# Monopolistically Competitive Search Equilibrium*

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**Abstract**

This paper introduces a monopolistically-competitive recruiting (intermediated) market in a standard (non-intermediated) search and matching model to explore the implications of intermediated labor markets, whose importance in new job creation is rising. We analytically show that: (1) the surplus to recruiters from successful monopolistic intermediation appears *directly and additively* in the surplus-sharing condition between newly matched workers and firms; (2) the surplus that accrues to monopolistic recruiters arises due to aggregate increasing returns in matching; (3) deviations from efficient wage setting (Nash-Hosios) in non-intermediated random-search markets spill over into recruiter creation and matching via intermediated markets, but deviations from efficient matching aggregation in recruiting markets have *no* impact on non-intermediated markets; and (4) in general equilibrium, the aggregate increasing returns in matching expands the aggregate resource frontier. We quantitatively show how the implications of wage distortions in non-intermediated markets on aggregate unemployment and labor force participation depend on the existence of intermediated labor markets.

**Keywords:** labor markets, hiring costs, search and matching, recruiting
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# 1 Introduction

This paper introduces a monopolistically-competitive recruiting sector with endogenous entry of recruiters alongside a standard (random-) search and matching model of labor markets. The core of our model builds on the work of Moen (1997) and Shimer (1996), who are the first to characterize competitive search equilibrium, and on the monopolistically-competitive endogenous entry model of Bilbiie, Ghironi, and Melitz (2012).[1] The two main questions our framework allows us to simultaneously address are: 1) What is the nature of the surplus sharing condition in the recruiting sector? 2) What is the role that aggregate increasing returns plays, in both a partial equilibrium view of labor markets as well as in general equilibrium? Our model also allows us to characterize several informative comparative static experiments between outcomes in intermediated recruiter matching and non-intermediated random search and matching. The results we obtain are important for labor market analysis and contribute to the quickly growing recent literature on directed search and intermediation in labor markets.

The central result of our work is the analytical characterization of the surplus sharing condition between labor suppliers and labor demanders in intermediated matching markets. Surplus sharing conditions are important because they embed the payoffs, or incentives, of both suppliers and demanders in labor markets. The surplus sharing condition that arises in our model is novel because, in addition to the standard pair of payoffs accruing to a newly-matched worker and a newly-filled job vacancy, it also contains a *third, additive* positive payoff received by the monopolistically-competitive recruiter for successful intermediation. To illustrate the economic rationale behind the novel surplus sharing rule, consider the (qualitative) surplus sharing condition

[(1-Share) x (Share) x Recruiter Surplus] + [(1-Share) x Employee Surplus] = [(Share) x Employer Surplus],

in which the "Share" term is a scalar between zero and one that measures the percentage of the total *aggregate* surplus from a newly-created job match.

The emphasis on the total *aggregate* surplus is crucial and is due to the second main analytical result, which is that the monopolistic recruiting sector exhibits *aggregate increasing returns to scale* in matching. Intuitively, aggregate increasing returns in matching expands (in symmetric equilibrium) the total surplus of every new job created by the monopolistically-competitive recruiting sector. More precisely, the total surplus generated by a new job match is *not* just the sum of the employee surplus and the employer surplus. Instead, the total surplus is the sum of the employee surplus, the employer surplus, and, due to aggregate increasing returns in new job creation, the recruiting agency's surplus. Aggregate increasing returns in matching arises naturally in the presence of positive sunk costs of entry, which in turn leads to an endogenous measure of

[1] The application of Bilbiie, Ghironi, and Melitz (2012) is to product markets.

monopolistically-competitive recruiting agencies.[2] The idea of aggregate increasing returns is well known ever since at least Romer (1990). However, to the best of our knowledge, this well-known concept of aggregate increasing returns in *production* models has not been applied to *labor* models in the way that our framework does.[3]

The two main results described so far arise in the partial equilibrium of the labor market. Embedding the recruiting market structure in a general equilibrium environment allows us to characterize three more analytical results. First, the aggregate increasing returns to scale in matching carries over to the economy's general equilibrium goods resource constraint. Second, we prove that deviations from efficient wage setting in *non*-intermediated random-search matching spill over into and distort the development of new specialized recruiters and hence matching via monopolistically-competitive recruiting markets. However, as a corollary, distortions that emerge from inefficient aggregation in monopolistic recruiting markets have *no* impact on non-intermediated random-search markets. Spillovers are thus asymmetric in nature and only occur in one direction; the economic rationale behind the corollary is that the measure of goods-producing firms is exogenously fixed.[4]

The third general-equilibrium analytical result is that changes in the costs of research and development (or other components) in creating a new monopolistically-competitive recruiting agency do *not* have any effect at all on the endogenous equilibrium measure of recruiting agencies. This result at first seems surprising because a natural conjecture would be that (for example) a reduction in the cost of entry for potential new recruiters would lead to an increase in the number of recruiting firms. However, due to the presence of the other, non-intermediated random-matching channel, a substitution of both vacancies and active job search from intermediated recruiting markets to the random search-and-matching channel occurs. This substitution occurs due to equilibration of job-filling and job-finding probabilities across the two channels via which new job matches are created.

As noted above, our results contribute to the small, but quickly growing, recent literature on directed search and intermediation in labor markets. Following the Rubinstein and Wolinksy (1987) article on intermediation, several recent papers that, in different ways, enrich upon and connect to their seminal work are Masters (2007), Wright and Wong (2014), Nosal, Wong, and Wright (2015), Gautier, Hu, and Watanabe (2016), Chang and Zhang (2016), Gregor and Menzio (2016), Farboodi, Jarosch, and Shimer (2017), and Wright, Kircher, Julien, and Guerrieri (2017). This list is admittedly not exhaustive, which might partly be due to how rapidly the literature has been

[2]Or, more generally, *any* type of departure from *perfectly*-competitive recruiting agencies. We use monopolistic competition, which we think is a natural, simple departure from perfect competition, but other "imperfectly"-competitive market structures can also be used.

[3]The labor-market model constructed by Masters (2007) highlights the fact that matching technologies with increasing returns imply that intermediaries can bring about welfare gains; however, it is difficult to compare our results and the results obtained by Masters (2007) because the primitives of the two models are very different.

[4]More precisely, the measure of goods-producing firms is $[0, 1]$.

evolving. Section 6 elaborates further on how our work fits into this growing literature. For now, we simply state that this set of papers is mostly theoretical, as is ours.

On the empirical front, labor market intermediaries play an important role in helping firms meet their employment needs and job seekers find employment opportunities. The services and reach of these intermediaries has grown over time, especially (as shown by Nakamura, Shaw, Freedman, Nakamura, and Pyman (2009) and Bagues and Sylos Labini (2009)) with a dramatic expansion of online recruiting sites and their services since the middle of the 1990s. Survey evidence from the Society for Human Resource Management for 2007 suggests that more than 40 percent of new hires in both the public and private sectors originated from online recruiting agencies. Several other empirical papers on labor market intermediation are Autor, Katz, and Krueger (1998), Kuhn and Skuterud (2004), Stevenson (2008), and Kroft and Pope (2014); Appendix F provides a more in-depth discussion of the empirical results of this set of papers.

The rest of the paper is organized as follows. Section 2 describes the structure of recruiting market and characterizes the surplus-sharing function that arises when monopolistically-competitive recruiters intermediate frictional labor demand and labor supply. Section 3 then embeds the recruiting sector in a general equilibrium framework. Section 4 provides several general-equilibrium analytical results, and Section 5 adds quantitative richness to the analytical results. Section 6 places our main results within the context of existing work on intermedaition and matching frictions, and Section 7 concludes. Many of the algebraic derivations are provided in a detailed set of Appendices, as is a discussion of some of the work on the empirical aspects of labor market intermediation.

# 2 Recruiters — Partial Equilibrium

We begin with a partial equilibrium model of the imperfectly competitive recruiting sector with endogenous entry.

## 2.1 Recruiting Market $j$

There is a continuum $[0, 1]$ of perfectly-competitive recruiting markets. As shown in Figure 1, in each recruiting market $j \in [0, 1]$, perfectly-competitive recruiting agencies purchase differentiated submarket $ij$ matches and aggregate them using a technological aggregator. As shown in Figure 1, in each recruiting market $j \in [0, 1]$, perfectly-competitive recruiting agencies purchase differentiated submarket $ij$ matches and aggregate them using a technological aggregator. Table 1 shows the several matching aggregators considered in the theoretical and quantitative analysis, and, for reference, Table 2 provides definitions of notation used in the partial equilibrium analysis.

The representative labor-market $j$ recruiting agency is modeled as being a "large" recruiting

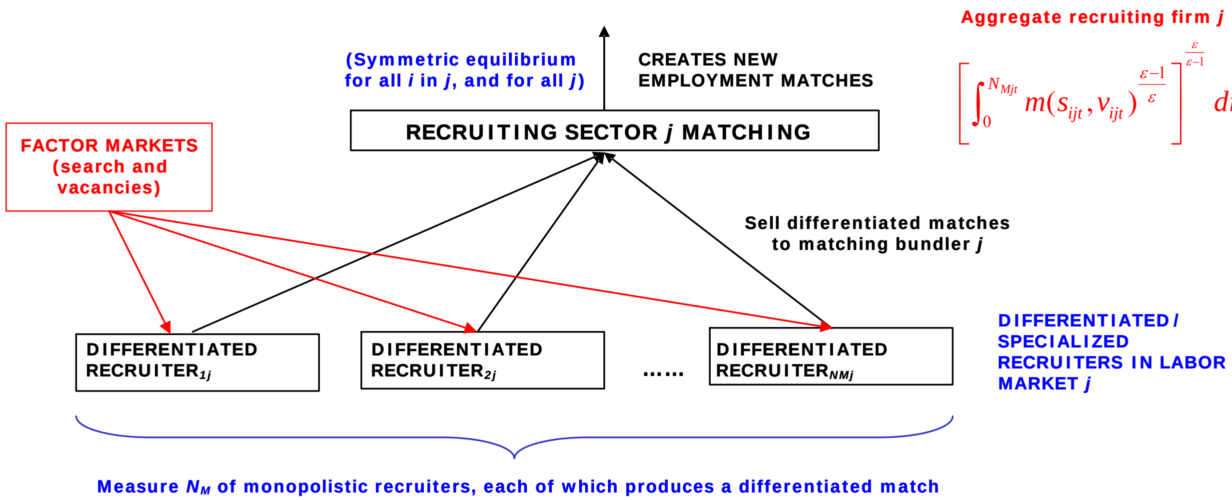


Figure 1: **Structure of Matching Markets.** Differentiated recruiting agencies produce specialized matches in their particular submarkets, which are then aggregated by perfectly-competitive recruiting agencies in labor market $j$. In each labor market $j$, there are $N_M$ differentiated recruiting agencies. The matching aggregator displayed (as but one example) is the Dixit-Stiglitz technology, in which the parameter $\varepsilon$ measures the elasticity of substitution between any pair of differentiated matches.

agency that develops "many" differentiated recruiting agencies. The labor-market $j$ recruiting agency is "large" in the sense that it produces multiple recruiting agencies, but the assumption of a continuum of recruiting firms ensures that each is small relative to the overall labor market, and hence does not internalize the effects of its decisions on the outcomes in matching-market $j$. Thus, we are assuming that recruiting agency $j$'s decisions regarding the development of new differentiated matching agencies do not internalize the fact that by creating new differentiated matching agencies the profits of *any* existing agencies within the firm are adversely affected (which is dubbed the "profit destruction externality"). This can be rationalized by assuming that new differentiated matching agencies are developed by independent recruiting line managers who communicate little with each other or are even encouraged to compete with each other.[5]

This rationale allows us to independently characterize the entry of new recruiters in labor market $j$ and the demand for each differentiated recruiter $i$'s match $m(s_{ijt}, v_{ijt})$ in labor market $j$, to which we now turn.

[5]This assumption is standard in the Bilbiie, Ghironi, and Melitz (2012) class of models on which our recruiting sector builds.

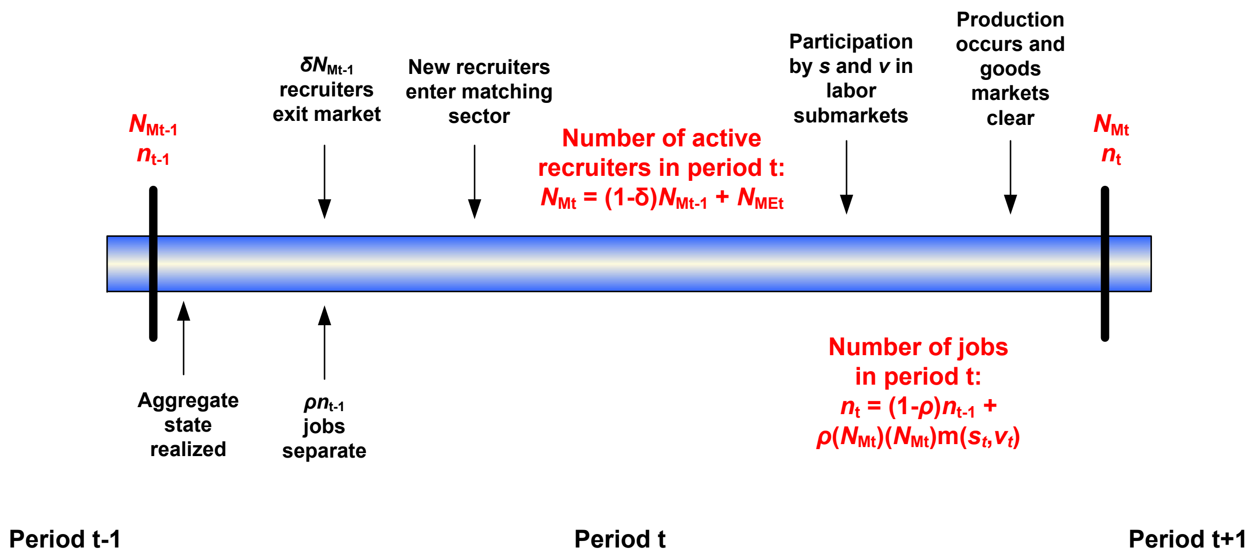


Figure 2: **Ordering of events in intermediated labor markets.** Newly-developed monopolistic recruiting agencies begin operations in period $t$, and newly-created job matches in period $t$ begin producing goods in period $t$. The $\rho(N_{Mt})N_{Mt}$ term measures the increasing returns to scale in matching.

**Entry of New Recruiters.**

Expressed in real terms (that is, in units of consumption goods), the intertemporal profit function of the representative recruiter in labor market $j$ is

$$E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left[ \left( \int_0^{N_{Mjt}} (\rho_{ijt} - mc_{jt}) \cdot m(s_{ijt}, v_{ijt}) \, di \right) - mc_{jt} \cdot \Gamma_{Et} \cdot N_{MEjt} \right], \tag{1}$$

in which $\rho_{ijt}$ is the price, stated in units of consumption goods, of a job match and $\Xi_{t|0}$ is the period-zero discount factor of the ultimate owners of the recruiting firm.[6] Entry of a new recruiter in period $t$ entails the endogenous cost $mc_{jt}\Gamma_{Et}$, in which the exogenous component $\Gamma_{Et}$ is denominated in units of consumption goods and is identical across all potential entrants.

The total number of new recruiters in labor market $j$ is $N_{MEjt}$. The law of motion for the total number of monopolistic recruiters in labor market $j$ is

$$N_{Mjt} = (1-\omega)N_{Mjt-1} + N_{MEjt}, \tag{2}$$

which is a constraint on recruiter $j$'s optimization problem. Given this constraint, recruiter $j$ maximizes its intertemporal profit function (1) by choosing $N_{Mjt}$ and $N_{MEjt}$. The first-order

[6] As will be clear in the general equilibrium model in Section 3, the ultimate owner of recruiting firms and hence any flow profits they earn is the representative household.

| Dixit-Stiglitz | Benassy | Translog |
|---|---|---|
| $\mu(N_M) = \mu = \frac{\varepsilon}{\varepsilon-1}$ | $\mu(N_M) = \mu = \frac{\varepsilon}{\varepsilon-1}$ | $\mu(N_M) = 1 + \frac{1}{\sigma N_M}$ |
| $\rho(N_M) = N_M^{\mu-1} = N_M^{\frac{1}{\varepsilon-1}}$ | $\rho(N_M) = N_M^{\varphi}$ | $\rho(N_M) = \exp\left(-\frac{1}{2}\frac{\tilde{N}_M - N_M}{\sigma \tilde{N} N_M}\right)$ |
| $\epsilon(N_M) = \mu - 1$ | $\epsilon(N_M) = \varphi$ | $\epsilon(N_M) = \frac{1}{2\sigma N_M} = \frac{1}{2}(\mu(N_M) - 1)$ |

Table 1: **Matching aggregators.** The markup, relative price of symmetric good, and aggregate increasing returns as functions of the number of recruiters for the Dixit-Stiglitz, Benassy, and translog variety aggregators. The Benassy aggregator nests the Dixit-Stiglitz aggregator if $\varphi = \frac{\varepsilon}{\varepsilon-1} - 1$, in which $\varphi$ characterizes, in terms of elasticity, the welfare benefits of increasing returns. $\tilde{N}_M$ denotes the mass of potential submarket recruiters for the translog aggregator. For the translog aggregator, the symmetric price elasticity of demand is the time-varying $-(1 + \sigma N_{Mt})$, whereas for both the Dixit-Stiglitz and the Benassy aggregators, the symmetric price elasticity of demand is the time-invariant $\varepsilon$.

conditions with respect to $N_{Mjt}$ and $N_{MEjt}$ yield the matching-market $j$ free-entry condition

$$mc_{jt} \cdot \Gamma_{Et} = (\rho_{ijt} - mc_{jt}) \cdot m(s_{ijt}, v_{ijt}) + (1-\omega) E_t \left\{ \Xi_{t+1|t} \left( mc_{jt+1} \cdot \Gamma_{Et+1} \right) \right\}, \ i = N_{Mjt}. \tag{3}$$

Intuitively, the free-entry condition equates the endogenous cost of entering submarket $j$ to the expected marginal benefit, which, in turn, depends on the flow of marginal profits and, conditional on the Poisson exit rate $\omega$, the continuation term. The entry condition can, via forward iteration, equivalently be expressed as

$$\begin{aligned} mc_{jt}\Gamma_{Et} &= (\rho_{ijt} - mc_{jt})\, m(s_{ijt}, v_{ijt}) \\ &+ \quad E_t \sum_{s=1}^{\infty} (1-\omega)^s \Xi_{t+s|t} \left( \rho_{jt+s} - mc_{jt+s} \right) m(s_{jt+s}, v_{jt+s}), \ i = N_{Mjt}. \end{aligned} \tag{4}$$

Regardless of formulation, the free-entry condition can be thought of as pinning down the endogenous measure of newly-entered monopolistic recruiters $N_{MEjt}$; it is the analogue of the entry condition in Bilbiie, Ghironi, and Melitz (2012) except, in the context of our model, now applied to production of differentiated job matches instead of production of differentiated goods.

**Demand Function for $m(s_{ijt}, v_{ijt})$.**

Next, we characterize the representative labor-market $j$ recruiter's demand for submarket $ij$ new job matches $m(s_{ijt}, v_{ijt})$. For ease of exposition, we assume that the recruiting-market $j$ aggregator is of Dixit-Stiglitz form

$$\left[ \int_0^{N_{Mjt}} m(s_{ijt}, v_{ijt})^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} di \tag{5}$$

(in which the parameter $\varepsilon$ measures the elasticity of substitution between any pair of specialized

| Variable Name | Definitions/Notes |
|---|---|
| $N_{Mjt}$ | Stock of recruiting agencies in submarket $ij$ |
| $N_{MEjt}$ | New recruiting agencies in submarket $ij$ |
| $\rho_{ijt}$ | Relative price of new job match $ij$ (in units of consumption goods) |
| $w_{ijt}$ | Wage for newly-hired employees in submarket $ij$ |
| $v_{ijt}$ | Vacancies directed to submarket $ij$ |
| $s_{ijt}$ | Active job search directed to submarket $ij$ |
| $\theta_{ijt}$ | Labor-market tightness ($\equiv v_{ijt}/s_{ijt}$) in submarket $ij$ |
| $k^f(\theta_{ijt})$ | Probability of job filling in submarket $ij$ |
| $k^h(\theta_{ijt})$ | Probability of job finding in submarket $ij$ |
| $\mathbf{W}(w_{ijt})$ | Value of active job search participating in submarket $ij$ that successfully finds an employer |
| $\mathbf{U}_t$ | Value of active job search in submarket $ij$ that fails to find a job |
| $\mathbf{J}(w_{ijt})$ | Value of job vacancy in submarket $ij$ that successfully finds an employee |
| $\Gamma_{Et}$ | Exogenous cost of developing a specialized recruiting agency and entering the recruiting market (in units of consumption goods) |
| $\omega$ | Exogenous Poisson exit rate of recruiting agencies |

Table 2: **Notation.** Partial equilibrium model of labor market.

matches), but the results also hold for other aggregators.[7]

Characterization of demand functions for submarket $ij$ new job matches requires a reformulation of the profit function stated in (1), the rationale for which is, as described above, the "autonomous" recruiting line managers within the "large" recruiting agency $j$. The reformulated profit function is the "static" profit function

$$\left[\int_0^{N_{Mjt}} m(s_{ijt}, v_{ijt})^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} di - \int_0^{N_{Mjt}} \rho_{ijt} \cdot m(s_{ijt}, v_{ijt})\ di. \tag{6}$$

Optimization yields the demand functions

$$m(s_{ijt}, v_{ijt}) = \rho_{ijt}^{-\varepsilon} \cdot \left[\int_0^{N_{Mjt}} m(s_{ijt}, v_{ijt})^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} di \tag{7}$$

for each underlying differentiated matching firm $ij$. Rewriting the demand function to isolate $\rho_{ijt}$ gives

$$\rho_{ijt} = m(s_{ijt}, v_{ijt})^{-\frac{1}{\varepsilon}} \cdot \left[\left[\int_0^{N_{Mjt}} m(s_{ijt}, v_{ijt})^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} di\right]^{\frac{1}{\varepsilon}}. \tag{8}$$

## 2.2 Monopolistically-Competitive Surplus Sharing

We now turn to the optimization problem of a differentiated recruiter $i$ in labor market $j$ which in turn leads to the novel surplus-sharing condition that arises in monopolistically-competitive recruiting markets.

**Profit Maximization.**

As standard in monopolistically competitive models, a differentiated firm (in our application, a differentiated recruiting agency) maximizes profits by choosing its price based on its demand function. Because the matching function $m(s_{ijt}, v_{ijt})$ is constant-returns-to-scale, it is sufficient to describe its cost-per-match in terms of the marginal cost $mc_{jt}$, which is independent across submarkets.[8] Recruiting agency $ij$'s period-$t$ profits are thus given by

$$\rho_{ijt} \cdot m(s_{ijt}, v_{ijt}) - mc_{jt} \cdot m(s_{ijt}, v_{ijt}). \tag{9}$$

[7] Such as the Benassy aggregator and the translog aggregator.

[8] In equilibrium, the factor prices will be equated to the marginal cost of creating a new job match, which is $mc_{jt} = p_{sjt}/m_s(\cdot) = p_{v_{jt}}/m_v(\cdot)$.

