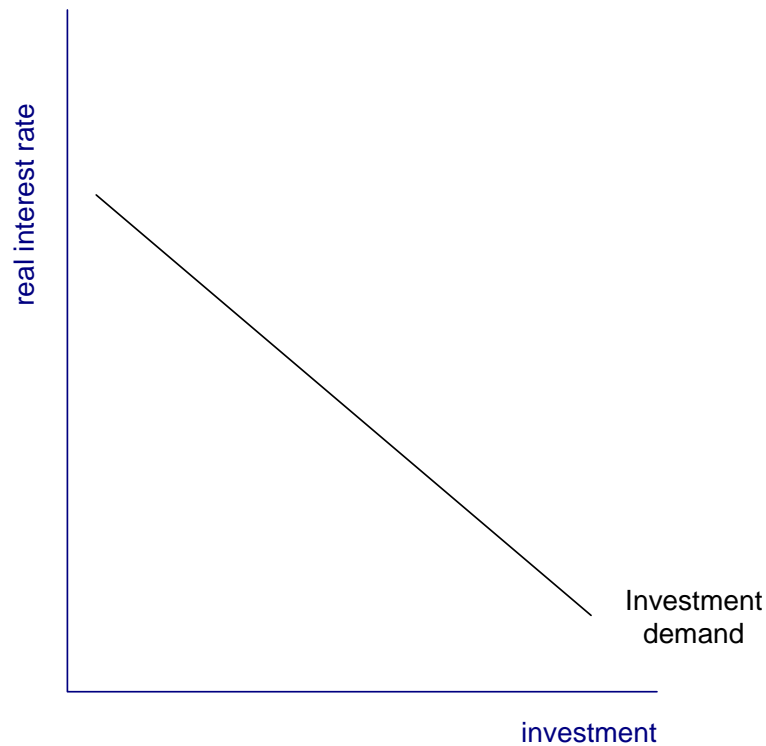


Figure 27. Investment demand is a negative function of the price of capital, which is the real interest rate.

We quickly note two extensions of the preceding analysis. First, in Figure 27, we can superimpose the upward-sloping aggregate private savings function we derived in the consumption-savings model. The intersection of the savings function and the investment function then determines the equilibrium real interest rate in the economy.

Second, recall from above (and our discussion of the Fisher Equation) that the real interest rate is related to the nominal interest rate and the current and future price level by

$$1 + r = \frac{P_1(1+i)}{P_2} .,$$

which shows that if i and P_2 are held constant, the real interest rate and the price level in period 1 are directly related to each other. That is, any rise in P_1 leads to a rise in r , as long as i and P_2 are held constant. Thus, an alternative view of Figure 27 is the downward-sloping function in Figure 28, in which investment is plotted against the price level in period 1. Note that this diagram is **not** what is usually meant by the term “investment function.” However, it is useful because we can now horizontally sum the function in Figure 28 with the consumption demand function we derived in the

consumption-savings model. Doing so (almost) gives us the aggregate demand function.⁴¹