Continuing with the Dixit-Stiglitz matching aggregator shown in (5), substitution of the Dixit-Stiglitz demand function (7) allows us to rewrite recruiting agency $ij$'s period-$t$ profits as

$$\left(\rho_{ijt}^{1-\varepsilon} - \rho_{ijt}^{-\varepsilon} \cdot mc_{jt}\right) \cdot \left[\int_0^{N_{Mjt}} m(s_{ijt}, v_{ijt})^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} di. \tag{10}$$

The first-order condition of (10) with respect to $\rho_{ijt}$ yields the Dixit-Stiglitz pricing condition

$$\rho_{ijt} = \left(\frac{\varepsilon}{\varepsilon-1}\right) mc_{jt}, \tag{11}$$

in which $\mu_t = \frac{\varepsilon}{\varepsilon-1}$ is the constant gross markup that emerges from the Dixit-Stiglitz aggregator.[9]

**Monopolistic Surplus Sharing.**

In terms of the ordering of events (refer to Figure 2), recruiter $ij$ has already maximized profits (and thus minimized costs) before the posting phase $(w_{ijt}, \theta_{ijt})$ that attract both suppliers and demanders to submarket $ij$. Due to the ordering of events, the posting phase only requires use of recruiter $ij$'s *marginal profit*.[10] More precisely, define the value function associated with the recruiter $ij$ problem as

$$\mathbf{V}_{Mij}(s_{ijt}, v_{ijt}; \cdot) = \max \left\{\rho_{ijt} \cdot m(s_{ijt}, v_{ijt}) - mc_{jt} \cdot m(s_{ijt}, v_{ijt})\right\}, \tag{12}$$

which implies there are two associated marginal profit conditions. The marginal profit condition with respect to $s_{ijt}$ is

$$\begin{aligned} \frac{\partial \mathbf{V}_{Mij}(s_{ijt}, v_{ijt}; \cdot)}{\partial s_{ijt}} &= \rho_{ijt} \cdot m_s(s_{ijt}, v_{ijt}) - mc_{jt} \cdot m_s(s_{ijt}, v_{ijt}) \\ &= (\rho_{ijt} - mc_{jt}) \cdot \xi \cdot k^h(\theta_{ijt}), \end{aligned} \tag{13}$$

in which the second line follows from the properties of the Cobb-Douglas matching function $m(s, v) = s^{\xi} v^{1-\xi}$ and $k^h(\theta_{ijt})$ denotes the probability that an active job searcher in submarket

[9] More generally (referring to Table 1), the pricing condition can be expressed as $\rho(N_{Mjt}) = \mu(N_{Mjt}) \cdot mc(N_{Mjt})$.

[10] Our use of "marginal profit" is due to the difference between "large" firms (each of the monopolistic recruiters in our model is "large" in the sense that it can create more than one new job match) often used in general-equilibrium models and "small" firms (as in, say, Moen (1997), in which a given recruiter can only create at most one new job match) that might be more partial equilibrium in nature. If one particular recruiting firm can only create one job match, then one could say that the "marginal" concept of profits for that particular recruiter is harder, or impossible, to understand than simply describing it as "total" profit.

$ij$ successfully gains employment. Analogously, the marginal profit condition with respect to $v_{ijt}$ is

$$\begin{aligned}\frac{\partial \mathbf{V}_{Mij}(s_{ijt}, v_{ijt}; \cdot)}{\partial v_{ijt}} &= \rho_{ijt} \cdot m_v(s_{ijt}, v_{ijt}) - mc_{jt} \cdot m_v(s_{ijt}, v_{ijt}) \\ &= (\rho_{ijt} - mc_{jt}) \cdot (1-\xi) \cdot k^f(\theta_{ijt}), \qquad (14)\end{aligned}$$

in which the second line follows from the properties of the Cobb-Douglas matching function $m(s, v) = s^{\xi} v^{1-\xi}$ and $k^f(\theta_{ijt})$ denotes the probability that a job opening in submarket $ij$ is successfully filled.

Similar to Moen (1997), recruiter $ij$ has to incentivize both labor suppliers and labor demanders to participate in submarket $ij$. The incentive mechanism for recruiter $ij$ is to take as constraints the participation conditions of labor suppliers and labor demanders. We detail the foundations of the participation constraints in Section 3; for now, though, we simply take them as given. Referring to the definitions in Table 2, the participation constraint of a labor supplier is

$$k^h(\theta_{ijt}) \cdot \mathbf{W}(w_{ijt}) + \Big(1 - k^h(\theta_{ijt})\Big) \mathbf{U} = \mathbf{X}^H \qquad (15)$$

and the participation constraint of a labor demander is

$$k^f(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}) = \mathbf{X}^F. \qquad (16)$$

Expression (15) states that the value of a labor supplier that directs search towards submarket $ij$ must be the same as the value $\mathbf{X}^H$ of directing search to any other submarket. Analogously, expression (16) states that the value of a labor demander that directs its job openings towards submarket $ij$ must be the same as the value $\mathbf{X}^H$ of directing its job openings to any other submarket.

Regardless of whether the marginal profit condition (13) or (14) is used, the following surplus-sharing rule (the proof for which appears in Appendix A) arises.

**Proposition 1.** ***Monopolistic Surplus Sharing.*** *The surplus-sharing rule between labor suppliers and labor demanders that meet via monopolistically-competitive labor-market intermediation is*

$$\xi \cdot (1-\xi) \cdot (\rho_{ijt} - mc_{jt}) + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}), \ \forall i \in j \qquad (17)$$

*in which* $\xi \in (0,1)$ *denotes the elasticity of total matches with respect to the measure of active job searchers.*

*Proof.* See Appendix A. □

Intuitively, there are *three* parties involved in the surplus, and each of the three parties earns a positive share of the overall surplus. The three parties are the newly-employed worker (whose value is $\mathbf{W}(\cdot) - \mathbf{U}$), the newly-filled job opening (the value of which is $\mathbf{J}(\cdot)$), and the monopolistic recruiter $ij$ (whose value is $\mathbf{V}_{Mij}(\cdot)$) that matches the other two parties. Casual observation of the monopolistic surplus sharing rule shows that the percentage of the total surplus received by workers $(1-\xi)$ and the percentage of the total surplus received by goods-producing firms $(\xi)$ sum to 100%. This observation naturally leads to the question of the source of the *extra* resources needed to provide monopolists the *positive economic profit* $\rho_{ijt} - mc_{jt}$.

## 2.3 Increasing Aggregate Returns in Matching

The ultimate source is the *increasing returns* that arise in the *aggregate match*. Continuing to use the Dixit-Stiglitz aggregator (5) for the sake of simplicity, the *perfectly-competitive* aggregate recruiter $j$ constructs, in decentralized labor-market $j$, new job matches via the technology stated in (5). However, if its matching technology were the more general

$$\tilde{m}\left(N_{Mjt}, m(s_{jt}, v_{jt})\right) = \left[\int_0^{N_{Mjt}} m(s_{ijt}, v_{ijt})^{\frac{\varepsilon-1}{\varepsilon}} \, di\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

there are constant returns to scale for the intermediate $ij$ matches in producing the final labor-market match $j$ for a given measure of differentiated recruiters $N_{Mjt}$. However, there are increasing returns to scale once $N_{Mjt}$ is treated as an input argument to production of market-$j$ matches, which implies that operating this $\tilde{m}(.)$ technology in the *perfectly-competitive* labor market $j$ is infeasible. To see the increasing returns more clearly, imposing symmetry across $i$ yields

$$\begin{aligned}
\tilde{m}\left(N_{Mjt}, m(s_{jt}, v_{jt})\right) &= \left[m(s_{jt}, v_{jt})^{\frac{\varepsilon-1}{\varepsilon}} \int_0^{N_{Mjt}} 1 \, di\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[m(s_{jt}, v_{jt})^{\frac{\varepsilon-1}{\varepsilon}} \cdot N_{Mjt}\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= m(s_{jt}, v_{jt}) \cdot N_{Mjt}^{\frac{\varepsilon}{\varepsilon-1}} \\
&= m(s_{jt}, v_{jt}) \cdot \rho(N_{Mjt}) \cdot N_{Mjt}, \qquad (18)
\end{aligned}$$

from which it is clear that, given $\varepsilon < \infty$, aggregate increasing returns arises in matching. Based on this Dixit-Stiglitz example, Table 1 informs us that the increasing returns term is $N_{Mjt}^{\frac{\varepsilon}{\varepsilon-1}} = \rho(N_{Mjt}) \cdot N_{Mjt}$. But aggregate increasing returns in matching is not restricted to this particular function. Thus, the general formulation of the increasing returns component is $\rho(N_{Mt})N_{Mt}$, which takes into account other aggregators (Table 1 shows two other commonly-used aggregators).

**Labor Supply and Labor Demand.**

To determine the monopolistic wage implicit in Proposition 1, the value expressions $\mathbf{W}(w_{ijt})$, $\mathbf{U}_t$, and $\mathbf{J}(w_{ijt})$ require characterization. These values can be defined here, from which we can characterize the increasing-returns equilibrium in the labor market. However, we defer characterizing these value expressions until we describe the general equilibrium model in Section 3, which analytically shows how aggregate increasing returns in the recruiting sector affects the aggregate goods resource frontier.

# 3 General Equilibrium

We now place the partial equilibrium recruiting model into a general equilibrium framework. The general equilibrium framework characterizes the foundations of the directed search constraints faced by monopolistic recruiters. The general equilibrium framework also relaxes the assumption that recruiting is the only channel by which new job matches are created by introducing a second process for new job creation, which is the well-known Pissarides (1985) non-intermediated *random search* matching process that has become common in macroeconomic models that use the labor search and matching structure. For reference, Table 3 provides definitions of notation for the general equilibrium model.

## 3.1 Households

There is a continuum $[0, 1]$ of identical households. In each household, there is a continuum $[0, 1]$ of family members. In period $t$, each family member in the representative household has a labor-market status of employed, unemployed and actively seeking a job, or being outside the labor force. Regardless of which labor-market status a particular family member is in, each family member receives the same exact amount of consumption $c_t$ due to full risk-sharing within each household (see Andolfatto (1996) for formal details).

The representative household maximizes lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(c_t) - h \left( n_t + \underbrace{(1 - k_{Nt}^h) \cdot s_{Nt}}_{=ue_t^N} + \int_0^1 \left( \int_0^{N_{Mjt}} \underbrace{(1 - k_{ijt}^h) \cdot s_{ijt}}_{=ue_{ijt}} \; di \right) dj \right) \right] \tag{19}$$

subject to the budget constraint

$$\begin{aligned} c_t + k_{t+1} + T_t = & \; (1 + r_t - \delta) k_t + w_t (1 - \rho) n_{t-1} + w_{Nt} \cdot k_{Nt}^h \cdot s_{Nt} \\ + & \int_0^1 \int_0^{N_{Mjt}} w_{ijt} \cdot k_{ijt}^h \cdot s_{ijt} \; di \; dj + \int_0^1 \int_0^{N_{Mjt}} p_{s_{jt}} \cdot s_{ijt} \; di \; dj \\ + & \; (1 - k_{Nt}^h) \cdot s_{Nt} \chi + \int_0^1 \int_0^{N_{Mjt}} \left(1 - k_{ijt}^h\right) \cdot s_{ijt} \chi \; di \; dj + \int_0^1 \Pi_{jt}^M \; dj + \Pi_t^F, \end{aligned} \tag{20}$$

| Variable Name | Definitions/Notes |
|---|---|
| $v_{Nt}$ | Vacancies posted in non-intermediated matching market (i.e., random search) |
| $s_{Nt}$ | Active job search in non-intermediated matching market (i.e., random search) |
| $\gamma$ | Vacancy posting cost for submarket $ij$ |
| $\gamma_N$ | Vacancy posting cost for non-intermediated matching market |
| $w_{Nt}$ | Wage for employees hired in non-intermediated matching market |
| $\theta_{Nt}$ | Labor-market tightness ($\equiv v_{Nt}/s_{Nt}$) in non-intermediated matching market |
| $k^f(\theta_{Nt})$ | Probability of $v_{Nt}$ matching in non-intermediated matching market |
| $k^h(\theta_{Nt})$ | Probability of $s_{Nt}$ matching in non-intermediated matching market |
| $p_{v_{jt}}$ | Income per job vacancy $v_{ijt}$ posted in labor market $j$ |
| $p_{s_{jt}}$ | Income per unit search $s_{ijt}$ directed towards labor market $j$ |
| $k_t$ | Physical capital |
| $r_t$ | Real interest rate |
| $\chi$ | Government-provided unemployment benefits |
| $\Pi_{jt}^M$ | Period-$t$ flow profits, recruiting firms in labor market $j$ |
| $\Pi_t^F$ | Period-$t$ flow profits, goods-producing firms |

Table 3: **Notation.** General equilibrium model.

in which $\Pi^M_{jt}$ is the period-$t$ flow profits from labor market $j$ that the household receives lump-sum, and $\Pi^F_t$ is the period-$t$ flow profits from the goods-producing firms that the household receives lump-sum. The representative household also faces the period-$t$ perceived law of motion of employment, which is

$$n_t = (1-\rho)n_{t-1} + k^h_{Nt} \cdot s_{Nt} + \int_0^1 \int_0^{N_{Mjt}} k^h_{ijt} \cdot s_{ijt} \; di \; dj. \tag{21}$$

Constraint (21) captures new employment relationships from both sources of matching (intermediated and non-intermediated) in one perceived law of motion.

The optimality conditions (the details of which are provided in Appendix C) that emerge are the standard Euler expression for the supply of physical capital

$$1 = E_t \left\{ \Xi_{t+1|t} \left(1 + r_{t+1} - \delta\right) \right\}, \tag{22}$$

in which $\Xi_{t+1|t} \equiv \beta u'(c_{t+1})/u'(c_t)$ denotes the stochastic discount factor, and a set of labor-force participation conditions

$$\begin{aligned} \frac{h'(lfp_t)}{u'(c_t)} &= (1-k^h_{Nt})\chi \\ &+ \quad k^h_{Nt}\left[w_{Nt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\left(\frac{1-k^h_{Nt+1}}{k^h_{Nt+1}}\right)\cdot\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\}\right] \end{aligned} \tag{23}$$

and

$$\begin{aligned} \frac{h'(lfp_t)}{u'(c_t)} &= p_{s_{jt}} + (1-k^h_{ijt})\chi \\ &+ \quad k^h_{ijt}\left[w_{ijt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\left(\frac{1-k^h_{jt+1}}{k^h_{jt+1}}\right)\cdot\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - p_{s_{jt+1}}\right)\right\}\right] \;\; \forall ij. \end{aligned} \tag{24}$$

The participation function (23) characterizes endogenous, but *random*, job search in the non-intermediated labor market, whereas the set of participation functions (24) characterize endogenous *directed* job search towards intermediated labor submarket $ij$. Given the household-level envelope conditions, around the optimum, active job search in all submarkets must yield the same value $k^h(\theta_{ijt}) \cdot \mathbf{W}(w_{ijt}) + \left(1 - k^h(\theta_{ijt})\right) \cdot \mathbf{U}(\cdot) = k^h(\theta_{kjt}) \cdot \mathbf{W}(w_{kjt}) + \left(1 - k^h(\theta_{kjt})\right) \cdot \mathbf{U}(\cdot), \forall i \neq k.$

### 3.2 Firms

There is a continuum $[0,1]$ of identical goods-producing firms. The representative goods-producing firm's lifetime profit function is

$$
\begin{aligned}
& E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left\{ \; z_t f(k_t, n_t) - r_t k_t - \gamma_N \cdot_{Nt} \; \right\} \\
- \quad & E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left\{ \int_0^1 \int_0^{N_{Mjt}} \gamma \cdot v_{ijt} \; di \; dj - \int_0^1 \int_0^{N_{Mjt}} p_{v_{jt}} \cdot v_{ijt} \; di \; dj \right\} \\
- \quad & E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left\{ w_t \cdot (1-\rho) n_{t-1} + w_{Nt} \cdot k^f_{Nt} \cdot v_{Nt} + \int_0^1 \int_0^{N_{Mjt}} w_{ijt} \cdot k^f_{ijt} \cdot v_{ijt} \; di \; dj \right\}
\end{aligned} \tag{25}
$$

subject to the period-$t$ perceived law of motion of employment

$$
n_t = (1-\rho) n_{t-1} + k^f_{Nt} \cdot v_{Nt} + \int_0^1 \int_0^{N_{Mjt}} k^f_{ijt} \cdot v_{ijt} \; di \; dj. \tag{26}
$$

Profit-maximization (see Appendix B for the formal analysis) leads to the set of job-creation conditions

$$
\gamma_N = k^f_{Nt} \cdot \underbrace{\left( z_t f_n(k_t, n_t) - w_{Nt} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left( \frac{\gamma_N}{k^f_{Nt+1}} \right) \right\} \right)}_{\equiv \mathbf{J}(w_{Nt})}, \tag{27}
$$

and

$$
\gamma = p_{v_{jt}} + k^f_{ijt} \cdot \underbrace{\left( z_t f_n(k_t, n_t) - w_{ijt} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left( \frac{\gamma - p_{v_{jt+1}}}{k^f_{jt+1}} \right) \right\} \right)}_{\equiv \mathbf{J}(w_{ijt})} \; \forall ij. \tag{28}
$$

The job-creation condition (27) characterizes endogenous, but *random*, vacancy postings in the non-intermediated labor market, whereas the set of job-creation condition (28) characterize endogenous *directed* vacancy postings in intermediated labor submarkets $ij$. Around the optimum, the firm is indifferent between directing new job vacancies to intermediated submarket $i$ or intermediated submarket $k$, $k^f(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}) = k^f(\theta_{kjt}) \cdot \mathbf{J}(w_{kjt})$, $\forall i \neq k$.

### 3.3 Wage Determination

**Wages in Intermediated Labor Market (Directed Search).**

With the foundations of the value expressions $\mathbf{W}(w_{ijt})$, $\mathbf{U}_t$, and $\mathbf{J}(w_{ijt})$ for wage determination now in place, we can express the wage implicit in the monopolistically-competitive surplus sharing condition in Proposition 1 in explicit form. Substitution of the (symmetric equilibrium) value expressions $\mathbf{W}(w_t)$, $\mathbf{U}_t$, and $\mathbf{J}(w_t)$ into (17) yields the (symmetric equilibrium) explicit-form wage

$$\begin{aligned} w_t &= \xi z_t f_n(k_t, n_t) + (1-\xi)\chi + \xi(1-\rho)E_t\left\{\Xi_{t+1|t} \cdot \theta_{t+1} \cdot \left(\gamma - p_{v_{t+1}}\right)\right\} \\ &- \xi(1-\xi)\left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) \\ &+ \xi(1-\xi)(1-\rho)E_t\left\{\Xi_{t+1|t} \cdot (1 - k^h(\theta_{t+1})) \cdot \left(\rho(N_{Mt+1}) - \frac{\rho(N_{Mt+1})}{\mu(N_{Mt+1})}\right)\right\}, \end{aligned} \tag{29}$$

the algebraic details for which appear in Appendix D. If the recruiting market were perfectly competitive ala Moen (1997), then $\rho(N_{Mt}) = \frac{\rho(N_{Mt})}{\mu(N_{Mt})} (= mc(N_{Mt}))\ \forall ij$ and $p_{v_t} = m_v(\cdot)$, in which case the real wage is characterized by the completely-standard first line of (29). However, if the recruiting market is monopolistically competitive, then it is not only the period-$t$ profits accruing to the recruiting sector that affect the period-$t$ wage, *period-*$(t+1)$ *profits also affect the period-*$t$ *wage*, despite the fact that monopolistic recruiters only make static decisions. The reason that recruiters' period-$t+1$ rents affect the period-$t$ wage is the long-lasting nature of employment relationships.

**Nash-Bargained Wages in Non-Intermediated Labor Market (Random Search).**

We assume that the wage model in the non-intermediated labor market is generalized Nash bargaining. Without going into details (which can easily be found in a textbook such as Pissarides (2000, Chapter 1)), the Nash surplus-sharing condition is

$$\mathbf{W}(w_{Nt}) - \mathbf{U}_t = \left(\frac{\eta}{1-\eta}\right)\mathbf{J}(w_{Nt}), \tag{30}$$

in which $\eta \in (0,1)$ denotes the potential new employee's generalized Nash bargaining power. Substitution of the value expressions $\mathbf{W}(w_{Nt})$, $\mathbf{U}_t$, and $\mathbf{J}(w_{Nt})$ yields the explicit-form wage

$$w_{Nt} = \eta \cdot z_t f_n(k_t, n_t) + (1-\eta) \cdot \chi + \eta(1-\rho)E_t\left\{\Xi_{t+1|t}\left(\gamma_N \cdot \theta_{Nt+1}\right)\right\} \tag{31}$$

in non-intermediated *random search* labor markets.

## 3.4 Aggregate Employment

The aggregate law of motion for employment

$$n_t = (1-\rho)n_{t-1} + m(s_{Nt}, v_{Nt}) + \rho(N_{Mt}) \cdot N_{Mt} \cdot m(s_t, v_t) \tag{32}$$

takes into account both new job matches produced by the intermediated labor market — which, as described in the partial equilibrium model in Section 2, leads to aggregate increasing returns in matching — and the non-intermediated labor market.

### 3.5 Government

The (symmetric equilibrium) flow budget constraint of the government is

$$T_t = (1 - k^h(\theta_t)) \cdot s_t \cdot N_{Mt} \cdot \chi + (1 - k^h(\theta_{Nt})) \cdot s_{Nt} \cdot \chi, \tag{33}$$

in which lump-sum taxes $T_t$ levied on households finance government-provided unemployment benefits.

### 3.6 Aggregate Goods Resource Constraint

The decentralized economy's aggregate goods resource constraint

$$\begin{aligned} & c_t + k_{t+1} - (1-\delta)k_t + \gamma \cdot v_t \cdot N_{Mt} \\ + \quad & \gamma_N \cdot v_{Nt} + \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) \Gamma_{Et} N_{MEt} = z_t f(k_t, n_t) + \rho(N_{Mt}) \cdot N_{Mt} \cdot m(s_t, v_t), \end{aligned} \tag{34}$$

the derivation of which appears in Appendix E. Note that the aggregate increasing returns term in matching, $\rho(N_{Mt}) N_{Mt} m(s_t, v_t)$, appears in the goods resource constraint.

### 3.7 Private-Sector Equilibrium

The period-$t$ state of the economy is $S_t \equiv [n_{t-1}, N_{Mt-1}, k_t, z_t]$. A symmetric private-sector general equilibrium is made up of seventeen endogenous state-contingent processes $\{c_t, n_t, lfp_t, k_{t+1}, N_{Mt}, N_{MEt}, s_t, v_t, \theta_t, w_t, s_{Nt}, v_{Nt}, \theta_{Nt}, w_{Nt}, mc_t, p_{v_t}, p_{s_t}\}_{t=0}^{\infty}$ that satisfy the following seventeen conditions: the aggregate resource constraint

$$\begin{aligned} & c_t + k_{t+1} - (1-\delta)k_t + \gamma \cdot v_t \cdot N_{Mt} \\ + \quad & \gamma_N \cdot v_{Nt} + \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) \Gamma_{Et} N_{MEt} = z_t f(k_t, n_t) + \rho(N_{Mt}) \cdot N_{Mt} \cdot m(s, v_t), \end{aligned} \tag{35}$$

the aggregate law of motion for labor

$$n_t = (1-\rho) n_{t-1} + m(s_{Nt}, v_{Nt}) + \rho(N_{Mt}) \cdot N_{Mt} \cdot m(s_t, v_t), \tag{36}$$

the definition of aggregate LFP

$$lfp_t = (1-\rho) n_{t-1} + s_{Nt} + s_t \cdot N_{Mt}, \tag{37}$$

the aggregate law of motion for recruiters

$$N_{Mt} = (1-\omega)N_{Mt-1} + N_{MEt}, \tag{38}$$

the capital Euler condition

$$1 = E_t \left\{ \Xi_{t+1|t} \left(1 + z_{t+1} f_k(k_{t+1}, n_{t+1}) - \delta\right) \right\}, \tag{39}$$

the free-entry condition for recruiters

$$\left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) \Gamma_{Et} = \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) \cdot m(s_t, v_t) + (1-\omega) E_t \left\{ \Xi_{t+1|t} \left(\frac{\rho(N_{Mt+1})}{\mu(N_{Mt+1})}\right) \Gamma_{Et+1} \right\}, \tag{40}$$

the vacancy creation condition directed towards monopolistically-competitive labor markets

$$\gamma = p_{v_t} + k_t^f \cdot \underbrace{\left( z_t f_n(k_t, n_t) - w_t + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left( \frac{\gamma - p_{v_{t+1}}}{k_{t+1}^f} \right) \right\} \right)}_{\equiv \mathbf{J}(w_t)}, \tag{41}$$

the vacancy creation condition for non-intermediated random search labor markets

$$\gamma_N = k_{Nt}^f \cdot \underbrace{\left( z_t f_n(k_t, n_t) - w_{Nt} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left( \frac{\gamma_N}{k_{Nt+1}^f} \right) \right\} \right)}_{\equiv \mathbf{J}(w_{Nt})}, \tag{42}$$

the active job search condition directed towards monopolistically-competitive labor markets

$$\begin{aligned} \frac{h'(lfp_t)}{u'(c_t)} &= p_{s_t} + (1 - k_t^h) \underbrace{\chi}_{\equiv \mathbf{U}} \\ &+ \quad k_t^h \left[ w_t + (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left( \frac{1 - k_{t+1}^h}{k_{t+1}^h} \right) \cdot \left( \frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - p_{s_{t+1}} \right) \right\} \right] \ \forall ij, \end{aligned} \tag{43}$$

the active job search condition for non-intermediated random search labor markets

$$\begin{aligned} \frac{h'(lfp_t)}{u'(c_t)} &= (1 - k_{Nt}^h) \underbrace{\chi}_{\equiv \mathbf{U}} \\ &+ \quad k_{Nt}^h \left[ w_{Nt} + (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left( \frac{1 - k_{Nt+1}^h}{k_{Nt+1}^h} \right) \cdot \left( \frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \right], \end{aligned} \tag{44}$$

the surplus-sharing rule that determines wages $w_t$ in monopolistic labor markets

$$\xi \cdot \left( \rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) + \mathbf{W}(w_t) - \mathbf{U}_t = \left( \frac{\xi}{1-\xi} \right) \mathbf{J}(w_t), \tag{45}$$

the surplus-sharing rule that determines Nash-bargained wages (with $\eta$ denoting the employee's Nash bargaining power) in non-intermediated labor markets

$$\mathbf{W}(w_{Nt}) - \mathbf{U}_t = \left( \frac{\eta}{1-\eta} \right) \mathbf{J}(w_{Nt}), \tag{46}$$

the monopolistic matching-market pricing condition

$$\rho(N_{Mt}) = \mu(N_{Mt}) \cdot mc(N_{Mt}), \tag{47}$$

the definition of labor-market tightness in monopolistic recruiting markets

$$\theta_t = \frac{v_t}{s_t}, \tag{48}$$

the definition of labor-market tightness in monopolistic recruiting markets

$$\theta_{Nt} = \frac{v_{Nt}}{s_{Nt}}, \tag{49}$$

along with the equilibrium input prices

$$\begin{aligned} p_{v_t} &= m_{vt} \cdot \left( \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \\ &= \underbrace{(1-\xi) \cdot k^f(\theta_t)}_{\text{C-D } m(.)} \cdot \underbrace{\left( \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right)}_{= mc(N_{Mt})}, \end{aligned} \tag{50}$$

and

$$\begin{aligned} p_{s_t} &= m_{st} \cdot \left( \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \\ &= \underbrace{\xi \cdot k^h(\theta_t)}_{\text{C-D } m(.)} \cdot \underbrace{\left( \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right)}_{= mc(N_{Mt})}, \end{aligned} \tag{51}$$

taking as given the exogenous stochastic process for $z_t$ and the initial conditions $n_{-1}, N_{M,-1}$, and $k_0$.

# 4 General Equilibrium I — Analytical Results

Well known by those who work in the labor search literature is that (sans other distortions) the Mortensen-Hosios condition $\eta = \xi$ for Nash-bargained wages supports efficient allocations because it eliminates congestion externalities.[11] Well known by those who work with monopolistically-competitive models with endogenous entry is that (sans other distortions) Dixit-Stiglitz aggregation is efficient in that it eliminates the competing effects of incentives for entry and aggregate increasing returns, leading to an *efficient* number of monopolistically-competitive firms.[12]

In our model, *both* search-based congestion externalities *and* possible misalignment of aggregate increasing returns and incentives for entry are possible *in the same market.* As is formally stated below, a congestion externality (i.e., $\eta \neq \xi$) in the random-search and bargaining channel of new job creation *directly* leads to an *inefficient* number of monopolistically-competitive recruiting agencies *even if Dixit-Stiglitz aggregation holds.* We note that the results in Proposition 2, Lemma 1, and Proposition 3 are for the static version of the model (in which the separation rates for employment and recruiters are, respectively, $\rho = 1$ and $\omega = 1$). Focusing on a static setting delivers a set of transparent results. As our quantitative analysis using a dynamic ($\rho < 1$ and $\omega < 1$) version in Section 5 confirms, the results in the static version hold in the steady state of the fully dynamic model.

## 4.1 Spillover Effects on Monopolistic Recruiting Markets

As stated, respectively, in Proposition 2 and Corollary 1 for the static model, efficient Nash-bargained wages in new job matches via the random-search channel create *no* distortions in efficient monopolistic recruiting markets, whereas *in*efficient Nash-bargained wages create *inefficiencies* in monopolistic recruiting markets even though matching aggregation is Dixit-Stiglitz.

**Proposition 2.** ***Efficient Non-Intermediated Labor ⇒ Efficient Intermediated Labor Markets.*** *Suppose the matching aggregator in monopolistic recruiting markets is Dixit-Stiglitz, and assume that $\rho = 1$ and $\omega = 1$. Both $N_M$ and $\theta$ are efficient and are maximized if wages are determined efficiently ($\eta = \xi$) in new job matches created through random search,*

$$\frac{\partial N_M^*}{\partial \eta} = \frac{\partial \theta^*}{\partial \eta} = 0 \text{ if } \eta = \xi \text{ (Hosios)} \tag{52}$$

*in which the asterisks denote maximized values.*

*Proof.* See Appendix G. □

[11] As in Section 3, $\eta \in (0, 1)$ denotes the generalized Nash-bargaining power of workers and $\xi \in (0, 1)$ denotes the elasticity of new matches with respect to the number of job seekers.

[12] A helpful recent review of this latter result is provided by Bilbiie, Ghironi, and Melitz (2008).

**Corollary 1.** ***Inefficient Non-Intermediated Labor $\Rightarrow$ Inefficient Intermediated Labor Markets.*** *Suppose the matching aggregator in monopolistic recruiting markets is Dixit-Stiglitz, and assume that $\rho = 1$ and $\omega = 1$. Both $N_M$ and $\theta$ are below their efficient, maximized values if wages are inefficiently low in new job matches created through random search,*

$$\frac{\partial N_M^*}{\partial \eta} > 0 \textit{ and } \frac{\partial \theta^*}{\partial \eta} > 0 \textit{ iff } \eta < \xi \tag{53}$$

*or are above their efficient, maximized levels if wages are inefficiently high in new job matches created through random search,*

$$\frac{\partial N_M^*}{\partial \eta} < 0 \textit{ and } \frac{\partial \theta^*}{\partial \eta} < 0 \textit{ iff } \eta > \xi, \tag{54}$$

*in which the asterisks denote maximized values.*

*Proof.* See Appendix G. □

From an economic standpoint, Proposition 2 and Corollary 1 shed much insight because the prime focus of the search and matching framework regards *new* job creation. The intuition behind the non-monotonic patterns in the number of recruiting firms and recruiting-market tightness that Proposition 2 and Corollary 1 jointly imply is as follows. As the bargaining power in the non-intermediated market increases from an initially low level (i.e., $\eta$ is lower than $\xi$), households prefer to direct their search towards the market where their share of the surplus is expanding (i.e., the non-intermediated market). As a result, search in intermediated markets falls.

To offset this fall, recruiting firm entry rises since firms continue to post vacancies across markets, with the end result being a rise in intermediated market tightness. As the workers' bargaining power becomes increasingly higher, it becomes increasingly difficult to find employment in non-intermediated markets. There are two forces at play. First, as workers' bargaining power gets closer to "take-it-or-leave-it" offers (i.e., $\eta = 1$), this encourages potential new employees to continue to search in non-intermediated markets. Second, this same fact simultaneously leads to decreased job-finding probabilities as firms further reduce their non-intermediated market vacancies. This latter effect pushes households to start increasing their search in intermediated markets, ultimately leading to an increase in intermediated-market household searchers.

A similar rationale holds when we consider the behavior of vacancies in these markets. As the bargaining power of workers initially increases, firms decide to hire via the market where the bargaining power is not affecting how the employment surplus is split, but for high levels of the bargaining power, the job-filling probabilities are increasingly influenced by the high measure of searchers, implying that firms do not need to post as many vacancies to generate a given number of matches. As a result, intermediated-market vacancies start to decline as the bargaining power of workers approaches. Finally, as the bargaining power of workers increases and gets closer to 1, unemployment increases, and recruiting firms find it less profitable to participate in matching markets. This ultimately leads to a decline in the number of recruiting firms.

### 4.2 Lack of Spillover Effects on Random Search

A next natural question is whether causality runs in the other direction. In our model, the causality of inefficiencies in Proposition 2 and Corollary 1 does *not* run in the opposite direction. More precisely, suppose that Nash bargaining power is $\eta = \xi$. If the matching aggregator in monopolistic recruiting markets were *not* Dixit-Stiglitz — suppose it were instead the Benassy aggregator or the translog aggregator, in which the incentives for entry and the welfare benefits of aggregate increasing returns are misaligned[13] — the inefficient aggregation does *not* lead to an inefficiency in non-intermediated new job creation.

Intuitively, this uni-directional causality is due to the presence of endogenous (monopolistically-competitive) recruiting firm entry in intermediated markets, and incidentally, the *asymmetry* in the degree of competition between labor markets. Indeed, while both markets have vacancies and the measure of searchers as key margins of adjustment amid changes in the degree of congestion externalities in non-intermediated labor markets (and/or the degree of inefficiency in intermediated markets), intermediated markets have a third *critical* margin, which is the *endogenous* measure of recruiting firms. This implies that, in relative terms, the non-intermediated market will be less responsive to changes in intermediated markets. Of note, as suggested by our quantitative analysis based on a fully dynamic model, this uni-directional causality holds beyond a static environment.

### 4.3 Comparative Static Change in $\Gamma_E$

**Proposition 3.** ***Change in*** $\Gamma_E \Rightarrow$ ***No Change in*** $N_M^*$**.** *Suppose the matching aggregator in monopolistic recruiting markets is Dixit-Stiglitz, and assume that* $\rho = 1$ *and* $\omega = 1$*. A change in the exogenous component of entry costs* $\Gamma_E$ *does not affect the equilibrium number of monopolistic recruiters,*

$$\frac{\partial N_M^*}{\partial \Gamma_E} = 0, \tag{55}$$

*in which the asterisk denotes the optimal value.*

*Proof.* See Appendix H. □

The intuition behind Proposition 3 is the perfect substitutability between participating in intermediated monopolistically-competitive submarkets and random search markets. The perfect

[13] We again refer to Bilbiie, Ghironi, and Melitz (2008) for a useful review.

substitutability holds for both labor suppliers, due to the nature of the household marginal rate of substitution $\frac{h'(lfp)}{u'(c)}$, which, around the household's optimality conditions, is independent of which labor market (intermediated or non-intermediated) the marginal family member actively searches in, and labor demanders, due to the linearity of total vacancy posting costs in both intermediated matching and non-intermediated matching markets. This substitution occurs due to equilibration of job-filling and job-finding probabilities across the two channels in which new job matches are created. Instead, if total vacancy posting costs for either or both intermediated and non-intermediated matching markets displayed curvature, then a change — say, a decline — in the entry cost $\Gamma_E$ would cause, all else equal, a change in the measure $N_M$ of monopolistic recruiters. However, since our work focuses only on steady-state outcomes, it is without loss of generality to maintain linear vacancy posting costs.[14]

As mentioned above, the results in Proposition 2, Corollary 1, and Proposition 3 are for the static version of the model, not for the steady state of the fully dynamic model. Proving the results for the steady state of the fully dynamic model is more difficult, but the economic insights of the "static" analytical results remain intact, as shown quantitatively in Section 5.

# 5 General Equilibrium II — Quantitative Results

This section provides some numerical support for the Propositions stated in Section 4 as well as brief discussion on the Beveridge Curve the model yields. The calibration described next follows that of general equilibrium labor search and matching models.

## 5.1 Empirical Targets and Calibration

We assume log utility with respect to consumption, $u(c) = \log c$. In turn, the disutility from participation is given by $h(lfp) = \kappa \cdot lfp^{1+\frac{1}{\iota}} / \left(1 + \frac{1}{\iota}\right)$, where $\kappa, \iota > 0$. The goods production function is Cobb-Douglas, $f(k, n) = k^{\alpha} n^{1-\alpha}$, with $0 < \alpha < 1$. All matching functions are also Cobb Douglas, $m(s, v) = \overline{m} \cdot s^{\xi} v^{1-\xi}$ and $m(s_N, v_N) = \overline{m}_N \cdot (s_N)^{\xi} (v_N)^{1-\xi}$, in which $\xi$ is the matching elasticity with respect to active jobs searchers and $\overline{m}$ and $\overline{m}_N$ denote, respectively, the exogenous matching efficiency parameters in the intermediated and non-intermediated labor market. This implies that the matching probabilities in the non-intermediated labor market are given by $k_N^h = m(s_N, v_N)/s_N$ and $k_N^f = m(s_N, v_N)/v_N$. The corresponding matching probabilities in the intermediated labor market take into account the increasing-returns-to-scale nature of the market, so that $k^h = m(s, v)/s$ and $k^f = m(s, v)/v$.

[14] In business-cycle analysis, the departure from the average cost and the marginal cost of posting the marginal vacancy can matter in matching cyclical components of labor markets — this point (to the best of our knowledge) was first emphasized by Pissarides (2009).

A period in the model represents a quarter. Following the search and matching and macroeconomics literatures, we set the capital share to $\alpha = 0.40$, the subjective discount factor to $\beta = 0.99$, the capital depreciation rate $\delta = 0.02$, and the participation elasticity parameter $\iota = 0.18$ (Arseneau and Chugh (2012)). Turning to the labor market parameters, we set the quarterly exogenous separation probability to $\rho = 0.10$, the matching elasticity $\xi = 0.40$, and the Nash bargaining power for workers in non-intermediated labor markets to $\eta = 0.40$. We normalize steady-state aggregate productivity $z$ to 1.

The novel block of the model is monopolistically-competitive intermediation in one of the two matching markets. We set the exit rate of recruiting firms to $\omega = 0.05$. For the matching aggregator function, we set the elasticity of substitution $\varepsilon = 6$ (which results in a 20 percent steady-state markup in the recruiting sector) when using Dixit-Stiglitz and, when using the translog case, we calibrate the translog parameter $\sigma$ to target the same 20 percent steady-state net markup, and the potential space of the universe of recruiting agencies $\tilde{N}_M$ to a sufficiently high value so that it affects neither the model's steady state nor its dynamics.[15]

Regarding cost parameters, we set $\gamma = \gamma_N$ as the baseline case. For the baseline case, we calibrate $\gamma(= \gamma_N)$, $\chi$, $\kappa$, $\overline{m}$, and $\overline{m}_N$ to jointly match the following steady-state targets: a job-finding probability in the non-intermediated market of 0.6, a job-filling probability in the non-intermediated market of 0.70, a labor-force participation rate of 0.74, a value for unemployment benefits that is 40 percent of average wages, and a share of intermediated-market matches in total matches of 40 percent.[16] Finally, the steady-state parameter $\Gamma_E$, as discussed in Section 2 regarding the free-entry condition in the recruiting sector (and along the lines of BGM), we set, without loss of generality, $\Gamma_E = 1$. The calibration of these three costs implies that the total resource cost from vacancy postings and recruiting-firm creation is close to 5 percent of total output. For ease of reference, Table 4 summarizes the baseline parameters. A point that will be discussed further in Section 6 is that the baseline parameters are *not* chosen in a way that purposefully allows for endogeneity of the intermediated labor market. Endogeneity of the monopolistic recruiting sector is an inherent property of the model (both in general equilibrium and in the partial equilibrium labor market).

### 5.2 Steady-State Analysis

To understand how intermediated and non-intermediated matching interact with each other and affect labor market and macro outcomes, we briefly consider several comparative static experiments

[15]The precise settings are $\sigma = 52.4$ and $\tilde{N}_M = 10^8$; the latter orders of magnitude larger than needed so that its precise setting does affect the model's steady state.

[16]The benchmark values for the posting cost that arises for Dixit-Stiglitz aggregation and translog aggregation are very similar – we obtain $\gamma = 3.57$ for Dixit-Stiglitz aggregation and $\gamma = 3.58$ for translog aggregation.

| Parameter | Value | Description |
|---|---|---|
| | | Utility |
| $\beta$ | 0.99 | Quarterly subjective discount factor |
| $\iota$ | 0.18 | Wage elasticity of $lfp$ |
| | | Goods Production |
| $\alpha$ | 0.40 | Elasticity of Cobb-Douglas goods production function $f(k, n)$ with respect to $k$ |
| $\delta$ | 0.02 | Quarterly depreciation rate of physical capital |
| | | Labor Market |
| $\rho$ | 0.10 | Quarterly exogenous separation of jobs |
| $\xi$ | 0.40 | Elasticity of Cobb-Douglas matching technology $m(s, v)$ with respect to $s$ |
| $\eta$ | 0.40 | Generalized Nash bargaining power for workers in non-intermediated labor markets |

Table 4: **Baseline Parameters.**

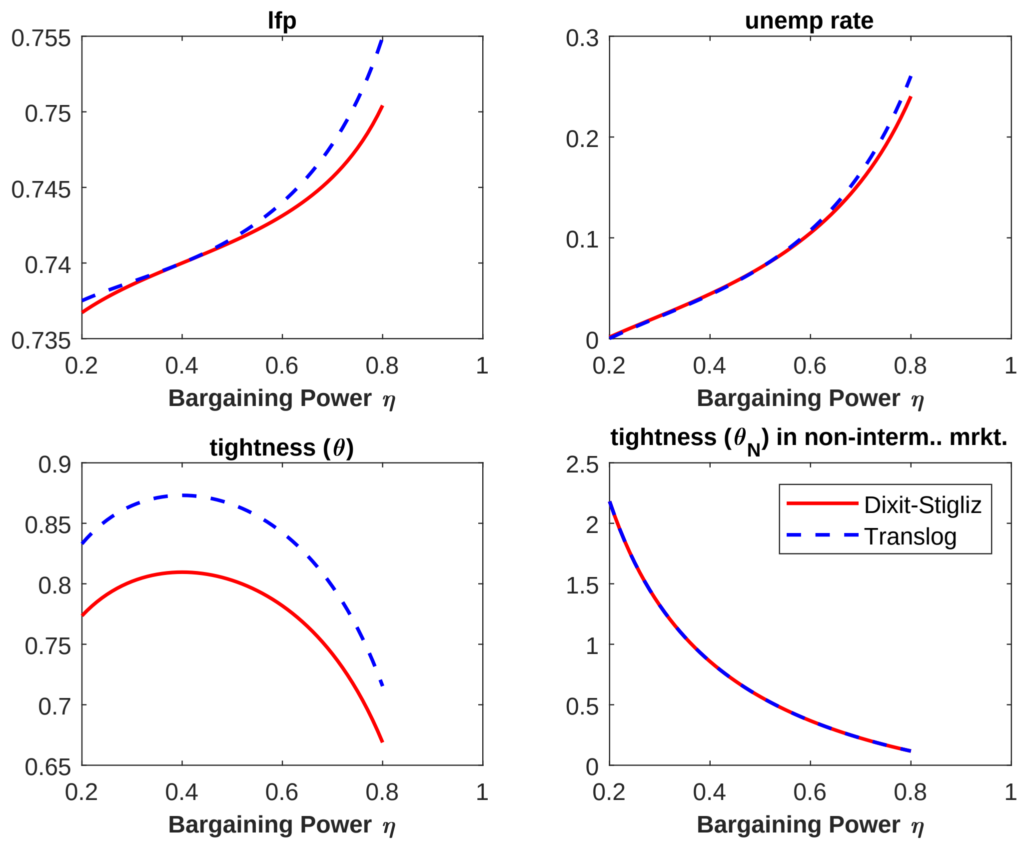


Figure 3: **Steady state as function of worker Nash wage bargaining power $\eta$ in non-intermediated labor market I.** Solid red line is Dixit-Stiglitz aggregation, dashed blue line is translog aggregation. All other parameters besides $\eta$ are held at their baseline values.

in key sets of parameters: the Nash bargaining power of workers in non-intermediated matches, the nature of the long-run Beveridge Curve and how increasing returns in matching acts as an endogenous shifter, and the implications of a secular decline in the cost of entering monopolistically competitive recruiting markets.

**Nash Bargaining Power of Workers — Non-Intermediated Matching.**

As shown in Figures 3, 4, and 5, larger bargaining power for workers generates monotonic increases in unemployment and search in non-intermediated labor markets as well as monotonic reductions in vacancy postings and market tightness. These results are well known from standard search models and are intuitive: higher bargaining power implies that households extract a larger share of the surplus from employment relationships, which leads to not only increased household search behavior but also to a reduction in firms' incentive to create vacancies and ultimately sectoral market tightness.