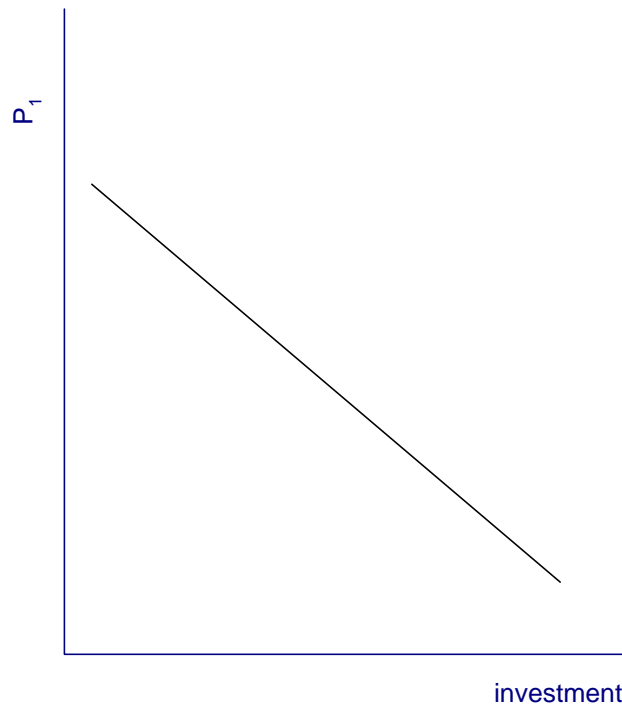


Figure 28. Because of the Fisher Equation, the investment demand function can also be represented as a negative function of the price level in period 1. Note that this diagram is **not** what you should usually think of as the “investment function.”

Aggregate Supply

With our description of the firms’ side of the macroeconomy, it is straightforward to derive the aggregate supply function of the economy. At any point in time, the capital stock of the economy is fixed. Thus, in the short-run, the only way for firms to produce more goods is to hire more labor.

Recalling that firms’ demand for labor is a derived demand, a rise in the price level P , with the nominal wage W held constant, shifts the demand for labor outwards.⁴² If the economy is in the upward-sloping region of the labor supply curve, labor market equilibrium means that more total hours of labor n are hired. A larger number of total hours means that total output, given by the production function $f(k, n)$, rises. Thus,

⁴¹ Only “almost” because we have not considered how government spending varies with the price level. It turns out that it essentially does not (i.e., government spending is inelastic), so that horizontally summing Figure 28 with the consumption demand function does in fact give us the aggregate demand function.

⁴² In other words, we are assuming here that wages do not adjust immediately when there is a change in the price level – there is some “stickiness” in wages. Empirical evidence generally supports the view that changes in wages lag changes in prices of other goods and services.

holding all else constant, a rise in the price level P induces firms to want to produce more output in order to increase their profits. Thus our model of firms presented here very simply generates an upward-sloping aggregate supply function, which is depicted in Figure 29.

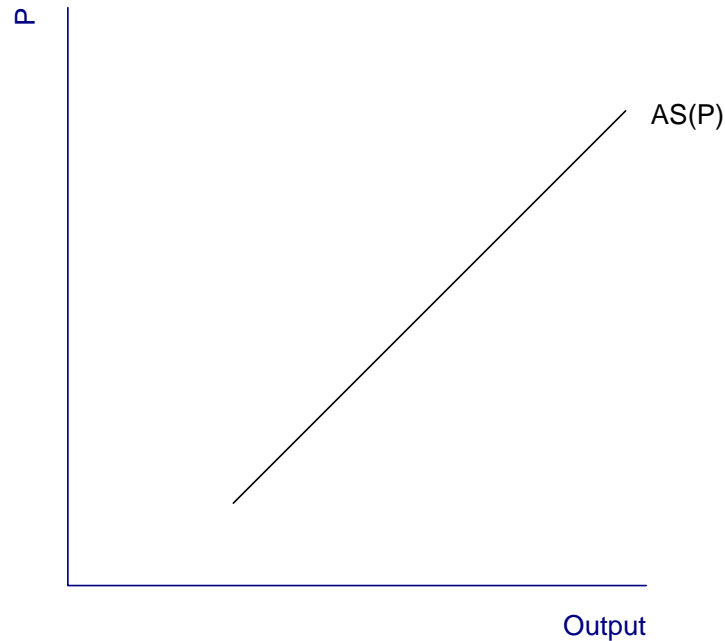


Figure 29. The upward-sloping aggregate supply function $AS(P)$ can be derived by the principle of profit-maximization by the representative firm.

Production Function Shocks

Just as we extended our models of consumer theory by supposing that an individual's utility function sometimes suffers unexpected shocks, we can extend our model of firms to suppose that the aggregate production function sometimes suffers unexpected shocks as well.⁴³ The introduction of a shock to the production function has the effect that for any given amount of both capital and labor, total output depends on the level of the shock. Such a shock is most commonly interpreted as a “technology shock.”

The most usual way of introducing production function shocks is to suppose that it simply multiplies the production function. Letting A denote this shock affecting the production function, we would now write the production function as $A \cdot f(k, n)$, where A is simply some constant over which a firm has no control but may change over time. It should be clear that if we set $A = 1$ always, then we recover the model we have just discussed.

⁴³ In fact, this is a very common theoretical modeling approach, much more so than that of preference shocks. For reasons beyond the scope of this text, however, this approach has failed to capture at a theoretical level some important features of macroeconomic data, especially regarding the behavior of inflation. As such, this remains a very active area of debate and research amongst academic and government economists.

If A rises, then the production function depicted in Figure 24 is modified as in Figure 30. This technology shock to the production function will be important to our later study of real business cycle theory.

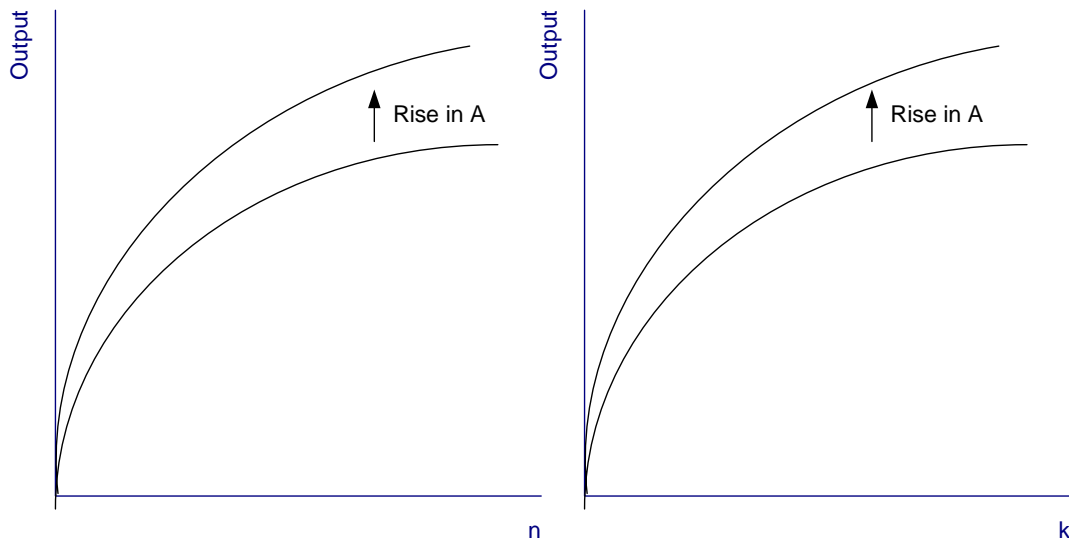


Figure 30. A rise in A causes output to rise for any given quantity of labor and capital.

Appendix: Alternative Derivation of Investment Function

Here, we present the firm's investment decision in slightly different terms, distinguishing between gross investment and net investment. In the following, we will consider the benefits and costs facing a single firm when it is faced with investment opportunities. We will then appeal to the basic result from microeconomics that optimal decisions occur at points at which marginal benefit equals marginal cost, to determine the optimal amount of investment this single firm should undertake. From this, we will extrapolate to firms in aggregate so that we can speak of investment demand in the entire economy.