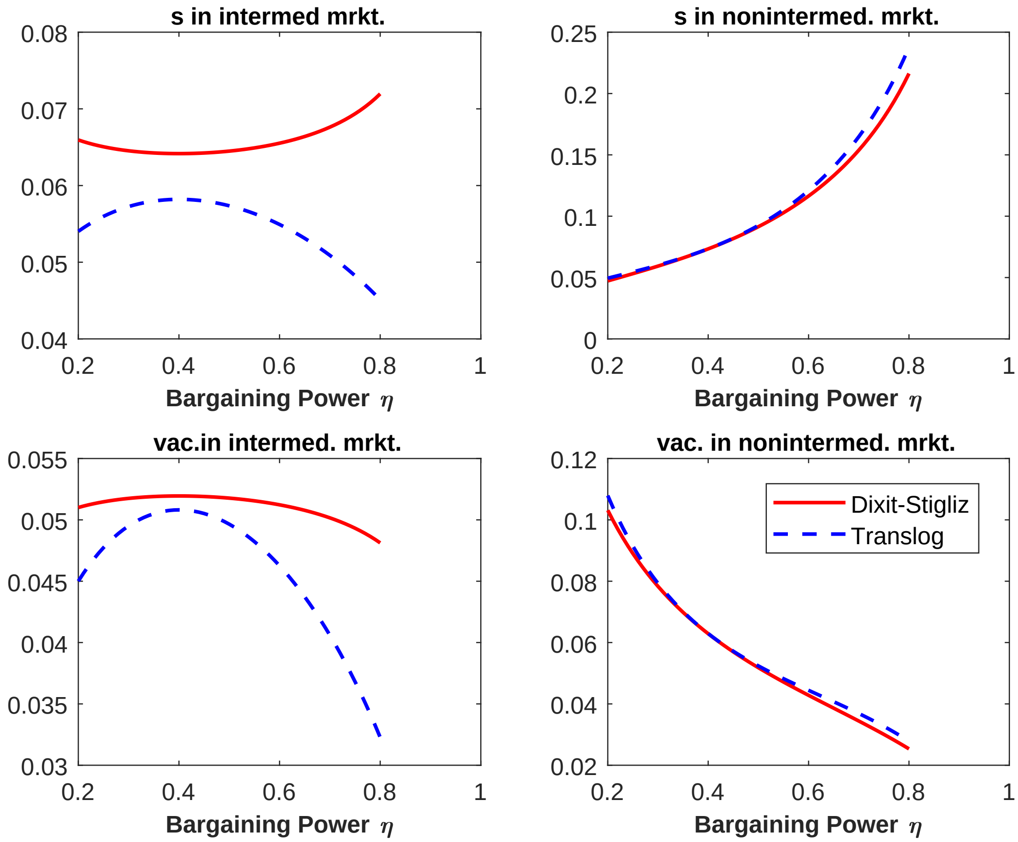


Figure 4: **Steady state as function of worker Nash wage bargaining power $\eta$ in non-intermediated labor market II.** Solid red line is Dixit-Stiglitz aggregation, dashed blue line is translog aggregation. All other parameters besides $\eta$ are held at their baseline values.

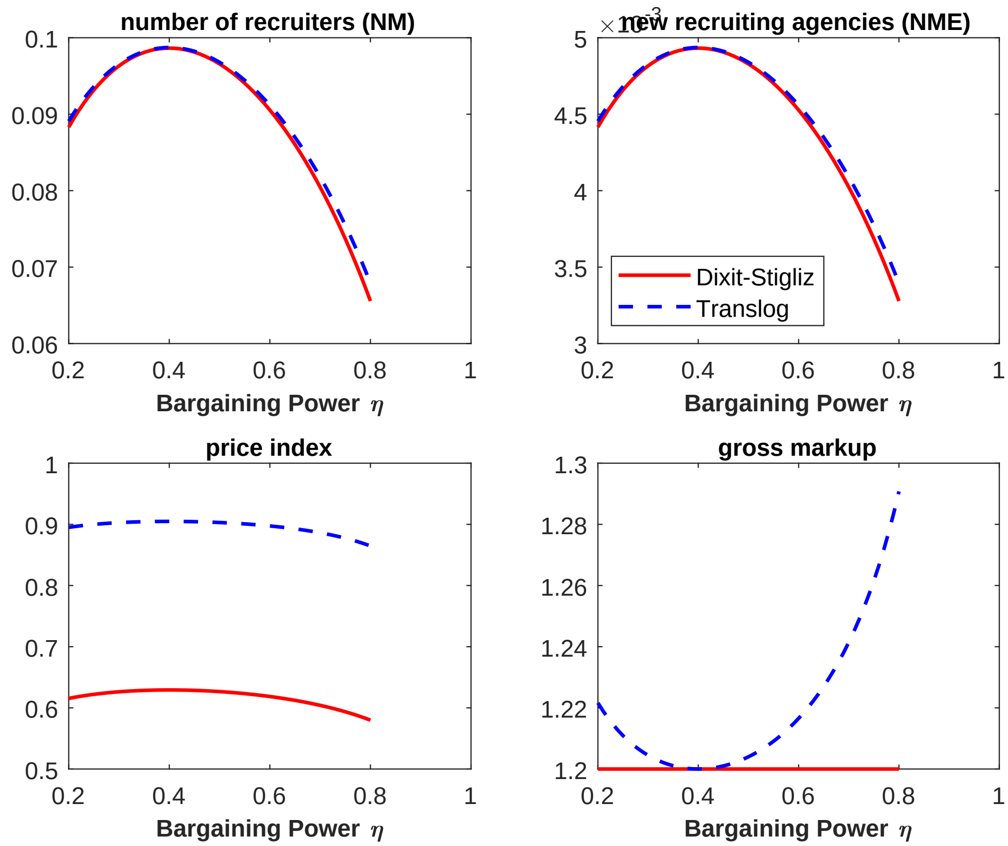


Figure 5: **Steady state as function of worker Nash wage bargaining power $\eta$ in non-intermediated labor market III.** Solid red line is Dixit-Stiglitz aggregation, dashed blue line is translog aggregation. All other parameters besides $\eta$ are held at their baseline values.

In contrast, with the exception of labor force participation, the intermediated labor market exhibits non-monotonic changes in its corresponding variables. More importantly, the sign of the slope changes *when the Hosios condition in the non-intermediated market holds* ($\xi = \eta = 0.4$), as was formalized (albeit in a static framework) in Proposition 2 and Corollary 1. As discussed in Section 4, for low bargaining power, the number of recruiting firms, labor-market tightness intermediated markets, and vacancies are all increasing in the bargaining power of workers. Conversely, for high levels of bargaining power, all three of these variables are decreasing in bargaining power.

**Beveridge Curve.**

A common view in the macro-search literature is that inclusion of endogenous labor force participation leads to a counterfactual *positive* relationship between actively searching unemployed individuals and job vacancies.[17] As pointed out by Arseneau and Chugh (2012, p. 944), the intuition behind this *misleading* view is that an expansion in long-run productivity $z$ creates incentives for both individuals to increase job search activities and firms to increase recruiting activities. However, it is not the case that all models that allow for endogeneity in all three pools of individuals — employed, unemployed but actively searching, and outside the labor force — lead to a counterfactual Beveridge Curve. Instead, it is the particular way in which endogenous participation is modeled. Our model of endogenous participation, which is exactly the same as in Arseneau and Chugh (2012), leads to a downward-sloping locus in $v - ue$ space, as shown in Figure 6. We think this point is an important one for the macro-labor literature.

Given the existence of the Beveridge Curve in our model, changes in the measure of recruiters $N_M$ act as a shifter of the Beveridge function. Stated differently, endogenous developments in the recruiting sector act like "matching efficiency" shocks, one consequence of which is movements of the Beveridge Curve.

**Costs of Entry in Recruiting Market.**

Proposition 3 states that there is no effect on the measure of monopolistically-competitive recruiting firms given a change in the exogenous component $\Gamma_E$ of costs of entering the recruiting market. The intuition behind the result is the perfect substitutability of participation between intermediated and non-intermediated matching. Figure 7 visualizes this intuition. More precisely, the changes in both $s$ and $v$ occur in such a way that labor market tightness in both the intermediated market and the non-intermediated market — respectively, $\theta$ and $\theta_N$, which are shown in the third row of Figure 7 — remain constant, which in turn implies that job-filling and job-finding probabilities, which depend only on market tightness given constant returns in the matching technology, also remain constant.

# 6 Discussion

**Existence of Middlemen.**

One potential criticism of our model is that it does not endogenize the emergence of potentially "costly" labor-market intermediation. This criticism is somewhat misleading given the aggregate

[17] An early study that made this point was Tripier (2003).

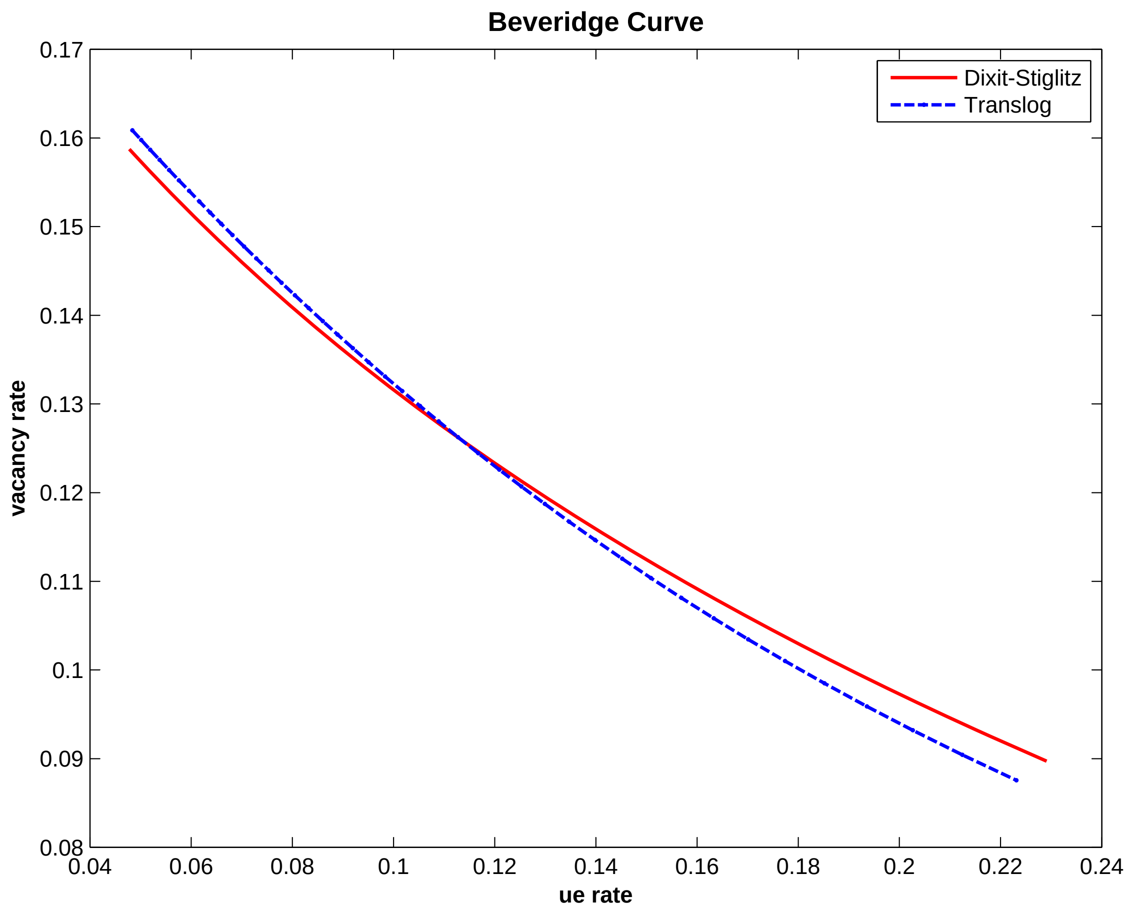


Figure 6: **Steady-State Beveridge Curve.** As the outside option $\chi$ (government-provided unemployment benefits) varies (in the range of 10 percent of wages to 80 percent of wage), a steady-state Beveridge Curve is traced out. Using the Bureau of Labor Statistics' definition of the vacancy rate (plotted on the vertical axis) the vacancy rate is defined as $\frac{v \cdot N_M \cdot m(s,v) + v_N \cdot m(s_N, v_N)}{n + v \cdot N_M \cdot m(s,v) + v_N \cdot m(s_N, v_N)}$. Solid red line is Dixit-Stiglitz aggregation, dashed blue line is translog aggregation.

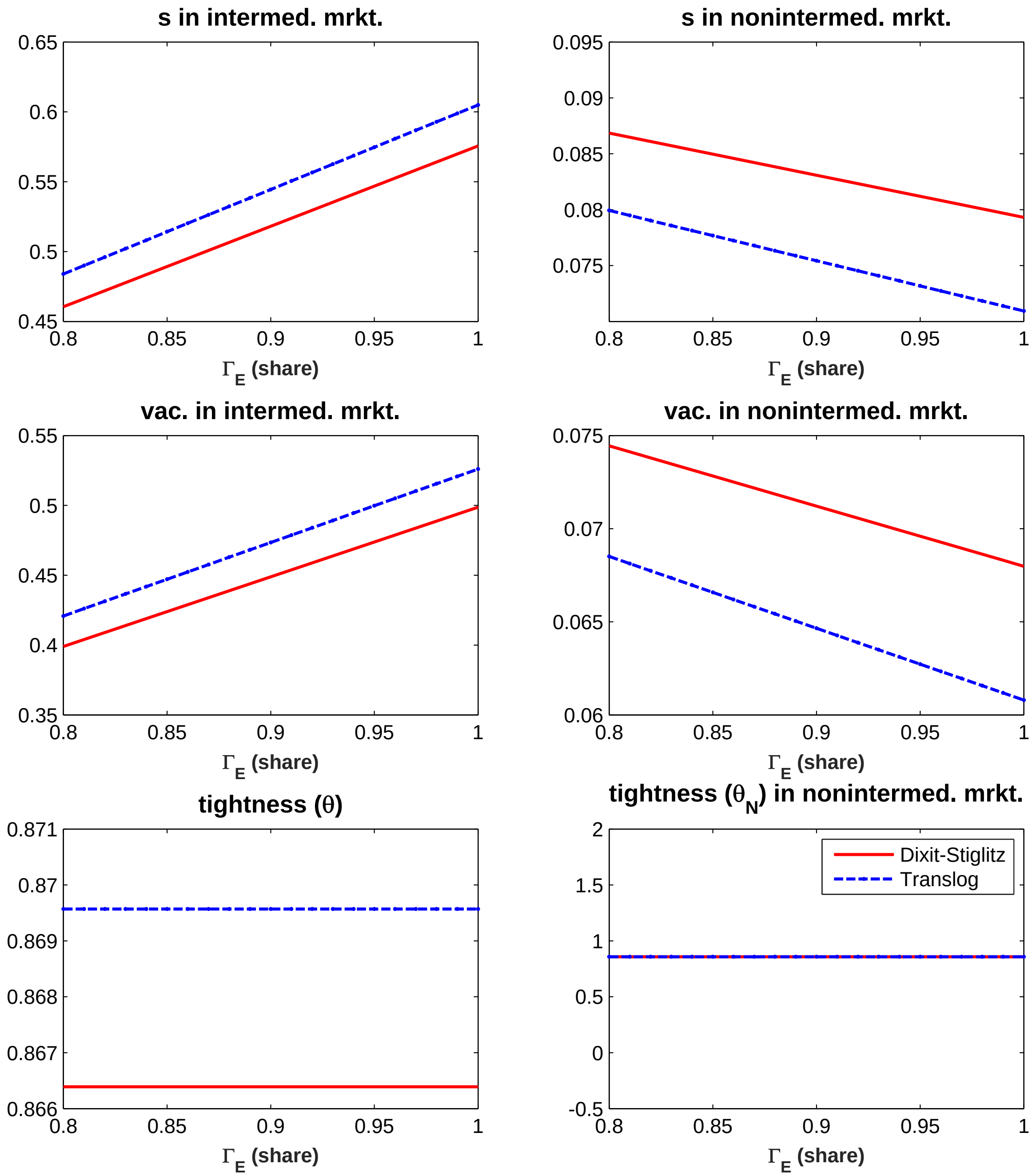


Figure 7: **Steady state as function of $\Gamma_E$.** As the exogenous component $\Gamma_E$ of the entry cost declines to 80 percent of its benchmark value (left side of the horizontal axis), both active search and vacancy postings shift from the intermediated recruiting market to the random matching market in such a way that labor market tightness (shown in the third row) in both the intermediated (bottom left panel) and the non-intermediated (bottom right panel) market remains constant. Solid red line is Dixit-Stiglitz aggregation, dashed blue line is translog aggregation.

increasing returns in intermediated matching. There is no reason that *both* an intermediated labor market that features aggregate increasing returns *and* a non-intermediated labor market cannot co-exist *as long as matching probabilities appropriately adjust* between intermediated and non-intermediated labor markets. Matching probabilities across intermediated and non-intermediated labor markets *do* adjust in our model.[18] As but one example in which probabilities do not appropriately adjust, suppose that, for unmodeled reasons outside the scope of this framework, wages in the intermediated sector are "rigid" over time. The wage rigidity would cause a failure in matching probabilities in the intermediated sector to appropriately adjust. In this case, it is clear that (as long as outcomes such as, say, labor rationing do not occur) the existence of the "middlemen" sector is a waste of resources and would therefore shut down.

**Relation to Literature.**

Relative to the existing literature on intermediaries — several prominent examples of which are Rubinstein and Wolinksy (1987), Masters (2007), Wright and Wong (2014), Nosal, Wong, and Wright (2015), Gautier, Hu, and Watanabe (2016), and Farboodi, Jarosch, and Shimer (2017) — we stress our focus on market-structure imperfections in intermediated labor markets and endogenous entry among recruiting firms in the latter in a general equilibrium environment.[19] Importantly, our framework emphasizes endogenous entry among intermediaries, whereas the existing literature has generally modeled whether individuals become intermediaries instead of goods producers. Given our interest in labor markets, it is natural to consider the creation of labor market intermediaries through the lens of firm creation. As a result, our modeling approach centered on recruiting-firm entry complements existing theoretical work on middlemen and intermediaries.

More specifically, we put forth four main new results relative to existing work. First, the employment surplus between production firms and workers when matches take place via intermediated markets is influenced by the competitiveness of the recruiting sector, with important implications for wages and therefore the incentive to search and post vacancies. Second, the presence of endogenous entry in the monopolistically-competitive recruiting sector gives rise to increasing returns to scale in intermediated-based matching. While this is, in a broad sense, related to the environment in Masters (2007) where, under increasing returns, the matching rate is increasing in the number of people who participate in the market, our framework instead posits that the matching probabilities for production firms and workers in intermediated labor markets depends on the measure of

[18]Note that "appropriate" adjustment is not synonymous with "efficient" adjustment.

[19]Farboodi, Jarosch, and Shimer (2017) highlight conditions under which intermediaries may arise in equilibrium. Other related studies on intermediation include Hall and Rust (2002) and Hendrickson (2016), who rationalize the existence of a minimum wage in a model where unions arise as middlemen, Gautier, Hu, and Watanabe (2016), who show that middlemen can arise in a directed search environment, and, among others, Chang and Zhang (2016) and Gregor and Menzio (2016).

intermediaries (i.e., recruiting firms) in addition to the number of individuals in the market (i.e., searchers and firm vacancies). Then, the degree of increasing returns is intimately connected to both the cost of entering the recruiting sector and the cost of posting vacancies in intermediated markets. This differs from the environment in Masters (2007).

Third, focusing on our quantitative application, we stress that the behavior of intermediated labor markets is affected by the degree of efficiency in non-intermediated labor markets, with important implications for sectoral and overall labor market conditions (i.e., labor market tightness), unemployment, and participation.

Perhaps closest to our work is Nosal, Wong, and Wright (2015), who establish the conditions under which middlemen arise as well as the characteristics under which efficiency is obtained in an environment with production. More specifically, their framework describes the distinct cases under which intermediaries arise in equilibrium (and, importantly, the cases under which intermediaries may be irrelevant). Moreover, the authors show that an appropriate calibration of bargaining powers (a la Hosios) amid endogenous production can lead to efficiency, and that deviations from Hosios generally lead to inefficient outcomes. Our work differs from Nosal, Wong, and Wright (2015) in two main ways. Relative to their environment, we allow for the coexistence of intermediated and non-intermediated markets which, importantly, implies that the Hosios condition alone may be insufficient amid monopolistic competition (and positive entry costs) in intermediated matching markets. Second, our work explicitly highlights a surplus sharing rule where the surplus from employment relationships is directly influenced to the degree of competitiveness of intermediated recruiting markets. In particular, this result suggests interesting and potentially important implications for optimal policy as standard policy instruments that tackle inefficiencies may not be easily implementable (an issue we leave for future work).

# 7 Conclusion

Labor market intermediaries are playing an increasingly relevant role in job matching. This paper constructs a monopolistically-competitive intermediated recruiting sector with endogenous entry of recruiters to answer several important questions. A central result is that there are aggregate increasing returns in new job creation. This analytical result is derived in the partial equilibrium of the labor market; intuitively, the increasing returns expands (in symmetric equilibrium) the total surplus of every new job created by the monopolistically-competitive recruiting sector. In the general equilibrium environment, aggregate increasing returns expands not only the total surplus of every new job match created by the recruiting sector, but also — because the increasing returns term appears in the economy's goods resource frontier — the set of available resources.

We show that deviations from efficiency in non-intermediated markets have important implications for the behavior of unemployment and labor force participation, thereby highlighting the relevance of understanding the behavior of intermediated labor markets for aggregate labor market outcomes. Our framework is tractable enough to be used to explore several additional experiments, including the implications of an expanding recruiting sector for unemployment fluctuations, the role of differential changes in hiring costs across intermediated and non-intermediated markets for unemployment dynamics, and both labor market policy and optimal fiscal policy. We plan to explore these and other issues in future work.

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# A Surplus Sharing (Proof of Proposition 1)

This Appendix provides the proof for Proposition 1.

## A.1 Marginal Profit with respect to $s_{ijt}$

Recall that recruiting firm $ij$'s value function is given by

$$\mathbf{V}_{Mij}(s_{ijt}, v_{ijt}; \cdot) = \max \{\rho_{ijt} \cdot m(s_{ijt}, v_{ijt}) - mc_{jt} \cdot m(s_{ijt}, v_{ijt})\}. \tag{56}$$

Recruiting firm $ij$'s marginal profit condition with respect to $s_{ijt}$ is

$$\begin{aligned} \frac{\partial \mathbf{V}_{Mij}(s_{ijt}, v_{ijt}; \cdot)}{\partial s_{ijt}} &= \rho_{ijt} \cdot m_s(s_{ijt}, v_{ijt}) - mc_{jt} \cdot m_s(s_{ijt}, v_{ijt}) \\ &= (\rho_{ijt} - mc_{jt}) \cdot \xi \cdot k^h(\theta_{ijt}), \end{aligned} \tag{57}$$

in which the second line uses the Cobb-Douglas matching function.[20] As per Moen (1997), recruiting firm $ij$ chooses $w_{ijt}$ and $\theta_{ijt}$ to optimize

$$\begin{aligned} &(\rho_{ijt} - mc_{jt}) \cdot \xi \cdot k^h(\theta_{ijt}) + \varphi^f_{ijt} \cdot \left[\gamma - p_{v_{jt}} - k^f(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}) - \mathbf{X}^F\right] \\ &+ \ 1 \cdot \left[p_{s_{jt}} + k^h(\theta_{ijt}) \cdot \mathbf{W}(w_{ijt}) + (1 - k^h(\theta_{ijt})) \cdot \mathbf{U}_t - \mathbf{X}^H\right], \end{aligned} \tag{58}$$

with $\varphi^f_{ijt}$ and 1 being the respective Lagrange multipliers on attracting vacancies towards submarket $ij$ and on attracting actively searching individuals towards submarket $ij$.[21]

The first-order conditions with respect to $w_{ijt}$ and $\theta_{ijt}$ are

$$-\varphi^f_{ijt} \cdot k^f(\theta_{ijt}) \cdot \frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}} + k^h(\theta_{ijt}) \cdot \frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}} = 0, \tag{59}$$

and

$$(\rho_{ijt} - mc_{jt}) \cdot \xi \cdot \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} - \varphi^f_{ijt} \cdot \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \cdot \mathbf{J}(w_{ijt}) + \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = 0. \tag{60}$$

Noting that $\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}} = -1$ and $\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}} = 1$ in our model, the multiplier $\varphi^f_{ijt}$ is

$$\begin{aligned} \varphi^f_{ijt} &= -\frac{k^h(\theta_{ijt})}{k^f(\theta_{ijt})} \\ &= -\theta_{ijt}, \end{aligned} \tag{61}$$

[20]For ease of reference, the Cobb-Douglas matching function relationships are $m_s(s_{ijt}, v_{ijt}) = \xi \theta_{ijt}^{1-\xi}$, $k^f(\theta_{ijt}) = \theta_{ijt}^{-\xi}$, and $k^h(\theta_{ijt}) = \theta_{ijt}^{1-\xi}$.