Benefits of Investment

Recall our notion of the production function, $y = A \cdot f(k, n)$ (in this discussion, we include the technology parameter A from the beginning), where k denotes capital and n denotes labor. Capital goods are goods that businesses use in the production of other goods and services, for example, the trucks used by a delivery company, the hair dryers used in a beauty salon, and the manufacturing plants used to assemble cars. Such goods naturally wear out over time. The wearing out of capital goods is termed **depreciation**.⁴⁴

In the U.S. data show that roughly eight percent of the nation's capital stock depreciates every year and that this number has been fairly stable for a long period. Studies from other countries show that their depreciation rates are also fairly stable over time, though they may differ from the U.S. depreciation rate. We thus adopt the assumption that there is constant rate of depreciation each year and we denote this constant yearly depreciation rate by δ (the Greek letter "delta"), where $0 < \delta < 1$. Thus, if a business own k units of capital at the beginning of the year and purchases no capital goods at all during the year, at the end of the year it will own $(1 - \delta)k$ units of capital.

We now introduce two more pieces of terminology, gross investment and net investment. Suppose the business just mentioned was a "hair-drying salon" (all they do is dry hair!), and the only capital goods it uses is hair dryers. Say the business did purchase some new hair dryers during the course of the year. Specifically, suppose it purchased x units of hair dryers. We would call this absolute quantity of capital goods purchased **gross investment**. But it already had some hair dryers to begin with, some of which wore out during the course of the year. At the end of the year, the business would have $(1 - \delta)k + x$ hair dryers. We see that even though the salon purchased x hair dryers during the year, the net addition to the number of hair dryers during the course of the year was actually less than x due to depreciation. **Net investment** is gross investment minus

⁴⁴ This idea of wearing out of goods is the economic notion of depreciation, and has nothing to do with any types of depreciation rules you may have learned in an accounting class. Accounting standards and regulations are such that sometimes a company has some control over how to *report* its depreciation of capital goods (i.e., accelerated depreciation, straight-line depreciation). Our economic notion of depreciation has only to do with how quickly goods *actually* wear out over time and nothing to do with how a company may choose to *report* how quickly goods wear out.

total depreciation. In our example, total depreciation is δk , which represents the number of hair dryers that wore out. So we have in our example that gross investment by the salon was x while net investment was $x - \delta k$.

Now let's turn to the benefits of investment. The reason why our salon purchases hair dryers is because it helps them conduct business – it helps them to produce output, in other words. Recall from our initial look at the aggregate production function that total output displays diminishing marginal product in capital (as well as in labor), which is captured graphically by the fact that the slope of the curve in Figure 31 is always declining.

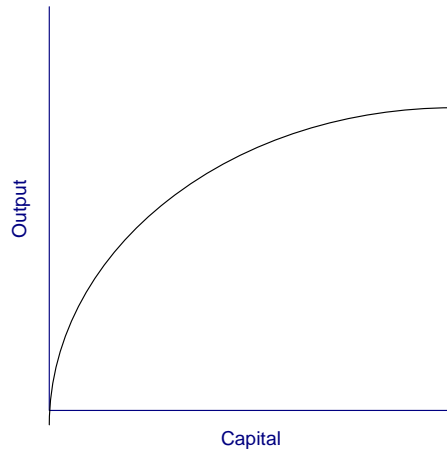


Figure 31

Appealing to your basic knowledge from microeconomics, the reason that a firm (our salon) would want to purchase *extra* capital is because of the *extra* output that it could produce with it. In more formal terminology, it is the **marginal product of capital** that is the benefit to a firm of purchasing capital – how much extra output a firm can produce by purchasing extra capital. This marginal product is diminishing as capital increases, as seen in Figure 32, which is simply a graph of the slope of the curve in Figure 31.