[21]It is without of generality to normalize one of the multipliers due to the constant-returns matching function.

in which the second line follows due to Cobb-Douglas matching. Substituting $\varphi^f_{ijt}$ in (60) gives

$$(\rho_{ijt} - mc_{jt}) \cdot \xi \cdot \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} + \theta_{ijt} \cdot \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \cdot \mathbf{J}(w_{ijt}) + \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = 0,$$

which, after substituting the Cobb-Douglas expressions $\frac{\partial k^h(\theta)}{\partial \theta}$ and $\frac{\partial k^f(\theta)}{\partial \theta}$ gives

$$(\rho_{ijt} - mc_{jt}) \cdot \xi \cdot (1-\xi)\theta_{ijt}^{-\xi} - \xi \theta_{ijt} \cdot \theta_{ijt}^{-\xi-1} \cdot \mathbf{J}(w_{ijt}) + (1-\xi)\theta_{ijt}^{-\xi} \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = 0,$$

Dividing this expression by $(1-\xi)\,\theta_{ijt}^{-\xi}$ and slightly rearranging gives the surplus sharing rule

$$\xi \cdot (1-\xi) \cdot (\rho_{ijt} - mc_{jt}) + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{62}$$

If the matching aggregator were of Dixit-Stiglitz form, the surplus-sharing condition is

$$\xi \cdot (1-\xi) \cdot \frac{1}{\varepsilon} \underbrace{N_{Mjt}^{\frac{1}{\varepsilon-1}}}_{=\rho(N_{Mjt})} + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{63}$$

If the matching aggregator were of Benassy form, the surplus-sharing condition is

$$\xi \cdot (1-\xi) \cdot \frac{1}{\varepsilon} \underbrace{N_{Mjt}^{\varphi}}_{=\rho(N_{Mjt})} + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{64}$$

If the matching aggregator were translog, the surplus-sharing condition is

$$\xi \cdot (1-\xi) \cdot \left( \frac{(\sigma N_{Mt})^{-1}}{1 + (\sigma N_{Mt})^{-1}} \right) \cdot \underbrace{\exp\left( -\frac{1}{2} \cdot \frac{\tilde{N}_M - N_{Mt}}{\sigma \tilde{N}_M N_{Mt}} \right)}_{=\rho(N_{Mjt})} + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{65}$$

## A.2 Marginal Profit with Respect to $v_{ijt}$

Recruiting firm $ij$'s marginal profit condition with respect to $v_{ijt}$ is

$$\begin{aligned}\frac{\partial \mathbf{V}_{ij}^{M}(s_{ijt}, v_{ijt}; \cdot)}{\partial v_{ijt}} &= \rho_{ijt} \cdot m_v(s_{ijt}, v_{ijt}) - mc_{jt} \cdot m_v(s_{ijt}, v_{ijt}) \\ &= (\rho_{ijt} - mc_{jt}) \cdot (1-\xi) \cdot k^f(\theta_{ijt}). \qquad (66)\end{aligned}$$

As per Moen (1997), recruiting firm $ij$ chooses $w_{ijt}$ and $\theta_{ijt}$ to optimize

$$\begin{aligned}&(\rho_{ijt} - mc_{jt}) \cdot (1-\xi) \cdot k^f(\theta_{ijt}) + 1 \cdot \left[\gamma - p_{v_{jt}} - k^f(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}) - \mathbf{X}^F\right] \\ &+ \ \varphi_{ijt}^h \cdot \left[p_{s_{jt}} + k^h(\theta_{ijt}) \cdot \mathbf{W}(w_{ijt}) + (1 - k^h(\theta_{ijt})) \cdot \mathbf{U}_t - \mathbf{X}^H\right], \qquad (67)\end{aligned}$$

with 1 and $\varphi_{ijt}^h$ the respective Lagrange multipliers on attracting vacancies towards submarket $ij$ and on attracting actively searching individuals towards submarket $ij$.[22]

The first-order conditions with respect to $w_{ijt}$ and $\theta_{ijt}$ are

$$-k^f(\theta_{ijt}) \cdot \frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}} + \varphi_{ijt}^h \cdot k^h(\theta_{ijt}) \cdot \frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}} = 0 \qquad (68)$$

and

$$(\rho_{ijt} - mc_{jt}) \cdot (1-\xi) \cdot \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} - \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \cdot \mathbf{J}(w_{ijt}) + \varphi_{ijt}^h \cdot \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = 0. \quad (69)$$

Noting that $\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}} = -1$ and $\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}} = 1$ in our model, the multiplier $\varphi_{ijt}^h$ is

$$\begin{aligned}\varphi_{ijt}^h &= -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})} \\ &= -\theta_{ijt}^{-1}, \qquad (70)\end{aligned}$$

in which the second line follows due to Cobb-Douglas matching.

Substituting $\varphi_{ijt}^h$ in (69) gives

$$(\rho_{ijt} - mc_{jt}) \cdot (1-\xi) \cdot \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} - \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \cdot \mathbf{J}(w_{ijt}) - \theta_{ijt}^{-1} \cdot \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = 0,$$

which, after substituting the Cobb-Douglas expressions $\frac{\partial k^h(\theta)}{\partial \theta}$ and $\frac{\partial k^f(\theta)}{\partial \theta}$ gives

$$-(\rho_{ijt} - mc_{jt}) \cdot (1-\xi) \cdot \xi \theta_{ijt}^{-\xi-1} + \xi \theta_{ijt}^{-\xi-1} \cdot \mathbf{J}(w_{ijt}) - (1-\xi) \cdot \theta_{ijt}^{-1} \cdot \theta_{ijt}^{-\xi} \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = 0.$$

[22] It is without of generality to normalize one of the multipliers due to the constant-returns matching function.

Dividing this expression by $(1-\xi)\,\theta_{ijt}^{-\xi-1}$ and slightly rearranging gives the surplus sharing rule

$$\xi \cdot (1-\xi) \cdot (\rho_{ijt} - mc_t) + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{71}$$

If the matching aggregator were of Dixit-Stiglitz form, the surplus-sharing condition is

$$\xi \cdot (1-\xi) \cdot \frac{1}{\varepsilon} \underbrace{N_{Mjt}^{\frac{1}{\varepsilon-1}}}_{=\rho(N_{Mjt})} + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{72}$$

If the matching aggregator were of Benassy form, the surplus-sharing condition is

$$\xi \cdot (1-\xi) \cdot \frac{1}{\varepsilon} \underbrace{N_{Mjt}^{\varphi}}_{=\rho(N_{Mjt})} + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{73}$$

If the matching aggregator were translog, the surplus-sharing condition is

$$\xi \cdot (1-\xi) \cdot \underbrace{\left( \frac{(\sigma N_{Mt})^{-1}}{1 + (\sigma N_{Mt})^{-1}} \right) \cdot \exp\left( -\frac{1}{2} \cdot \frac{\tilde{N}_M - N_{Mt}}{\sigma \tilde{N}_M N_{Mt}} \right)}_{=\rho(N_{Mjt})} + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{74}$$

# B Firms

There is a continuum $[0, 1]$ of identical goods-producing firms. Allowing (for generality) for total vacancy posting cost function to display curvature in total vacancies, the representative goods-producing firm's lifetime profit function is

$$E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left\{ z_t f(k_t, n_t) - r_t k_t - \gamma_N(v_{Nt}) - \int_0^1 \int_0^{N_{Mjt}} \gamma(v_{ijt}) \; di \; dj + \int_0^1 \int_0^{N_{Mjt}} p_{v_{jt}} v_{ijt} \; di \; dj \right\}$$
$$- \quad E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left\{ w_t \cdot (1-\rho) n_{t-1} + w_{Nt} \cdot k^f_{Nt} \cdot v_{Nt} + \int_0^1 \int_0^{N_{Mjt}} w_{ijt} \cdot k^f_{ijt} \cdot v_{ijt} \; di \; dj \right\} \quad (75)$$

subject to the period-$t$ perceived law of motion of employment

$$n_t = (1-\rho) n_{t-1} + k^f_{Nt} \cdot v_{Nt} + \int_0^1 \int_0^{N_{Mjt}} k^f_{ijt} \cdot v_{ijt} \; di \; dj. \quad (76)$$

Defining the Lagrange multiplier on the perceived law of motion (76) as $\mu_t$, the first-order conditions with respect to $k_t$, $v_{ijt}$, $v_{Nt}$, and $n_t$ are

$$z_t f_k(k_t, n_t) - r_t = 0, \quad (77)$$

$$\mu_t \cdot k^f_{ijt} - \gamma'(v_{ijt}) + p_{v_{jt}} - w_{ijt} \cdot k^f_{ijt} = 0 \; \forall ij, \quad (78)$$

$$\mu_t \cdot k^f_{Nt} - \gamma'_N(v_{Nt}) - w_{Nt} \cdot k^f_{Nt} = 0, \quad (79)$$

and

$$-\mu_t + z_t f_n(k_t, n_t) + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left( \mu_{t+1} - w_{t+1} \right) \right\} = 0. \quad (80)$$

Isolating the multiplier $\mu_t$ from expression (79) gives

$$\mu_t = w_{Nt} + \frac{\gamma'_N(v_{Nt})}{k^f_{Nt}}, \quad (81)$$

and isolating the multiplier $\mu_t$ from expression (78) gives

$$\mu_t = w_{ijt} + \frac{\gamma'(v_{ijt}) - p_{v_{jt}}}{k^f_{ijt}} \; \forall ij. \quad (82)$$

Substituting the value for $\mu_t$ from (82) into (80) gives

$$\frac{\gamma'(v_{ijt})}{k^f_{ijt}} = z_t f_n(k_t, n_t) - w_{ijt} + \frac{p_{v_{jt}}}{k^f_{ijt}} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left( \frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f_{jt+1}} + w_{jt+1} - w_{t+1} \right) \right\} \; \forall ij. \quad (83)$$

Next, substituting the value for $\mu_t$ from (81) into (80) gives

$$\frac{\gamma_N'(v_{Nt})}{k_{Nt}^f} = z_t f_n(k_t, n_t) - w_{Nt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma_N'(v_{Nt+1})}{k_{Nt+1}^f} + w_{Nt+1} - w_{t+1}\right)\right\}. \tag{84}$$

## B.1 Job-Creation Conditions

Without loss of generality, assuming that wages for incumbent employees in the periods after they were first hired (regardless of whether they were first hired through intermediated or non-intermediated labor markets) are identical simplifies the pair of expressions above to

$$\gamma'(v_{ijt}) = p_{v_{jt}} + k_{ijt}^f \cdot \underbrace{\left(z_t f_n(k_t, n_t) - w_{ijt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k_{jt+1}^f}\right\}\right)}_{\equiv \mathbf{J}(w_{ijt})} \quad \forall ij \tag{85}$$

and

$$\gamma_N'(v_{Nt}) = k_{Nt}^f \cdot \underbrace{\left(z_t f_n(k_t, n_t) - w_{Nt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\gamma_N'(v_{Nt+1})}{k_{Nt+1}^f}\right\}\right)}_{\equiv \mathbf{J}(w_{Nt})}, \tag{86}$$

which characterize, respectively, costly job vacancies directed towards any intermediated labor submarket $ij$ and costly job vacancies in the non-intermediated labor market. Around the optimum, the firm is indifferent between directing new job vacancies to intermediated submarket $i$ or intermediated submarket $k$, $k^f(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}) = k^f(\theta_{kjt}) \cdot \mathbf{J}(w_{kjt})$, $\forall i \neq k$.

# C Households

There is a continuum $[0,1]$ of identical households. In each household, there is a continuum $[0,1]$ of family members. In period $t$, each family member in the representative household has a labor-market status of employed, unemployed and actively seeking a job, or being outside the labor force. Regardless of which labor-market status a particular family member is in, each family member receives the same exact amount of consumption $c_t$ due to full risk-sharing within each household (see Andolfatto (1996) for formal details).

The representative household maximizes lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(c_t) - h\left( n_t + \underbrace{(1-k_{Nt}^h)\cdot s_{Nt}}_{=ue_t^N} + \int_0^1 \left( \int_0^{N_{Mjt}} \underbrace{(1-k_{ijt}^h)\cdot s_{ijt}}_{=ue_{ijt}}\ di \right) dj \right) \right], \tag{87}$$

subject to the budget constraint

$$\begin{aligned} c_t + k_{t+1} + T_t = {} & (1+r_t-\delta)k_t + w_t(1-\rho)n_{t-1} + w_{Nt}\cdot k_{Nt}^h \cdot s_{Nt} \\ + & \int_0^1 \int_0^{N_{Mjt}} w_{ijt}\cdot k_{ijt}^h \cdot s_{ijt}\ di\ dj + \int_0^1 \int_0^{N_{Mjt}} p_{s_{jt}} \cdot s_{ijt}\ di\ dj \\ + & (1-k_{Nt}^h)\cdot s_{Nt}\chi + \int_0^1 \int_0^{N_{Mjt}} \left(1-k_{ijt}^h\right)\cdot s_{ijt}\chi\ di\ dj + \int_0^1 \Pi_{jt}^M\ dj + \Pi_t^F, \end{aligned} \tag{88}$$

and the period-$t$ perceived law of motion of employment

$$n_t = (1-\rho)n_{t-1} + k_{Nt}^h \cdot s_{Nt} + \int_0^1 \int_0^{N_{Mjt}} k_{ijt}^h \cdot s_{ijt}\ di\ dj. \tag{89}$$

Defining the Lagrange multiplier on the flow budget constraint as $\lambda_t$ and on the perceived law of motion as $\mu_t$, the first-order conditions with respect to $c_t$, $k_{t+1}$, $n_t$, $s_{Nt}$, and $s_{ijt}$ are

$$u'(c_t) - \lambda_t = 0, \tag{90}$$

$$-\lambda_t + \beta E_t \left\{ \lambda_{t+1}\left(1 + r_{t+1} - \delta\right) \right\} = 0, \tag{91}$$

$$-\mu_t - h'(lfp_t) + \beta(1-\rho)E_t\left\{ \lambda_{t+1} w_{t+1} + \mu_{t+1} \right\} = 0, \tag{92}$$

$$-(1-k_{Nt}^h)\cdot h'(lfp_t) + \lambda_t \cdot \left( k_{Nt}^h \cdot w_{Nt} + (1-k_{Nt}^h)\cdot \chi \right) + \mu_t \cdot k_{Nt}^h = 0, \tag{93}$$

and

$$-(1-k_{ijt}^h)\cdot h'(lfp_t) + \lambda_t \cdot \left( k_{ijt}^h \cdot w_{ijt} + (1-k_{ijt}^h)\cdot \chi \right) + \lambda_t \cdot p_{s_{jt}} + \mu_t \cdot k_{ijt}^h = 0\ \forall ij. \tag{94}$$

Isolating the multiplier $\mu_t$ from (93) gives

$$\frac{\mu_t}{u'(c_t)} = \left(\frac{1-k^h_{Nt}}{k^h_{Nt}}\right)\cdot\left(\frac{h'(lfp_t)}{u'(c_t)} - \chi\right) - w_{Nt}, \tag{95}$$

and isolating the multiplier $\mu_t$ from (94) gives

$$\frac{\mu_t}{u'(c_t)} = \left(\frac{1-k^h_{ijt}}{k^h_{ijt}}\right)\cdot\left(\frac{h'(lfp_t)}{u'(c_t)} - \chi\right) - \frac{p_{s_{jt}}}{k^h_{ijt}} - w_{ijt}\ \forall ij, \tag{96}$$

in which both of these expressions have substituted the marginal utility of income $\lambda_t = u'(c_t)$ from (90).

Substituting the multiplier as stated in expression (95) into (92) yields

$$\begin{aligned}\left(\frac{1-k^h_{Nt}}{k^h_{Nt}}\right)\cdot\left(\frac{h'(lfp_t)}{u'(c_t)} - \chi\right) - w_{Nt} &= -\frac{h'(lfp_t)}{u'(c_t)} \\ + \quad (1-\rho)E_t &\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(w_{t+1} + \left(\frac{1-k^h_{Nt+1}}{k^h_{Nt+1}}\right)\cdot\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right) - w_{Nt+1}\right)\right\}.\end{aligned}$$

Cancelling the $-h'(lfp_t)/u'(c_t)$ terms and multiplying by $k^h_{Nt}$ gives

$$\begin{aligned}\frac{h'(lfp_t)}{u'(c_t)} &= k^h_{Nt}w_{Nt} + (1-k^h_{Nt})\chi \\ + \quad k^h_{Nt}(1-\rho)E_t &\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(w_{t+1} - w_{Nt+1} + \left(\frac{1-k^h_{Nt+1}}{k^h_{Nt+1}}\right)\cdot\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right)\right\}.\end{aligned}$$

Next, substituting the multiplier as stated in expression (96) into (92) and following the same steps of algebra as above yields

$$\begin{aligned}\frac{h'(lfp_t)}{u'(c_t)} &= p_{s_{jt}} + k^h_{ijt}w_{ijt} + (1-k^h_{ijt})\chi \\ + \quad k^h_{ijt}(1-\rho)E_t &\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(w_{t+1} - w_{jt+1} + \left(\frac{1-k^h_{jt+1}}{k^h_{jt+1}}\right)\cdot\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - p_{s_{jt+1}}\right)\right)\right\}\ \forall ij.\end{aligned}$$

## C.1 Labor Force Participation Conditions

Without loss of generality, assuming that wages for incumbent employees in the periods after they were first hired (regardless of whether they were first hired through intermediated or non-intermediated labor markets) are identical simplifies the pair of expressions above to

$$\frac{h'(lfp_t)}{u'(c_t)} = k^h_{Nt}\left[w_{Nt} + (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(\frac{1-k^h_{Nt+1}}{k^h_{Nt+1}}\right)\cdot\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\}\right] + (1-k^h_{Nt})\chi \tag{97}$$

and

$$\begin{aligned}\frac{h'(lfp_t)}{u'(c_t)} &= p_{s_{jt}} + (1-k^h_{ijt})\chi \\ &+ \quad k^h_{ijt}\left[w_{ijt} + (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(\frac{1-k^h_{jt+1}}{k^h_{jt+1}}\right)\cdot\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - p_{s_{jt+1}}\right)\right\}\right] \ \forall ij,\end{aligned} \tag{98}$$

which characterize, respectively, active job search in the non-intermediated labor market and active job search directed towards intermediated labor submarket $ij$. Given the household-level envelope conditions, around the optimum, active job search in all submarkets must yield the same value $k^h(\theta_{ijt})\cdot\mathbf{W}(w_{ijt}) + \left(1-k^h(\theta_{ijt})\right)\cdot\mathbf{U}(\cdot) = k^h(\theta_{kjt})\cdot\mathbf{W}(w_{kjt}) + \left(1-k^h(\theta_{kjt})\right)\cdot\mathbf{U}(\cdot), \forall i \neq k$.

For use in Appendix D, the participation conditions (97) and (98) can, respectively, be equivalently expressed as

$$\frac{h'(lfp_t) - u'(c_t)\chi}{k^h_{Nt}\cdot u'(c_t)} = w_{Nt} - \chi + (1-\rho)E_t\left\{\Xi_{t+1|t}(1-k^h_{Nt+1})\left(\frac{h'(lfp_{t+1}) - u'(c_{t+1})\chi}{k^h_{Nt+1}\cdot u'(c_{t+1})}\right)\right\} \tag{99}$$

and

$$\begin{aligned}\frac{h'(lfp_t) - u'(c_t)\chi - u'(c_t)\cdot p_{s_{jt}}}{k^h_{ijt}\cdot u'(c_t)} &= w_{ijt} - \chi \\ &+ \quad (1-\rho)E_t\left\{\Xi_{t+1|t}(1-k^h_{jt+1})\left(\frac{h'(lfp_{t+1}) - u'(c_{t+1})\chi - u'(c_{t+1})\cdot p_{s_{jt+1}}}{k^h_{jt+1}\cdot u'(c_{t+1})}\right)\right\} \ \forall ij.\end{aligned} \tag{100}$$

# D Derivation of Real Wage in Intermediated Market

Recall that the labor force participation condition can be written as

$$\begin{aligned} \frac{h'(lfp_t)}{u'(c_t)} - p_{s_{jt}} &= k^h(\theta_{ijt}) \left[ w_{ijt} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \frac{\mu_{jt+1}}{u'(c_{t+1})} \right\} \right] + (1 - k^h(\theta_{ijt})) \cdot \chi \\ &= k^h(\theta_{ijt}) \cdot \mathbf{W}(w_{ijt}) + (1 - k^h(\theta_{ijt})) \cdot \mathbf{U}_t, \end{aligned} \tag{101}$$

and

$$\mathbf{W}(w_{ijt}) - \mathbf{U}_t = \frac{h'(lfp_t) - u'(c_t) \cdot \chi - u'(c_t) \cdot p_{s_{jt}}}{k^h(\theta_{ijt}) \cdot u'(c_t)}. \tag{102}$$

In turn, the job creation condition is given by

$$\frac{\gamma'(v_{ijt}) - p_{v_{jt}}}{k^f(\theta_{ijt})} = \underbrace{z_t f_n(k_t, n_t) - w_{ijt} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right\}}_{=\mathbf{J}(w_{ijt})}. \tag{103}$$

In recursive form, the surplus earned by the household is

$$\mathbf{W}(w_{ijt}) - \mathbf{U}_t = w_{ijt} - \chi + (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot (\mathbf{W}(w_{jt+1}) - \mathbf{U}_{t+1}) \right\}, \tag{104}$$

and the surplus earned by the goods-producing firm is

$$\mathbf{J}(w_{ijt}) = z_t f_n(k_t, n_t) - w_{ijt} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \mathbf{J}(w_{jt+1}) \right\}. \tag{105}$$

Inserting expression (104) into the surplus-sharing condition

$$\xi \cdot (\rho_{ijt} - mc_t) + \mathbf{W}(w_{ijt}) - \mathbf{U}_t = \left( \frac{\xi}{1-\xi} \right) \cdot \mathbf{J}(w_{ijt}) \tag{106}$$

gives

$$\begin{aligned} &\xi \cdot (\rho_{ijt} - mc_t) + w_{ijt} - \chi \\ &+ (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot (\mathbf{W}(w_{jt+1}) - \mathbf{U}_{t+1}) \right\} = \left( \frac{\xi}{1-\xi} \right) \mathbf{J}(w_{ijt}). \end{aligned} \tag{107}$$

Next, using the period-$t+1$ sharing rule gives

$$\begin{aligned} &\xi \cdot (\rho_{ijt} - mc_t) + w_{ijt} - \chi \\ &+ (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left[ \left( \frac{\xi}{1-\xi} \right) \mathbf{J}(w_{jt+1}) - \xi \cdot (\rho_{jt+1} - mc_{t+1}) \right] \right\} \\ &= \left( \frac{\xi}{1-\xi} \right) \mathbf{J}(w_{ijt}). \end{aligned} \tag{108}$$