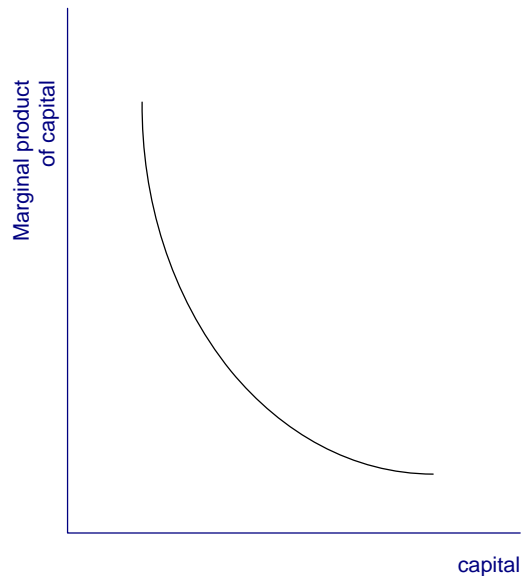


Figure 32

Costs of Investment

The relevant cost of investment is a little more subtle to recognize. The relevant cost of investment is *not* the price of the capital goods being purchased. Rather, it is the *opportunity cost* to the firm. Continuing our hair-drying salon example, rather than purchase new hair dryers, the firm could alternatively have put its funds in some sort of savings device (a bank account that pays interest, say) and earned interest on it. The relevant interest rate is the real interest rate, rather than the nominal interest rate that the account specifies. But we can easily compute the real interest rate from the nominal interest rate using the Fisher equation if we know something about the inflation rate.⁴⁵ By buying hair dryers rather than putting its money in some interest-bearing account, the salon is forgoing the interest it could have earned – hence the real interest rate is the relevant cost of investment.

The real interest rate is independent of the amount of capital a firm owns, in contrast to the way in which the marginal product of capital depends on the amount of capital (see Figure 32). Thus, we have Figure 33.

⁴⁵ If you're unsure about the mechanics or intuition behind the Fisher equation, now is the time to review that topic.

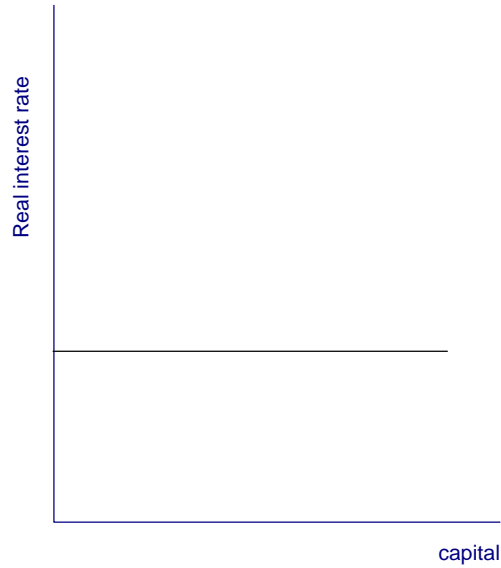


Figure 33

Optimal Investment

To determine optimal investment by a firm, we put Figure 32, which describes the benefit of investment, together with Figure 33, which describes the cost of investment, to generate Figure 34.

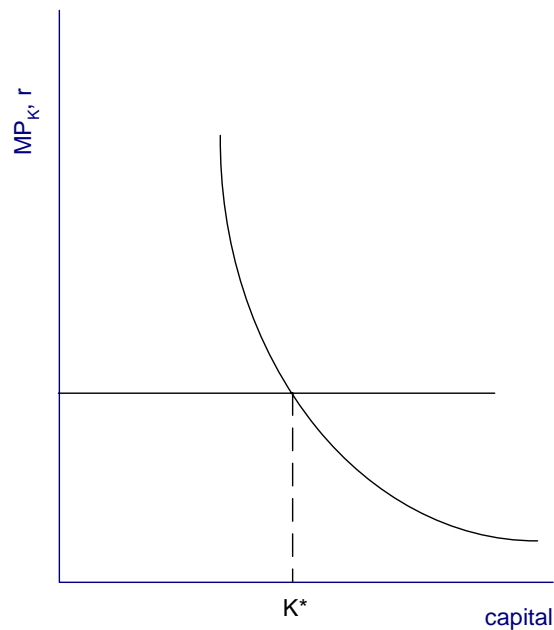


Figure 34

In Figure 34, the quantity k^* denotes the optimal quantity of capital the firm would like to have. This quantity of capital is optimal because it is the amount of which the marginal benefit of capital (the marginal product) equals the marginal cost of capital (the real interest rate). A basic lesson from microeconomics is that optimal decisions occur at points at which marginal benefits balance marginal costs – this is exactly the principle here.