Substituting $\mathbf{J}(w_{ijt}) = \frac{\gamma'(v_{ijt}) - p_{v_{jt}}}{k^f(\theta_{ijt})}$ and $\mathbf{J}(w_{jt+1}) = \frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})}$ yields

$$\begin{aligned}
&\xi \cdot (\rho_{ijt} - mc_t) + w_{ijt} - \chi \\
&+ \quad (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left[ \left( \frac{\xi}{1-\xi} \right) \cdot \left( \frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) - \xi \cdot (\rho_{jt+1} - mc_{t+1}) \right] \right\} \\
&= \quad \left( \frac{\xi}{1-\xi} \right) \cdot \left( \frac{\gamma'(v_{ijt}) - p_{v_{jt}}}{k^f(\theta_{ijt})} \right).
\end{aligned} \tag{109}$$

Next, use the job-creation condition to substitute on the right-hand side, which gives

$$\begin{aligned}
&\xi \cdot (\rho_{ijt} - mc_t) + w_{ijt} - \chi \\
&+ \quad (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left[ \left( \frac{\xi}{1-\xi} \right) \cdot \left( \frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) - \xi \cdot (\rho_{jt+1} - mc_{t+1}) \right] \right\} \\
&= \quad \left( \frac{\xi}{1-\xi} \right) \cdot \left( z_t f_n(k_t, n_t) - w_{ijt} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left( \frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\} \right).
\end{aligned} \tag{110}$$

Grouping terms in $w_{ijt}$,

$$\begin{aligned}
&w_{ijt} \cdot \left( 1 + \frac{\xi}{1-\xi} \right) = \left( \frac{\xi}{1-\xi} \right) z_t f_n(k_t, n_t) + \chi - \xi \cdot (\rho_{ijt} - mc_t) \\
&- \quad (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left[ \left( \frac{\xi}{1-\xi} \right) \cdot \left( \frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) - \xi \cdot (\rho_{jt+1} - mc_{t+1}) \right] \right\} \\
&+ \quad \left( \frac{\xi}{1-\xi} \right) \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \left( \frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\}.
\end{aligned} \tag{111}$$

Rearranging,

$$\begin{aligned}
&w_{ijt} \cdot \left( \frac{1}{1-\xi} \right) = \left( \frac{\xi}{1-\xi} \right) z_t f_n(k_t, n_t) + \chi - \xi \cdot (\rho_{ijt} - mc_t) \\
&- \quad (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left[ \left( \frac{\xi}{1-\xi} \right) \cdot \left( \frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) - \xi \cdot (\rho_{jt+1} - mc_{t+1}) \right] \right\} \\
&+ \quad \left( \frac{\xi}{1-\xi} \right) \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \left( \frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\}.
\end{aligned} \tag{112}$$

Next, multiply by $(1-\xi)$, which gives

$$\begin{aligned}
&w_{ijt} = \xi \cdot z_t f_n(k_t, n_t) + (1-\xi) \cdot \chi - (1-\xi) \cdot \xi \cdot (\rho_{ijt} - mc_t) \\
&- \quad (1-\xi) \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left[ \left( \frac{\xi}{1-\xi} \right) \cdot \left( \frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) - \xi \cdot (\rho_{jt+1} - mc_{t+1}) \right] \right\} \\
&+ \quad \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \left( \frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\}.
\end{aligned} \tag{113}$$

Expanding the terms that appear in the second line yields

$$\begin{aligned} w_{ijt} &= \xi \cdot z_t f_n(k_t, n_t) + (1-\xi) \cdot \chi - (1-\xi) \cdot \xi \cdot (\rho_{ijt} - mc_t) \\ &- \ \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left( \frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\} \\ &+ \ \xi \cdot (1-\xi) \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot (\rho_{jt+1} - mc_{t+1}) \right\} \\ &+ \ \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \left( \frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\}. \end{aligned} \tag{114}$$

Next, collect the terms that contain the monopolistic term $(\rho_{ijt} - mc_t)$, which gives

$$\begin{aligned} w_{ijt} &= \xi \cdot z_t f_n(k_t, n_t) + (1-\xi) \cdot \chi \\ &- \ \xi \cdot (1-\xi) \cdot (\rho_{ijt} - mc_t) + \xi \cdot (1-\xi) \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot (\rho_{jt+1} - mc_{t+1}) \right\} \\ &- \ \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left( \frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\} \\ &+ \ \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \left( \frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\}. \end{aligned} \tag{115}$$

Expanding the term in the third line yields

$$\begin{aligned} w_{ijt} &= \xi \cdot z_t f_n(k_t, n_t) + (1-\xi) \cdot \chi \\ &- \ \xi \cdot (1-\xi) \cdot (\rho_{ijt} - mc_t) + \xi \cdot (1-\xi) \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot (\rho_{jt+1} - mc_{t+1}) \right\} \\ &- \ \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \left( \frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\} + \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot \frac{k^h(\theta_{jt+1})}{k^f(\theta_{jt+1})} \cdot \left( \gamma'(v_{jt+1}) - p_{v_{jt+1}} \right) \right\} \\ &+ \ \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \left( \frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\}. \end{aligned} \tag{116}$$

After cancelling terms in the third and fourth lines and using the Cobb-Douglas functional form $\frac{k^f(\theta)}{k^f(\theta)} = \theta$, the submarket $ij$ wage is

$$\begin{aligned} w_{ijt} &= \xi z_t f_n(k_t, n_t) + (1-\xi)\chi + \xi(1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot \theta_{jt+1} \cdot \left( \gamma'(v_{jt+1}) - p_{v_{jt+1}} \right) \right\} \\ &- \ \xi(1-\xi) \left( \rho_{ijt} - mc_t \right) + \xi(1-\xi)(1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot (\rho_{jt+1} - mc_{t+1}) \right\}. \end{aligned} \tag{117}$$

# E Aggregation

The (symmetric equilibrium) flow budget constraint of the government is

$$T_t = (1 - k^h(\theta_t)) \cdot s_t \cdot N_{Mt} \cdot \chi + (1 - k^h(\theta_{Nt})) \cdot s_{Nt} \cdot \chi, \tag{118}$$

in which lump-sum taxes $T_t$ levied on households finance government-provided unemployment benefits.

## E.1 Aggregate Goods Resource Constraint

To construct the aggregate symmetric equilibrium household budget constraint, begin with expression (88), which, for convenience, is repeated here:

$$\begin{aligned} c_t + k_{t+1} + T_t &= (1 + r_t - \delta)k_t + w_t(1-\rho)n_{t-1} + w_{Nt} \cdot k^h_{Nt} \cdot s_{Nt} \\ &+ \int_0^1 \int_0^{N_{Mjt}} w_{ijt} \cdot k^h_{ijt} \cdot s_{ijt} \; di \; dj + \int_0^1 \int_0^{N_{Mjt}} p_{s_{jt}} \cdot s_{ijt} \; di \; dj \\ &+ (1 - k^h_{Nt}) \cdot s_{Nt}\chi + \int_0^1 \int_0^{N_{Mjt}} \left(1 - k^h_{ijt}\right) \cdot s_{ijt}\chi \; di \; dj + \int_0^1 \Pi^M_{jt} \; dj + \Pi^F_t, \end{aligned} \tag{119}$$

Integrating over the $i$ intermediated submarkets in each labor market $j$ gives

$$\begin{aligned} c_t + k_{t+1} + T_t &= (1 + r_t - \delta)k_t + w_t(1-\rho)n_{t-1} + w_{Nt} \cdot k^h_{Nt} \cdot s_{Nt} \\ &+ \int_0^1 N_{Mjt} \cdot w_{jt} \cdot k^h_{jt} \cdot s_{jt} \; dj + \int_0^1 N_{Mjt} \cdot p_{s_{jt}} \cdot s_{jt} \; dj \\ &+ (1 - k^h_{Nt}) \cdot s_{Nt}\chi + \int_0^1 N_{Mjt} \cdot \left(1 - k^h_{jt}\right) \cdot s_{jt}\chi \; dj + \int_0^1 \Pi^M_{jt} \; dj + \Pi^F_t. \end{aligned}$$

Next, integrating over the measure $j \in (0, 1)$ of recruiting markets gives the symmetric equilibrium household budget constraint

$$\begin{aligned} c_t + k_{t+1} + T_t &= (1 + r_t - \delta)k_t + w_t(1-\rho)n_{t-1} + w_{Nt} \cdot k^h_{Nt} \cdot s_{Nt} \\ &+ w_t \cdot k^h_t \cdot s_t \cdot N_{Mt} - p_{s_t} \cdot s_t \cdot N_{Mt} + (1 - k^h_{Nt}) \cdot s_{Nt}\chi + N_{Mt} \cdot \left(1 - k^h_t\right) \cdot s_t\chi + \Pi^M_t + \Pi^F_t. \end{aligned}$$

Combining this with the government budget (118) gives

$$\begin{aligned} c_t + k_{t+1} + (1 - \delta)k_t &= w_t(1-\rho)n_{t-1} + w_{Nt} \cdot k^h_{Nt} \cdot s_{Nt} \\ &+ w_t \cdot k^h_t \cdot s_t \cdot N_{Mt} + p_{s_t} \cdot s_t \cdot N_{Mt} + r_t k_t + \Pi^M_t + \Pi^F_t. \end{aligned} \tag{120}$$

In symmetric equilibrium, the period-$t$ aggregate flow profits for goods-producing firms $\Pi_t^F$ are

$$\begin{aligned}\Pi_t^F &= z_t f(k_t, n_t) - w_t(1-\rho)n_{t-1} - w_{Nt} \cdot k_{Nt}^f \cdot v_{Nt} \\ &- \quad w_t \cdot k_t^f \cdot v_t \cdot N_{Mt} + p_{v_t} \cdot v_t \cdot N_{Mt} - r_t k_t - \gamma(v_t) \cdot N_{Mt} - \gamma_N(v_{Nt}) \end{aligned} \tag{121}$$

and aggregate recruiting-firm profits $\Pi_t^M$ are

$$\begin{aligned}\Pi_t^M &= [\rho(N_{Mt}) \cdot m(s_t, v_t) - mc(N_{Mt}) \cdot m(s_t, v_t)] \cdot N_{Mt} - mc(N_{Mt})\Gamma_{Et} N_{MEt} \\ &= [\rho(N_{Mt}) \cdot m(s_t, v_t) - p_{s_t} s_t - p_{v_t} v_t] \cdot N_{Mt} - mc(N_{Mt})\Gamma_{Et} N_{MEt} \\ &= \rho(N_{Mt}) \cdot N_{Mt} \cdot m(s_t, v_t) - (p_{s_t} s_t + p_{v_t} v_t) \cdot N_{Mt} - mc(N_{Mt})\Gamma_{Et} N_{MEt}. \end{aligned} \tag{122}$$

Substituting $\Pi_t^F$ into (120) gives

$$\begin{aligned} c_t + k_{t+1} + (1-\delta)k_t &= w_t(1-\rho)n_{t-1} + w_{Nt} \cdot k_{Nt}^h \cdot s_{Nt} \\ &+ \quad w_t \cdot k_t^h \cdot s_t \cdot N_{Mt} + p_{s_t} \cdot s_t \cdot N_{Mt} + r_t k_t + \Pi_t^M \\ &+ \quad z_t f(k_t, n_t) - w_t(1-\rho)n_{t-1} - w_{Nt} \cdot k_{Nt}^f \cdot v_{Nt} - w_t \cdot k_t^f \cdot v_t \cdot N_{Mt} \\ &+ \quad p_{v_t} \cdot v_t \cdot N_{Mt} - r_t k_t - \gamma(v_t) \cdot N_{Mt} - \gamma_N(v_{Nt}). \end{aligned}$$

Next, cancelling several terms and grouping the remaining terms informatively gives

$$\begin{aligned} c_t + k_{t+1} + (1-\delta)k_t + \gamma(v_t) \cdot N_{Mt} + \gamma_N(v_{Nt}) &= z_t f(k_t, n_t) \\ + \quad w_{Nt} \cdot \underbrace{\left(k_{Nt}^h s_{Nt} - k_{Nt}^f v_{Nt}\right)}_{=0} + w_t \cdot N_{Mt} \cdot \underbrace{\left(k_t^h s_t - k_t^f v_t\right)}_{=0} &+ (p_{s_t} s_t + p_{v_t} v_t) \cdot N_{Mt} + \Pi_t^M. \end{aligned}$$

Due to matching-market clearing in both the non-intermediated labor market and the intermediated labor market ($k_{Nt}^h s_{Nt} = k_{Nt}^f v_{Nt}$ and $k_t^h s_t = k_t^f v_t$, respectively), the second and third terms on the right-hand side vanish. Next, substituting aggregate recruiting-sector profits $\Pi_t^M$ from (122) gives

$$\begin{aligned} c_t + k_{t+1} + (1-\delta)k_t + \gamma(v_t) \cdot N_{Mt} + \gamma_N(v_{Nt}) &= z_t f(k_t, n_t) \\ + \quad (p_{s_t} s_t + p_{v_t} v_t) \cdot N_{Mt} + \rho(N_{Mt}) \cdot m(s_t, v_t) \cdot N_{Mt} &- (p_{s_t} s_t + p_{v_t} v_t) \cdot N_{Mt} - mc(N_{Mt})\Gamma_{Et} N_{MEt}. \end{aligned}$$

Cancelling terms gives the decentralized economy's aggregate goods resource constraint

$$\begin{aligned} & c_t + k_{t+1} + (1-\delta)k_t + \gamma(v_t) \cdot N_{Mt} + \gamma_N(v_{Nt}) \\ + & \quad \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) \Gamma_{Et} N_{MEt} = z_t f(k_t, n_t) + \rho(N_{Mt}) \cdot N_{Mt} \cdot m(s_t, v_t). \end{aligned} \tag{123}$$

## E.2 Private-Sector Equilibrium

A symmetric private-sector general equilibrium is made up of seventeen endogenous state-contingent processes $\{c_t, n_t, lfp_t, k_{t+1}, N_{Mt}, N_{MEt}, s_t, v_t, \theta_t, w_t, s_{Nt}, v_{Nt}, \theta_{Nt}, w_{Nt}, mc_t, p_{v_t}, p_{s_t}\}_{t=0}^{\infty}$ that satisfy the following seventeen sequences of conditions: the aggregate resource constraint

$$\begin{aligned} & c_t + k_{t+1} + (1-\delta)k_t + \gamma(v_t)\cdot N_{Mt} + \gamma_N(v_{Nt}) \\ + \quad & \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\Gamma_{Et}N_{MEt} = z_t f(k_t, n_t) + \rho(N_{Mt})\cdot N_{Mt}\cdot m(s_t, v_t). \end{aligned} \tag{124}$$

the aggregate law of motion for labor

$$n_t = (1-\rho)n_{t-1} + m(s_{Nt}, v_{Nt}) + \rho(N_{Mt})\cdot N_{Mt}\cdot m(s_t, v_t), \tag{125}$$

the definition of aggregate LFP

$$lfp_t = (1-\rho)n_{t-1} + s_{Nt} + s_t\cdot N_{Mt}, \tag{126}$$

the aggregate law of motion for recruiters

$$N_{Mt} = (1-\omega)N_{Mt-1} + N_{MEt}, \tag{127}$$

the capital Euler condition

$$1 = E_t\left\{\Xi_{t+1|t}\left(1 + z_{t+1}f_k(k_{t+1}, n_{t+1}) - \delta\right)\right\}, \tag{128}$$

the free-entry condition for recruiters

$$\left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\Gamma_{Et} = \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)m(s_t, v_t) + (1-\omega)E_t\left\{\Xi_{t+1|t}\left(\frac{\rho(N_{Mt+1})}{\mu(N_{Mt+1})}\right)\Gamma_{Et+1}\right\}, \tag{129}$$

the vacancy creation condition for intermediated labor markets

$$\gamma'(v_t) = p_{v_t} + k_t^f\cdot\underbrace{\left(z_t f_n(k_t, n_t) - w_t + (1-\rho)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma'(v_{t+1}) - p_{v_{t+1}}}{k_{t+1}^f}\right)\right\}\right)}_{\equiv \mathbf{J}(w_t)}, \tag{130}$$

the vacancy creation condition for non-intermediated labor markets

$$\gamma_N'(v_{Nt}) = k_{Nt}^f \cdot \underbrace{\left( z_t f_n(k_t, n_t) - w_{Nt} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \frac{\gamma_N'(v_{Nt+1})}{k_{Nt+1}^f} \right\} \right)}_{\equiv \mathbf{J}(w_{Nt})}, \tag{131}$$

the active job search condition for non-intermediated labor markets

$$\frac{h'(lfp_t)}{u'(c_t)} = k_{Nt}^h \left[ w_{Nt} + (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left( \frac{1-k_{Nt+1}^h}{k_{Nt+1}^h} \right) \cdot \left( \frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \right] + (1-k_{Nt}^h)\chi, \tag{132}$$

the active job search condition directed towards intermediated labor markets

$$\begin{aligned} \frac{h'(lfp_t)}{u'(c_t)} &= p_{s_t} + (1-k_t^h)\chi \\ &+ \quad k_t^h \left[ w_t + (1-\rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[ \left( \frac{1-k_{t+1}^h}{k_{t+1}^h} \right) \cdot \left( \frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi - p_{s_{t+1}} \right) \right] \right\} \right], \end{aligned} \tag{133}$$

the surplus-sharing rule that determines wages $w_t$ in monopolistic recruiting markets

$$\xi \cdot \left( \rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) + \mathbf{W}(w_t) - \mathbf{U}_t = \left( \frac{\xi}{1-\xi} \right) \mathbf{J}(w_t), \tag{134}$$

the surplus-sharing rule that determines Nash-bargained wages (with $\eta$ denoting the employee's Nash bargaining power) in non-intermediated labor markets

$$\mathbf{W}(w_{Nt}) - \mathbf{U}_t = \left( \frac{\eta}{1-\eta} \right) \mathbf{J}(w_{Nt}), \tag{135}$$

the definition of labor-market tightness in monopolistic recruiting markets

$$\theta_t = \frac{v_t}{s_t}, \tag{136}$$

the definition of labor-market tightness in non-intermediated matches

$$\theta_{Nt} = \frac{v_{Nt}}{s_{Nt}}, \tag{137}$$

along with the equilibrium input prices

$$p_{v_t} = m_v(s_t, v_t) \cdot \left( \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \tag{138}$$

and

$$p_{s_t} = m_s(s_t, v_t) \cdot \left( \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right), \tag{139}$$

taking as given the stochastic processes $\{z_t, \Gamma_{Et}\}$ and the initial conditions $n_{-1}, N_{M_{-1}}, k_0$.

# F Recruiting Sector Empirics

Labor market intermediaries play an important role in helping firms meet their employment needs and job seekers find employment opportunities. The services and reach of these intermediaries has grown over time, especially with a dramatic expansion of e-recruiting firms and their services since the middle of the 1990s (Nakamura et al., 2009; Bagues and Sylos Labini, 2009). Examples of labor market intermediaries (LMIs, for short) include employment agencies and recruiting and staffing firms and job search engines and services. For an comprehensive summary of online job services, their proliferation, and their importance, see Nakamura et al. (2009). Among other things, labor market intermediaries build resume databases, provide services that centralize job applications, provide customized matching services for firms, and advertise open employment positions. Well-known providers of online (for-profit) e-recruiting services include Monster.com (one of the largest e-recruiting firms in the U.S. that started in 1995), Indeed.com, and CareerBuider.Similar services operate in the non-profit realm as well (for example, America's Job Bank). For the benefits of e-recruiting (which include the reduction of variable recruiting costs and processing costs, among others), see Nakamura et al. (2009). Survey evidence from the Society for Human Resource Management for 2007 suggests that more than 40 percent of new hires in both the public and private sectors originated from e-recruiting (Nakamura et al., 2009). Using data from iLogos Research, Nakamura et al. (2009) document a sharp expansion in the corporate website employment sections in global 500 companies: while in 1998 these sections represented 29 percent of corporate website use, these sections expanded to 94 percent of website use in the 2000s. For related work on e-recruiting and the labor market, see Autor, Katz, and Krueger (1998), Kuhn (2003), Kuhn and Skuterud (2004), and Stevenson (2008), among others. Figure 8 provides evidence on the growth of staffing firms as measured by the evolution of employment in these firms.

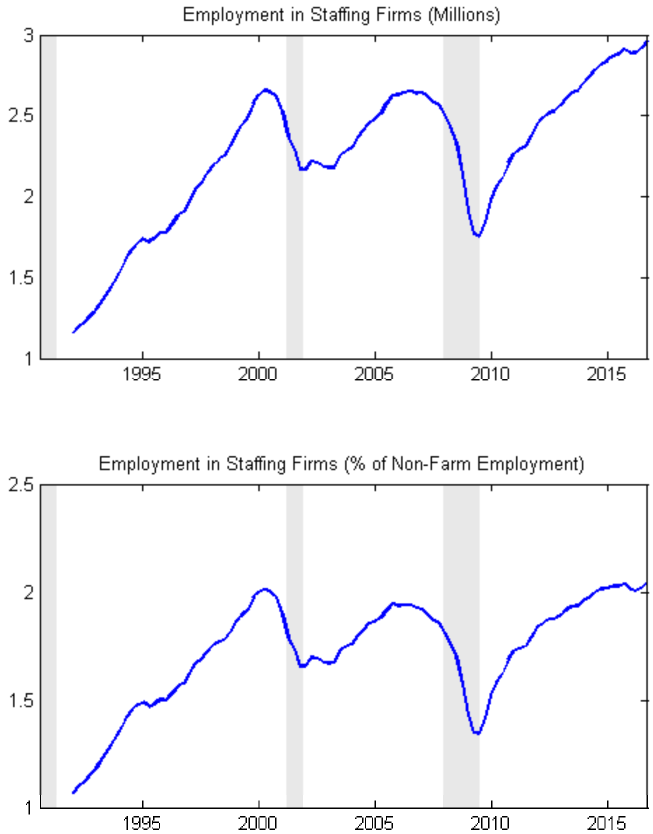


Figure 8: **Employment in Staffing Firms.** Source: https://americanstaffing.net/staffing-research-data/asa-data-dashboard/asa-employment-sales/. Similar patterns emerge if we consider temporary help employment (a common measure of employment in the recruiting sector)s.