We are not quite finished, however. Figure 34 describes the optimal amount of capital – we are interested in determining the optimal amount of investment. With the terminology we developed at the beginning of this section, it is easy to make this connection.

Suppose the firm currently has k units of capital, and its target level of capital one year from now is k^* , determined through a diagram like Figure 34. Further suppose that $k < k^*$. Then Figure 35 shows us the amount of gross investment as well as net investment the firm must undertake.⁴⁶

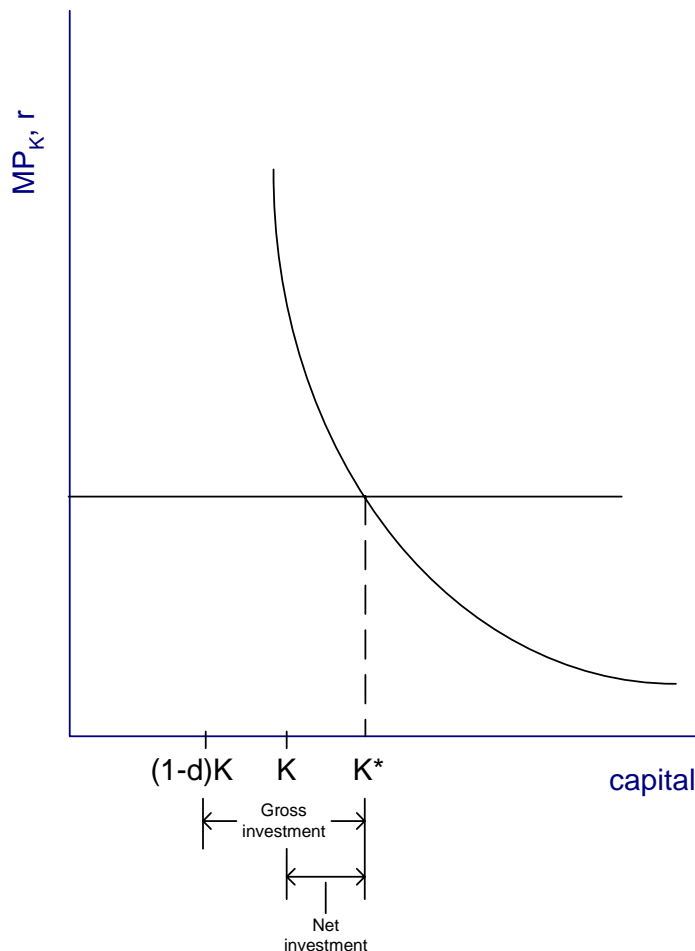


Figure 35

⁴⁶ A useful exercise is to confirm for yourself the relationships shown in Figure 35 using the definitions of investment we introduced earlier.

Macroeconomic Investment Demand

Now suppose there is a large number of firms just like the one we have studied, each of which faces the same costs and benefits of investment. Each firm would make optimal investment decisions according to the same principles. As we have described, the relevant cost of investment is the real interest rate. Now we will trace out the effect on investment (whether net or gross) of a change in the real investment. Doing so will generate the **investment demand** function.

Our analysis here uses Figure 35. Suppose that the real interest rate decreased, while current capital k , the depreciation rate δ , and the marginal product of capital function all remained unchanged. As Figure 36 shows, this would lead to a larger desired level of optimal capital, named k^{**} in Figure 36, and hence larger amount of optimal (both gross and net) investment, as can be seen by comparing with Figure 35.