# G Proofs of Proposition 2 and Corollary 1

This Appendix provides the proofs of Proposition 2 and Corollary 1. For simplicity, suppose $\rho = 1$ and $\omega = 1$ and that total vacancy posting costs are linear in vacancies. The following are the equilibrium conditions: aggregate LFP (which could potentially be fixed at $l\bar{f}p$) is

$$lfp_t = l\bar{f}p = s_{Nt} + s_t \cdot N_{Mt}, \tag{140}$$

the (symmetric equilibrium) LFP condition directed towards recruiting markets is

$$\frac{h'(lfp_t)}{u'(c_t)} = p_{s_t} + k^h(\theta_t) \cdot w_t + (1 - k^h(\theta_t)) \cdot \chi, \tag{141}$$

the LFP condition for matching via through random-search-and-bargaining is

$$\frac{h'(lfp_t)}{u'(c_t)} = k^h(\theta_{Nt}) \cdot w_{Nt} + (1 - k^h(\theta_{Nt})) \cdot \chi, \tag{142}$$

the (symmetric equilibrium) job-creation condition directed towards recruiting markets is

$$\gamma = p_{v_t} + k^f(\theta_t)\,(z_t - w_t)\,, \tag{143}$$

the job-creation condition for matching via random-search-and-bargaining is

$$\gamma = k^f(\theta_{Nt})\,(z_t - w_{Nt})\,, \tag{144}$$

the (symmetric equilibrium) wage in the monopolistically-competitive recruiting sector stated in explicit form is

$$\begin{aligned} w_t &= \xi z_t + (1-\xi)\chi - \xi(1-\xi)\left(\rho(N_{Mt}) - mc(N_{Mt})\right) \\ &= \xi z_t + (1-\xi)\chi - \xi(1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right), \end{aligned} \tag{145}$$

the input factor prices are

$$\begin{aligned} p_{v_t} &= m_v(s_t, v_t) \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) \\ &= (1-\xi) \cdot \theta_t^{-\xi} \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right), \end{aligned} \tag{146}$$

and

$$\begin{aligned} p_{s_t} &= m_s(s_t, v_t) \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) \\ &= \xi \cdot \theta_t^{1-\xi} \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right), \end{aligned} \tag{147}$$

the Nash wage stated in explicit form is

$$w_{Nt} = \eta \cdot z_t + (1 - \eta) \cdot \chi, \tag{148}$$

the free-entry condition for the monopolistically-competitive recruiting market is

$$\left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) \Gamma_{Et} = (\rho(N_{Mt}) - mc(N_{Mt})) \cdot m(s_t, v_t) \tag{149}$$

$$= \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) \cdot m(s_t, v_t), \tag{150}$$

the law of motion for recruiters is

$$N_{Mt} = (1 - \omega) N_{Mt-1} + N_{MEt}, \tag{151}$$

which, with $\omega = 1$, is

$$N_{Mt} = N_{MEt}, \tag{152}$$

the law of motion for labor is

$$n_t = m(s_{Nt}, v_{Nt}) + \rho(N_{Mt}) \cdot N_{Mt} \cdot m(s_t, v_t), \tag{153}$$

and the aggregate goods resource constraint is

$$c_t + \gamma \cdot v_t \cdot N_{Mt} + \gamma_N v_{Nt} + \Gamma_{Et} N_{MEt} - \rho(N_{Mt}) \cdot N_{Mt} \cdot m(s_t, v_t) = z_t n_t. \tag{154}$$

**Analysis.**

**Job-Creation Conditions.** The first step is to substitute the Nash wage (148) into (144), which gives

$$\begin{aligned} \gamma &= k^f(\theta_{Nt})\,(z_t - w_{Nt}) \\ &= m_N^{EFF} \cdot \theta_{Nt}^{-\xi} \cdot (1 - \eta) \cdot (z_t - \chi), \end{aligned} \tag{155}$$

in which the expression on the third line allows us to compute $\theta_{Nt}$

$$\theta_{Nt} = \left[\frac{\gamma}{m_N^{EFF} \cdot (1-\eta) \cdot (z_t - \chi)}\right]^{-1/\xi} \tag{156}$$

in closed form as a function of only exogenous parameters, which makes clear that $\partial\theta_{Nt}/\partial\eta < 0$.[23] Then, the second step is substitution of both the price $p_{v_t}$ from (146) and the recruiting-market wage (145) in the job-creation condition (143) which yields, after several steps of algebra,

$$\gamma = (1-\xi) \cdot m^{EFF} \cdot \theta_t^{-\xi} \cdot \left[\frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) + \xi \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\right], \tag{157}$$

which is an equilibrium restriction between recruiting-market tightness $\theta_t$ and the measure $N_{Mt}$ of monopolistic recruiters, which can equivalently be written in closed form as

$$\theta_t = \left(\frac{(1-\xi) \cdot m^{EFF} \cdot \left[\frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) + \xi \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\right]}{\gamma}\right)^{1/\xi}. \tag{158}$$

**Labor Force Participation Conditions.** Substitution of both the price $p_{s_t}$ from (147) and the recruiting-market wage (145) into the LFP condition (141) for recruiting markets gives

$$\begin{aligned}
\frac{h'(lfp_t)}{u'(c_t)} &= p_{s_t} + k^h(\theta_t)w_t + (1-k^h(\theta_t))\chi \\
&= m_s(s_t, v_t) \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) + m^{EFF} \cdot \theta_t^{1-\xi} \cdot \left[\xi z_t + (1-\xi)\chi - \xi(1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\right] \\
&\quad + \left(1 - m^{EFF} \cdot \theta_t^{1-\xi}\right)\chi \\
&= m_s(s_t, v_t) \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) + m^{EFF} \cdot \theta_t^{1-\xi} \cdot \left[\xi z_t - \xi(1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\right] \\
&\quad + m^{EFF} \cdot \theta_t^{1-\xi} \cdot \chi - \xi \cdot m^{EFF} \cdot \theta_t^{1-\xi} \cdot \chi + \chi - m^{EFF} \cdot \theta_t^{1-\xi} \cdot \chi \\
&= \xi \cdot k^h(\theta_t) \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) + m^{EFF} \cdot \theta_t^{1-\xi} \cdot \left[\xi(z_t - \chi) - \xi(1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\right] \\
&= \xi \cdot m^{EFF} \cdot \theta_t^{1-\xi} \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) + \xi \cdot m^{EFF} \cdot \theta_t^{1-\xi} \cdot (z_t - \chi) \\
&\quad - \xi \cdot m^{EFF} \cdot \theta_t^{1-\xi} \cdot (1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) \\
&= \xi \cdot m^{EFF} \cdot \theta_t^{1-\xi} \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) - (1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\right).
\end{aligned} \tag{159}$$

[23] In the analogous first step of the steady-state analysis of the dynamic model, it is also clear that $\partial\theta_{Nt}/\partial\eta < 0$, but the result arises from implicit differentiation because there is no closed-form solution; the lack of a closed-form solution in the very first step of the counterpart analysis greatly complicates matters.

Next, substituting the Nash wage (148) into the LFP condition (142) gives

$$\begin{aligned}\frac{h'(lfp_t)}{u'(c_t)} &= k^h(\theta_{Nt})\cdot w_{Nt} + (1-k^h(\theta_{Nt}))\cdot\chi \\ &= m_N^{EFF}\cdot {\theta_{Nt}}^{1-\xi}\cdot\eta\cdot(z_t-\chi). \end{aligned} \tag{160}$$

Substituting (156) into the $\theta_{Nt}$ term on the right-hand side gives

$$\begin{aligned}\frac{h'(lfp_t)}{u'(c_t)} &= m_N^{EFF}\cdot\left(\left[\frac{\gamma}{m_N^{EFF}\cdot(1-\eta)\cdot(z_t-\chi)}\right]^{-1/\xi}\right)^{1-\xi}\cdot\eta\cdot(z_t-\chi) \\ &= m_N^{EFF}\cdot\left[\frac{\gamma}{m_N^{EFF}\cdot(1-\eta)\cdot(z_t-\chi)}\right]^{-\frac{1-\xi}{\xi}}\cdot\eta\cdot(z_t-\chi). \end{aligned} \tag{161}$$

Then, dividing (159) by (161) gives

$$\begin{aligned}& m_N^{EFF}\cdot\left[\frac{\gamma}{m_N^{EFF}\cdot(1-\eta)\cdot(z_t-\chi)}\right]^{-\frac{1-\xi}{\xi}}\eta\cdot(z_t-\chi) \\ &= \xi\cdot m^{EFF}\cdot\theta_t^{1-\xi}\cdot\left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}+(z_t-\chi)-(1-\xi)\cdot\left(\rho(N_{Mt})-\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\right). \end{aligned} \tag{162}$$

Dividing by the expression on the left-hand side and dividing by $\theta_t^{1-\xi}$ gives

$$\theta_t^{\xi-1} = \frac{\xi\cdot m^{EFF}\cdot\left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}+(z_t-\chi)-(1-\xi)\cdot\left(\rho(N_{Mt})-\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\right)}{m_N^{EFF}\cdot\left[\frac{\gamma}{m_N^{EFF}\cdot(1-\eta)\cdot(z_t-\chi)}\right]^{-\frac{1-\xi}{\xi}}\eta\cdot(z_t-\chi)}, \tag{163}$$

which is an equilibrium restriction between recruiting-market tightness $\theta_t$ and the measure $N_{Mt}$ of monopolistic recruiters, which can equivalently be written in closed form as

$$\theta_t = \left(\frac{\xi\cdot m^{EFF}\cdot\left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}+(z_t-\chi)-(1-\xi)\cdot\left(\rho(N_{Mt})-\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\right)}{m_N^{EFF}\cdot\left[\frac{\gamma}{m_N^{EFF}\cdot(1-\eta)\cdot(z_t-\chi)}\right]^{-\frac{1-\xi}{\xi}}\eta\cdot(z_t-\chi)}\right)^{-1/(1-\xi)}. \tag{164}$$

**Summary.**

We have constructed two closed-form equilibrium restrictions between $\theta_t$ and $N_{Mt}$, one which arises from job-creation directed towards recruiting markets

$$\theta_t = \left(\left(\frac{1-\xi}{\gamma}\right)\cdot m^{EFF}\cdot\left[\frac{\rho(N_{Mt})}{\mu(N_{Mt})}+(z_t-\chi)+\xi\cdot\left(\rho(N_{Mt})-\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\right]\right)^{1/\xi} \tag{165}$$

and the other arises from labor-force participation directed towards recruiting markets

$$\theta_t = \left( \frac{\xi \cdot m^{EFF} \cdot \left( \frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) - (1-\xi) \cdot \left( \rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \right)}{m_N^{EFF} \cdot \left[ \frac{\gamma}{m_N^{EFF} \cdot (1-\eta) \cdot (z_t - \chi)} \right]^{-\frac{1-\xi}{\xi}} \eta \cdot (z_t - \chi)} \right)^{-1/(1-\xi)}, \tag{166}$$

which thus implies the condition

$$\left( \left( \frac{1-\xi}{\gamma} \right) \cdot m^{EFF} \cdot \left[ \frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) + \xi \cdot \left( \rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \right] \right)^{1/\xi} \tag{167}$$

$$= \left( \frac{\xi \cdot m^{EFF} \cdot \left( \frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) - (1-\xi) \cdot \left( \rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \right)}{m_N^{EFF} \cdot \underbrace{\left[ \frac{\gamma}{m_N^{EFF} \cdot (1-\eta) \cdot (z_t - \chi)} \right]^{-\frac{1-\xi}{\xi}}}_{=\theta_{Nt}} \eta \cdot (z_t - \chi)} \right)^{-1/(1-\xi)}$$

in which the only endogenous variable is $N_{Mt}$ and, note, in which the Nash-bargaining parameter $\eta$ appears.

Once $N_{Mt}$ is determined from expression (167), recruiting-market tightness $\theta_t$ is determined (from either (165) or (166)), which in turn jointly pin down the factor input prices $p_{s_t}$ and $p_{v_t}$ and the recruiting-market wage $w_t$.

The remaining variables to be determined are $s_t$, $s_{Nt}$, $v_t$, $v_{Nt}$, and $n_t$. Substituting (140) — which, as a reminder, is $lfp_t = l\bar{f}p = s_{Nt} + s_t \cdot N_{Mt}$ — in the marginal utility function $h'(lfp_t)$ in the LFP condition for random-search bargaining markets (142) gives

$$\frac{h'(l\bar{f}p)}{u'(c_t)} = m_N^{EFF} \cdot \left[ \frac{\gamma}{m_N^{EFF} \cdot (1-\eta) \cdot (z_t - \chi)} \right]^{-\frac{1-\xi}{\xi}} \cdot \eta \cdot (z_t - \chi), \tag{168}$$

which in turn gives

$$\frac{h'(s_{Nt} + s_t \cdot N_{Mt})}{u'(c_t)} = m_N^{EFF} \cdot \left[ \frac{\gamma}{m_N^{EFF} \cdot (1-\eta) \cdot (z_t - \chi)} \right]^{-\frac{1-\xi}{\xi}} \cdot \eta \cdot (z_t - \chi), \tag{169}$$

which (because $N_{Mt}$ has already been determined) is an equilibrium restriction between $s_t$ and $s_{Nt}$. Next, analogously, substitution of (140) in the LFP condition for recruiting markets (141) gives

$$\frac{h'(l\bar{f}p)}{u'(c_t)} = \xi \cdot m^{EFF} \cdot \theta_t^{1-\xi} \cdot \left( \frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) - (1-\xi) \cdot \left( \rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \right), \tag{170}$$

which in turn gives

$$\frac{h'(s_{Nt}+s_t\cdot N_{Mt})}{u'(c_t)}=\xi\cdot m^{EFF}\cdot\theta_t^{1-\xi}\cdot\left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}+(z_t-\chi)-(1-\xi)\cdot\left(\rho(N_{Mt})-\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\right), \tag{171}$$

which (because $N_{Mt}$ has already been determined) is a second equilibrium restriction between $s_t$ and $s_{Nt}$. The equilibrium restrictions (169) and (171) thus jointly determine $s_t$ and $s_{Nt}$. Given that $\theta_t$, $\theta_{Nt}$, $s_t$, and $s_{Nt}$ have now been determined, the determination of $v_t$, $v_{Nt}$, and $n_t$ easily follow from definitions of market tightness and the aggregate law of motion for employment. These results presume that $lfp_t=l\bar{f}p$, which in turn implies that disutility of participation $h'(l\bar{f}p)$ is fixed. To allow for endogenous $lfp_t$ (rather than fixed $l\bar{f}p$), relax the restriction of fixed disutility of participation $h'(\cdot)$. Finally, if we are considering general equilibrium, the determination of $c_t$ easily follows from the goods resource constraint.

**Dixit-Stiglitz Aggregation.**

For Dixit-Stiglitz aggregation, $\rho(N_{Mt})=N_{Mt}^{\frac{1}{\varepsilon-1}}$, $\mu(N_{Mt})=\frac{\varepsilon}{\varepsilon-1}$, and $\frac{\rho(N_{Mt})}{\mu(N_{Mt})}=\left(\frac{\varepsilon-1}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}$. Substituting these expressions in (167) gives

$$\left(\left(\frac{1-\xi}{\gamma}\right)\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)+\xi\cdot\left(N_{Mt}^{\frac{1}{\varepsilon-1}}-\left(\frac{\varepsilon-1}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}\right)\right)\right)^{1/\xi} \tag{172}$$

$$=\left(\frac{\xi\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)-(1-\xi)\cdot\left(N_{Mt}^{\frac{1}{\varepsilon-1}}-\left(\frac{\varepsilon-1}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}\right)\right)}{\eta\cdot m_N^{EFF}\cdot\underbrace{\left[\frac{\gamma}{m_N^{EFF}\cdot(1-\eta)\cdot(z_t-\chi)}\right]^{-\frac{1-\xi}{\xi}}}_{=\theta_{Nt}}(z_t-\chi)}\right)^{-1/(1-\xi)}.$$

Simplifying terms step-by-step for clarity, we first have

$$\left(\left(\frac{1-\xi}{\gamma}\right)\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)+\xi\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}\cdot\left(1-\frac{\varepsilon-1}{\varepsilon}\right)\right)\right)^{1/\xi} \tag{173}$$

$$=\left(\frac{\xi\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)-(1-\xi)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}\cdot\left(1-\frac{\varepsilon-1}{\varepsilon}\right)\right)}{\eta\cdot m_N^{EFF}\cdot\underbrace{\left[\frac{\gamma}{m_N^{EFF}\cdot(1-\eta)\cdot(z_t-\chi)}\right]^{-\frac{1-\xi}{\xi}}}_{=\theta_{Nt}}(z_t-\chi)}\right)^{-1/(1-\xi)}.$$

Second

$$\left(\left(\frac{1-\xi}{\gamma}\right)\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)+\xi\cdot\frac{1}{\varepsilon}\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}\right)\right)^{1/\xi} \tag{174}$$

$$= \left(\frac{\xi\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)-(1-\xi)\cdot\frac{1}{\varepsilon}\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}\right)}{\eta\cdot m_N^{EFF}\cdot(z_t-\chi)\cdot\underbrace{\left[\frac{\gamma}{m_N^{EFF}\cdot(1-\eta)\cdot(z_t-\chi)}\right]^{-\frac{1-\xi}{\xi}}}_{=\theta_{Nt}}}\right)^{-1/(1-\xi)}$$

Then,

$$\left(\left(\frac{1-\xi}{\gamma}\right)\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1+\xi}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)\right)^{1/\xi} \tag{175}$$

$$= \left(\frac{\xi\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1-(1-\xi)}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)}{\eta\cdot m_N^{EFF}\cdot\underbrace{\left[\frac{\gamma}{m_N^{EFF}\cdot(1-\eta)\cdot(z_t-\chi)}\right]^{-\frac{1-\xi}{\xi}}}_{=\theta_{Nt}}(z_t-\chi)}\right)^{-1/(1-\xi)}.$$

Define the implicit function

$$\Upsilon(N_{Mt},\eta;\cdot)\equiv\left(\left(\frac{1-\xi}{\gamma}\right)\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1+\xi}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)\right)^{\frac{1}{\xi}}$$

$$-\left(\frac{\xi\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1-(1-\xi)}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)}{\eta\cdot m_N^{EFF}\cdot(z_t-\chi)\cdot\underbrace{\left[\frac{\gamma}{m_N^{EFF}\cdot(1-\eta)\cdot(z_t-\chi)}\right]^{-\frac{1-\xi}{\xi}}}_{=\theta_{Nt}}}\right)^{\frac{1}{\xi-1}}$$

$$= 0. \tag{176}$$

The partial to be computed is
$$\frac{\partial N_{Mt}}{\partial\eta}=-\frac{\Upsilon_\eta(\cdot)}{\Upsilon_{N_{Mt}(\cdot)}}. \tag{177}$$

Before proceeding, we rewrite, for the sake of ease of calculation, the implicit function in a couple of steps. First,

$$\Upsilon(N_{Mt},\eta;\cdot) \equiv \left(\left(\frac{1-\xi}{\gamma}\right)\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1+\xi}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)\right)^{\frac{1}{\xi}} \tag{178}$$
$$- \left(\xi\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1-(1-\xi)}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)\right)^{\frac{1}{\xi-1}}\left(\eta\cdot m_N^{EFF}\cdot(z_t-\chi)\cdot\left[\frac{\gamma\cdot(1-\eta)^{-1}}{m_N^{EFF}\cdot(z_t-\chi)}\right]^{\frac{\xi-1}{\xi}}\right)^{\frac{1}{1-\xi}}$$
$$= 0.$$

Then,

$$\Upsilon(N_{Mt},\eta;\cdot) \equiv \left(\left(\frac{1-\xi}{\gamma}\right)\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1+\xi}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)\right)^{\frac{1}{\xi}}$$
$$- \underbrace{\left(\xi m^{EFF}\cdot\left(\left(\frac{\varepsilon-1-(1-\xi)}{\varepsilon}\right)N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)\right)^{\frac{1}{\xi-1}}}_{\equiv\mathbf{D}>0}\underbrace{\left(\eta\cdot m_N^{EFF}\cdot(z_t-\chi)\right)^{\frac{1}{1-\xi}}}_{\equiv f(\cdot)}\underbrace{\left(\frac{\gamma\cdot(1-\eta)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}}_{\equiv g(\cdot)}$$
$$= 0. \tag{179}$$

**Computation of $\Upsilon_{N_{Mt}}(\cdot)$.**

$$\Upsilon_{N_{Mt}}(\cdot) = \frac{1}{\xi}\left(\left(\frac{1-\xi}{\gamma}\right)m^{EFF}\left(\left(\frac{\varepsilon-1+\xi}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)\right)^{\frac{1}{\xi}-1}\left(\frac{1}{\varepsilon-1}\right)\left(\frac{1-\xi}{\gamma}\right)m^{EFF}\left(\frac{\varepsilon-1+\xi}{\varepsilon}\right)N_{Mt}^{\frac{1}{\varepsilon-1}-1}$$
$$+ \frac{\left(\frac{1}{1-\xi}\right)\left(\xi m^{EFF}\cdot\left(\left(\frac{\varepsilon-1-(1-\xi)}{\varepsilon}\right)N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)\right)^{\frac{1}{\xi-1}-1}\left(\frac{1}{\varepsilon-1}\right)\xi m^{EFF}\left(\frac{\varepsilon-1-(1-\xi)}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}-1}}{\left(\eta\cdot m_N^{EFF}\cdot(z_t-\chi)\right)^{-\frac{1}{1-\xi}}\left(\frac{\gamma\cdot(1-\eta)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{\frac{1}{\xi}}}$$
$$> 0 \tag{180}$$

Because the first term is strictly positive and the second term is also strictly positive (more precisely, both the numerator and the denominator of the second term are strictly positive), the partial $\Upsilon_{N_{Mt}}(\cdot)$ is unambiguously positive ($\Upsilon_{N_{Mt}}(\cdot) > 0$).

**Computation of $\Upsilon_{\eta}(\cdot)$.**

For the computation of the partial $\Upsilon_{\eta}(\cdot)$, it is only the second line of (179) that is needed because $\eta$ only appears in the second line. Moreover, for the sake of simplicity of notation, we use the term that is as defined $\mathbf{D}$ in the second line of (179) and define temporarily the functions

$$f(\cdot) = \left(\eta\cdot m_N^{EFF}\cdot(z_t-\chi)\right)^{\frac{1}{1-\xi}} > 0 \tag{181}$$

and

$$g(\cdot) = \left(\frac{\gamma\cdot(1-\eta)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}} > 0, \tag{182}$$

which imply the partials

$$f_{\eta}(\cdot) = \frac{m_N^{EFF} \cdot (z_t - \chi)}{1-\xi} \cdot \left(\eta \cdot m_N^{EFF} \cdot (z_t - \chi)\right)^{\frac{1}{1-\xi}-1} > 0 \tag{183}$$

and

$$\begin{aligned}
g_{\eta}(\cdot) &= -\frac{1}{\xi} \cdot \left(\frac{\gamma \cdot (1-\eta)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}-1} \cdot \left(\frac{\gamma \cdot (1-\eta)^{-2}}{m_N^{EFF}(z_t-\chi)}\right) \\
&= -\frac{1}{\xi} \cdot \left(\frac{\gamma}{(1-\eta) \cdot m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}} \cdot \left(\frac{\gamma}{(1-\eta) \cdot m_N^{EFF}(z_t-\chi)}\right)^{-1} \cdot \left(\frac{\gamma}{(1-\eta)^2 \cdot m_N^{EFF}(z_t-\chi)}\right) \\
&= -\frac{1}{\xi} \cdot \left(\frac{\gamma}{(1-\eta) \cdot m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}} \cdot \left(\frac{(1-\eta) \cdot m_N^{EFF}(z_t-\chi)}{\gamma}\right) \cdot \left(\frac{\gamma}{(1-\eta)^2 \cdot m_N^{EFF}(z_t-\chi)}\right) \\
&= -\frac{1}{\xi} \cdot \left(\frac{\gamma}{(1-\eta) \cdot m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}} \cdot (1-\eta)^{-1} \\
&= -\frac{1}{\xi} \cdot \left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}} \cdot (1-\eta)^{\frac{1}{\xi}} \cdot (1-\eta)^{-1} \\
&= -\frac{1}{\xi} \cdot \left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}} \cdot (1-\eta)^{\frac{1}{\xi}-1} \\
&= -\frac{1}{\xi} \cdot \left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}} \cdot (1-\eta)^{\frac{1-\xi}{\xi}} \\
&< 0.
\end{aligned} \tag{184}$$

The Nash bargaining parameter $\eta$ is intentionally emphasized in each of the functions $f(\cdot)$, $g(\cdot)$, $f_{\eta}(\cdot)$, and $g_{\eta}(\cdot)$, because it is the parameter around which the comparative static exercise is being conducted. With the natural restrictions on the model parameters ($\xi \in (0,1)$, $z_t > 0$, $m_N^{EFF} > 0$, $1 < \varepsilon < \infty$, $\gamma > 0$,, $z_t - \chi > 0$, $\eta \in (0,1)$), the functions $f(\cdot)$ and $g(\cdot)$ are both unambiguously positive, the partial $f_{\eta}(\cdot)$ is unambiguously positive, and the partial $g_{\eta}(\cdot)$ is unambiguously negative. Stated in terms of $\mathbf{D}$, $f(\cdot)$, $g(\cdot)$, $f_{\eta}(\cdot)$, and $g_{\eta}(\cdot)$, the partial of the implicit function $\Upsilon(\cdot)$ with respect to Nash bargaining power $\eta$ is

$$\begin{aligned}
\Upsilon_{\eta}(\cdot) &= -\underbrace{\mathbf{D}}_{>0} \cdot \left(\underbrace{f_{\eta}(\cdot)}_{>0} \cdot \underbrace{g(\cdot)}_{>0} + \underbrace{f(\cdot)}_{>0} \cdot \underbrace{g_{\eta}(\cdot)}_{<0}\right) \\
&\underbrace{=}_{?} 0,
\end{aligned} \tag{185}$$

whose sign depends on whether $\eta < \xi$, $\eta = \xi$, or $\eta > \xi$.