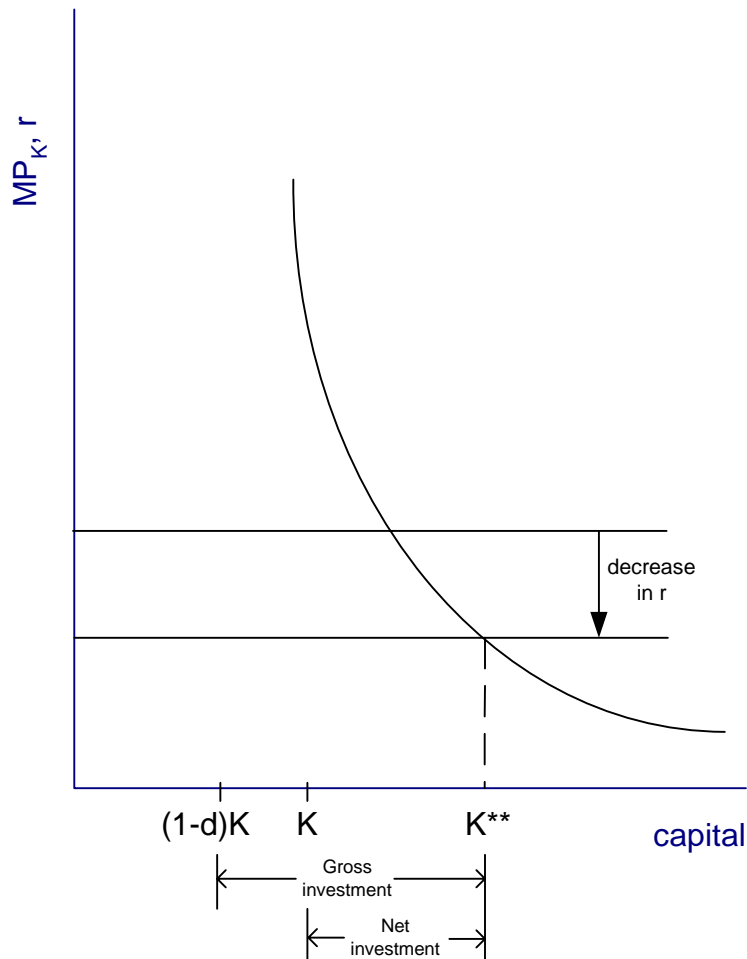


Figure 36

Similarly, a rise in the real interest rate would lead to a decline in investment. This relationship is captured in Figure 37, which is the macroeconomic investment demand curve. Notice that here we have finally put investment, rather than capital, on the axes of our graph.

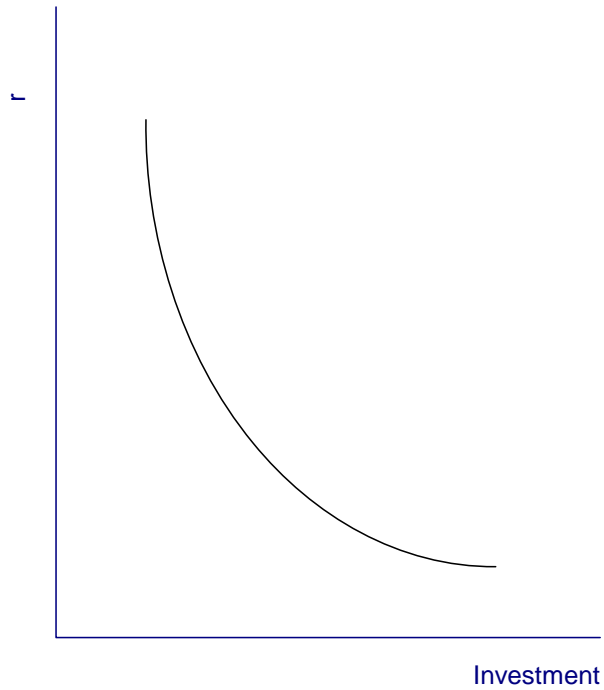


Figure 37