**Proof of Proposition 2.**

Starting with the simple case, evaluating the function (185) at $\eta = \xi$,

$$\begin{aligned}
f_\eta(\cdot)\cdot g(\cdot) + f(\cdot)\cdot g_\eta(\cdot) = & \underbrace{\frac{m_N^{EFF}(z_t-\chi)}{1-\xi}\cdot\left(\xi\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}-1}}_{=f_\eta(\cdot)}\cdot\underbrace{\left(\frac{\gamma(1-\xi)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}}_{=g(\cdot)} \\
& -\underbrace{\left(\xi\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}}}_{=f(\cdot)}\cdot\underbrace{\frac{1}{\xi}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1-\xi}{\xi}}}_{=-g_\eta(\cdot)} \\
= & \frac{m_N^{EFF}(z_t-\chi)}{1-\xi}\cdot\left(\xi\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}-1}\cdot\left(\frac{\gamma(1-\xi)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}} \\
& -\left(\xi\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}}\cdot\frac{1}{\xi}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1-\xi}{\xi}} \\
= & \frac{m_N^{EFF}(z_t-\chi)}{1-\xi}\cdot\left(\xi\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}}\cdot\left(\xi\cdot m_N^{EFF}(z_t-\chi)\right)^{-1}\cdot\left(\frac{\gamma(1-\xi)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}} \\
& -\left(\xi\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}}\cdot\frac{1}{\xi}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1-\xi}{\xi}} \\
= & \left(\xi\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}}\cdot\left[\left(\frac{m_N^{EFF}(z_t-\chi)}{1-\xi}\right)\cdot\left(\frac{1}{\xi\cdot m_N^{EFF}(z_t-\chi)}\right)\cdot\left(\frac{\gamma(1-\xi)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}-\frac{1}{\xi}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1-\xi}{\xi}}\right] \\
= & \left(\xi\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}}\cdot\left[\left(\frac{1}{1-\xi}\right)\cdot\left(\frac{1}{\xi}\right)\cdot\left(\frac{\gamma(1-\xi)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}-\frac{1}{\xi}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1-\xi}{\xi}}\right] \\
= & \left(\xi\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}}\cdot\left[\left(\frac{1}{1-\xi}\right)\cdot\left(\frac{1}{\xi}\right)\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1}{\xi}}-\frac{1}{\xi}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1}{\xi}}\cdot(1-\xi)^{-1}\right] \\
= & \left(\xi\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot\left[\left(\frac{1}{1-\xi}\right)\cdot\left(\frac{1}{\xi}\right)\cdot(1-\xi)^{\frac{1}{\xi}}-\frac{1}{\xi}\cdot(1-\xi)^{\frac{1}{\xi}}\cdot(1-\xi)^{-1}\right] \\
= & \left(\xi\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot\left[\left(\frac{1}{1-\xi}\right)\cdot\left(\frac{1}{\xi}\right)\cdot(1-\xi)^{\frac{1}{\xi}}-\left(\frac{1}{1-\xi}\right)\cdot\left(\frac{1}{\xi}\right)\cdot(1-\xi)^{\frac{1}{\xi}}\right] \\
= & \left(\xi\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1}{\xi}}\cdot\left[\left(\frac{1}{1-\xi}\right)\cdot\left(\frac{1}{\xi}\right)-\left(\frac{1}{1-\xi}\right)\cdot\left(\frac{1}{\xi}\right)\right] \\
= & \left(\xi\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1}{\xi}}\cdot\left[\frac{1}{(1-\xi)\cdot\xi}-\frac{1}{(1-\xi)\cdot\xi}\right] \\
= & \left(\xi\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1}{\xi}}\cdot\underbrace{\left[\frac{1}{\xi-\xi\cdot\xi}-\frac{1}{\xi-\xi\cdot\xi}\right]}_{=0} \\
= & \ 0
\end{aligned}$$

leads, as the last line clearly shows, to

$$\Upsilon_\eta^{\eta=\xi}(\cdot) = 0, \tag{186}$$

which in turns immediately implies

$$\frac{\partial N_{Mt}}{\partial \eta} = -\frac{\Upsilon_{\eta}(\cdot)}{\Upsilon_{N_{Mt}(\cdot)}} = 0. \tag{187}$$

The conclusion is that the equilibrium measure of monopolistically-competitive recruiters $N_{Mt}$ is maximized if Nash-bargained wages in the random-search channel lead to **efficient** outcomes in the sense of Mortensen (1982) and Hosios (1990).[24]

**Proof of Corollary 1.**

Replacing the highlighted $\xi$ terms that appear in expression (186) with $\eta$ gives

$$f_{\eta}(\cdot)\cdot g(\cdot)+f(\cdot)\cdot g_{\eta}(\cdot) = \underbrace{\frac{m_N^{EFF}(z_t-\chi)}{1-\xi}\cdot\left(\eta\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}-1}}_{=f_{\eta}(\cdot)}\cdot\underbrace{\left(\frac{\gamma(1-\xi)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}}_{=g(\cdot)}$$
$$-\underbrace{\left(\eta\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}}}_{=f(\cdot)}\cdot\underbrace{\frac{1}{\xi}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\eta)^{\frac{1-\xi}{\xi}}}_{=-g_{\eta}(\cdot)}$$
$$= \left(\eta\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\eta)^{\frac{1}{\xi}}\cdot\left[\frac{1}{\eta-\eta\cdot\xi}-\frac{1}{\xi-\eta\cdot\xi}\right], \tag{188}$$

from which it follows that the sign of $\Upsilon_{\eta}(\cdot)$, and hence the sign of $\frac{\partial N_{Mt}}{\partial \eta}$, depends only on whether the term in square brackets is positive or negative, which, in turn (and as is clear from observation of the term in square brackets) depends only on whether $\eta > \xi$ or $\eta < \xi$. If $\eta > \xi$, then, based on (185),

$$\frac{\partial N_{Mt}}{\partial \eta} = -\frac{\Upsilon_{\eta}(\cdot)}{\Upsilon_{N_{Mt}(\cdot)}} < 0, \tag{189}$$

whereas if $\eta < \xi$, then, based on (185),

$$\frac{\partial N_{Mt}}{\partial \eta} = -\frac{\Upsilon_{\eta}(\cdot)}{\Upsilon_{N_{Mt}(\cdot)}} > 0. \tag{190}$$

**Limiting Argument.**

This pair of results are limiting arguments that prove that $N_{Mt}$ is indeed maximized (rather than

[24]Or, potentially, minimized; we rule out the minimization outcome below through a limiting argument.

minimized) at $\eta = \xi$; formally, the fact that the left-hand limit

$$\lim_{\eta \to \xi^-} \frac{\partial N_{Mt}}{\partial \eta} = 0 \tag{191}$$

and the right-hand limit

$$\lim_{\eta \to \xi^+} \frac{\partial N_{Mt}}{\partial \eta} = 0 \tag{192}$$

are identical, $N_{Mt}$ is (at least locally) maximized at $\eta = \xi$.

**Recruiting Market Tightness and Remaining Variables.**

Having solved the comparative static results for $N_{Mt}$ with respect to $\eta$, $\frac{\partial N_{Mt}}{\partial \eta}$, the next step is to understand the comparative static results for (equilibrium) recruiting-market tightness $\theta_t$. The comparative static result requires computation of

$$\frac{\partial \theta_t}{\partial \eta} = \frac{\partial \theta_t}{\partial N_{Mt}} \cdot \frac{\partial N_{Mt}}{\partial \eta}. \tag{193}$$

Based on the closed-form equilibrium restriction between $\theta_t$ and $N_{Mt}$ that arises from job-creation directed towards recruiting markets that is stated in general form in expression (165), which, for the sake of convenience, is repeated here,

$$\theta_t = \left( \left( \frac{1-\xi}{\gamma} \right) \cdot m^{EFF} \cdot \left[ \frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) + \xi \cdot \left( \rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \right] \right)^{1/\xi}, \tag{194}$$

the Dixit-Stiglitz version[25] is

$$\theta_t = \left( \left( \frac{1-\xi}{\gamma} \right) \cdot m^{EFF} \cdot \left( \left( \frac{\varepsilon - 1 + \xi}{\varepsilon} \right) \cdot N_{Mt}^{\frac{1}{\varepsilon-1}} + (z_t - \chi) \right) \right)^{\frac{1}{\xi}}.$$

The partial with respect to $N_{Mt}$ is

$$\begin{aligned} &\frac{\partial \theta_t}{\partial N_{Mt}} \\ &= \frac{1}{\xi} \left( \left( \frac{1-\xi}{\gamma} \right) m^{EFF} \left( \left( \frac{\varepsilon - 1 + \xi}{\varepsilon} \right) \cdot N_{Mt}^{\frac{1}{\varepsilon-1}} + (z_t - \chi) \right) \right)^{\frac{1}{\xi}-1} \left( \frac{1}{\varepsilon - 1} \right) \left( \frac{1-\xi}{\gamma} \right) m^{EFF} \left( \frac{\varepsilon - 1 + \xi}{\varepsilon} \right) N_{Mt}^{\frac{1}{\varepsilon-1}-1} \\ &> 0, \end{aligned}$$

which, as stated in the last line, is strictly positive because each term is strictly positive. Thus, based on (193), the sign of $\frac{\partial \theta_t}{\partial \eta}$ is the same as the sign of $\frac{\partial N_{Mt}}{\partial \eta}$.

The signs of the remaining variables ($p_{s_t}$, $p_{v_t}$, $w_t$, $s_t$, $v_t$, $s_{Nt}$, $v_{Nt}$, $n_t$, $c_t$, and $lfp_t$) with respect to $\eta$ then easily follow from the conditions stated at the beginning of Appendix G.

[25] After substituting the Dixit-Stiglitz functions $\rho(N_{Mt}) = N_{Mt}^{\frac{1}{\varepsilon-1}}$, $\mu(N_{Mt}) = \frac{\varepsilon}{\varepsilon-1}$, and $\frac{\rho(N_{Mt})}{\mu(N_{Mt})} = \left(\frac{\varepsilon-1}{\varepsilon}\right) \cdot N_{Mt}^{\frac{1}{\varepsilon-1}}$.

# H Proof of Proposition 3

This Appendix provides the proof for Proposition 3. For simplicity, suppose $\rho = 1$ and $\omega = 1$ and that total vacancy posting costs are linear in both $v_{ij}$ and $v_N$. More precisely, suppose that the cost per vacancy in intermediated markets is $\gamma$ and in non-intermediated markets is $\gamma_N = \gamma + \gamma_{N_0}$, where $\gamma_{N_0} \gtreqless 0$.

Focusing on the Cobb-Douglas matching tedchnology and Dixit-Stiglitz aggregation, the (symmetric equilibrium) marginal cost of creating a new job match is

$$mc(N_M) = \frac{\rho(N_M)}{\mu(N_M)} = N_M^{\frac{1}{\varepsilon-1}} \left( \frac{\varepsilon - 1}{\varepsilon} \right), \tag{195}$$

and the (symmetric equilibrium) relative price of a new job match is

$$\rho(N_M) \left( 1 - \frac{1}{\mu(N_M)} \right) = N_M^{\frac{1}{\varepsilon-1}} \left( \frac{1}{\varepsilon} \right). \tag{196}$$

Based on Dixit-Stiglitz aggregation, the steady-state conditions are the pair of job creation conditions conditions

$$\gamma_N = k_N^f (z - w_N), \tag{197}$$

and

$$\gamma = k^f (z - w) + p_v, \tag{198}$$

the free-entry condition

$$mc(N_M) \cdot \Gamma_E = \rho(N_M) \left( 1 - \frac{1}{\mu(N_M)} \right) m(s, v), \tag{199}$$

the input prices

$$p_v = (1 - \xi) \cdot k^f \cdot mc(N_M) \tag{200}$$

and

$$p_s = \xi \cdot k^h \cdot mc(N_M), \tag{201}$$

the explicit-form real wage expressions in non-intermediated matching

$$w_N = \eta z + (1 - \eta)\chi \tag{202}$$

and in intermediated matching

$$w = \xi z + (1 - \xi)\chi - \xi(1 - \xi)\rho(N_M) \left( 1 - \frac{1}{\mu(N_M)} \right), \tag{203}$$

and the labor force participation conditions for intermediated matching

$$\frac{h'(lfp)}{u'(c)} = k^h w + (1 - k^h)\chi + p_s \tag{204}$$

and for non-intermediated matching.

$$\frac{h'(lfp)}{u'(c)} = k_N^h w_N + (1 - k_N^h)\chi. \tag{205}$$

### H.1 Step 1

To begin, substitute (195) and (196) in the free-entry condition, which gives

$$N_M^{\frac{1}{\varepsilon-1}} \left(\frac{\varepsilon - 1}{\varepsilon}\right) \Gamma_E = N_M^{\frac{1}{\varepsilon-1}} \left(\frac{1}{\varepsilon}\right) m(s, v). \tag{206}$$

Simplifying this expression yields

$$(\varepsilon - 1)\, \Gamma_E = m(s, v), \tag{207}$$

which implies that new job matches in intermediated markets are determined by the exogenous cost of recruiter entry $\Gamma_E$ and the Dixit-Stiglitz elasticity of substitution $\varepsilon$.

Next, recalling that $\gamma_N = \gamma + \gamma_{N_0}$, rewrite the job creation condition in non-intermediated markets as

$$\gamma = k_N^f (z - w_N) - \gamma_{N_0}. \tag{208}$$

Equating the job creation condition (208) for non-intermediated markets with the job creation for intermediated markets (198) gives

$$k_N^f (z - w_N) - \gamma_{N_0} = k^f (z - w) + p_v. \tag{209}$$

Next, substituting $w$ and $w_N$ using the wage expressions (202) and (203) and substituting $p_v$ using expression (200) allows us to rewrite (209) as

$$\begin{aligned} \frac{m(s_N, v_N)}{v_N}(1 - \eta)(z - \chi) - \gamma_{N_0} \quad = \quad & \frac{m(s, v)}{v} \left[ (1 - \xi)(z - \chi) + \xi(1 - \xi) N_M^{\frac{1}{\varepsilon-1}} \left(\frac{1}{\varepsilon}\right) \right] \\ & + (1 - \xi) \frac{m(s, v)}{v} N_M^{\frac{1}{\varepsilon-1}} \left(\frac{\varepsilon - 1}{\varepsilon}\right), \end{aligned} \tag{210}$$

which, after a couple of steps of algebraic rearrangement, can informatively be stated as

$$\frac{m(s_N, v_N)}{v_N}\frac{v}{m(s,v)}\left[\underbrace{(1-\eta)(z-\chi)}_{\text{constant}} - \gamma_{N_0}\frac{v_N}{m(s,v)_N}\right] = \underbrace{(1-\xi)(z-\chi)}_{\text{constant}} + N_M^{\frac{1}{\varepsilon-1}}\underbrace{(1-\xi)\left(\frac{\varepsilon-1+\xi}{\varepsilon}\right)}_{\text{constant}}. \tag{211}$$

Next, the pair of labor-force participation conditions (204) and (205) yield

$$k_N^h w_N + (1-k_N^h)\chi = k^h w + (1-k^h)\chi + p_s, \tag{212}$$

which can be expressed (proceeding in two steps) as

$$\begin{aligned}\left(\frac{m(s_N, v_N)}{s_N}\right)[\eta z + (1-\eta)\chi] + \left(1 - \frac{m(s,v)_N}{s_N}\right)\chi \quad = \quad & \left(\frac{m(s,v)}{s}\right)\left[\xi z + (1-\xi)\chi - \xi(1-\xi)N_M^{\frac{1}{\varepsilon-1}}\left(\frac{1}{\varepsilon}\right)\right] \\ & + \left(1 - \frac{m(s,v)}{s}\right)\chi + \xi\left(\frac{m(s,v)}{s}\right)N_M^{\frac{1}{\varepsilon-1}}\left(\frac{\varepsilon-1}{\varepsilon}\right)\end{aligned}$$

or, equivalently, as

$$\begin{aligned}\left(\frac{m(s,v)_N}{s_N}\right)[\eta\,(z-\chi)+\chi] - \left(\frac{m(s,v)_N}{s_N}\right)\chi \quad = \quad & \left(\frac{m(s,v)}{s}\right)\left[\xi\,(z-\chi) + \chi - \xi(1-\xi)N_M^{\frac{1}{\varepsilon-1}}\left(\frac{1}{\varepsilon}\right)\right] \\ & - \left(\frac{m(s,v)}{s}\right)\chi + \xi\left(\frac{m(s,v)}{s}\right)N_M^{\frac{1}{\varepsilon-1}}\left(\frac{\varepsilon-1}{\varepsilon}\right).\end{aligned}$$

Multiplying both sides by $s/m(s,v)$ and cancelling terms gives

$$\left(\frac{m(s,v)_N}{s_N}\frac{s}{m(s,v)}\right)\underbrace{\eta(z-\chi)}_{\text{constant}} = \underbrace{\xi(z-\chi)}_{\text{constant}} + N_M^{\frac{1}{\varepsilon-1}}\underbrace{\xi\left(\frac{\varepsilon-1-(1-\xi)}{\varepsilon}\right)}_{\text{constant}}. \tag{213}$$

## H.2 Step 2

To summarize, the algebra above delivers three main equations, which are the entry condition in recruiting markets

$$(\varepsilon-1)\,\Gamma_E = m(s,v), \tag{214}$$

the equilibrium labor demand expression

$$\frac{m(s_N, v_N)}{v_N}\frac{v}{m(s,v)}\left[\underbrace{(1-\eta)(z-\chi)}_{\text{constant}} - \gamma_{N_0}\frac{v_N}{m(s,v)_N}\right] = \underbrace{(1-\xi)(z-\chi)}_{\text{constant}} + N_M^{\frac{1}{\varepsilon-1}}\underbrace{(1-\xi)\left(\frac{\varepsilon-1+\xi}{\varepsilon}\right)}_{\text{constant}}, \tag{215}$$

and the equilibrium labor supply expression

$$\left(\frac{m(s_N,v_N)}{s_N}\frac{s}{m(s,v)}\right)\underbrace{\eta(z-\chi)}_{\text{constant}}=\underbrace{\xi(z-\chi)}_{\text{constant}}+N_M^{\frac{1}{\varepsilon-1}}\underbrace{\xi\left(\frac{\varepsilon-1-(1-\xi)}{\varepsilon}\right)}_{\text{constant}}. \tag{216}$$

Substituting (214) into (215) and (216) yields

$$\frac{m(s_N,v_N)}{v_N}\frac{v}{(\varepsilon-1)\,\Gamma_E}\left[\underbrace{(1-\eta)(z-\chi)}_{\text{constant}}-\gamma_{N_0}\frac{v_N}{m(s,v)_N}\right]=\underbrace{(1-\xi)(z-\chi)}_{\text{constant}}+N_M^{\frac{1}{\varepsilon-1}}\underbrace{(1-\xi)\left(\frac{\varepsilon-1+\xi}{\varepsilon}\right)}_{\text{constant}} \tag{217}$$

and

$$\left(\frac{m(s_N,v_N)}{s_N}\frac{s}{(\varepsilon-1)\,\Gamma_E}\right)\underbrace{\eta(z-\chi)}_{\text{constant}}=\underbrace{\xi(z-\chi)}_{\text{constant}}+N_M^{\frac{1}{\varepsilon-1}}\underbrace{\xi\left(\frac{\varepsilon-1-(1-\xi)}{\varepsilon}\right)}_{\text{constant}}. \tag{218}$$

Now, we can write these two conditions, respectively, as

$$\begin{aligned}N_M^{\frac{1}{\varepsilon-1}} &= -\frac{\varepsilon}{(\varepsilon-1+\xi)}(z-\chi)\\ &+\quad \frac{m(s_N,v_N)}{v_N}\frac{\varepsilon v}{(\varepsilon-1+\xi)\,(1-\xi)\,(\varepsilon-1)\,\Gamma_E}\left[(1-\eta)(z-\chi)-\gamma_{N_0}\frac{v_N}{m(s_N,v_N)}\right],\end{aligned} \tag{219}$$

and

$$N_M^{\frac{1}{\varepsilon-1}}=\left[\left(\frac{m(s_N,v_N)}{s_N}\frac{s}{(\varepsilon-1)\,\Gamma_E}\right)\eta-\xi\right]\frac{(z-\chi)\varepsilon}{\xi\,(\varepsilon-1-(1-\xi))}. \tag{220}$$

Equating expressions (219) and (220) yields

$$\begin{aligned}&\frac{m(s_N,v_N)}{v_N}\frac{\varepsilon v}{(\varepsilon-1)\,(\varepsilon-1+\xi)\,(1-\xi)\Gamma_E}\left[(1-\eta)(z-\chi)-\gamma_{N_0}\frac{v_N}{m(s_N,v_N)}\right]\\ &-\quad \frac{\varepsilon}{(\varepsilon-1+\xi)}(z-\chi)=\left[\left(\frac{m(s,v)_N}{s_N}\frac{s}{(\varepsilon-1)\,\Gamma_E}\right)\eta-\xi\right]\frac{(z-\chi)\varepsilon}{\xi\,(\varepsilon-1-(1-\xi))}.\end{aligned} \tag{221}$$

Note that for Cobb-Douglas matching, $\frac{m(s_N,v_N)}{v_N}=\theta_N^{-\xi}$ and $\frac{m(s_N,v_N)}{s_N}=\theta_N^{1-\xi}$; after substituting these Cobb-Douglas expressions and simplifying, we have

$$\begin{aligned}&\theta_N^{-\xi}\frac{\xi\,(\varepsilon-1-(1-\xi))\,v}{(\varepsilon-1)\,(\varepsilon-1+\xi)\,(1-\xi)\Gamma_E}\left[(1-\eta)-\frac{\gamma_{N_0}\theta_N^{-\xi}}{(z-\chi)}\right]\\ &-\quad \frac{\xi\,(\varepsilon-1-(1-\xi))}{(\varepsilon-1+\xi)}=\left(\theta_N^{1-\xi}\frac{s}{(\varepsilon-1)\,\Gamma_E}\right)\eta-\xi,\end{aligned} \tag{222}$$

from which it follows that $\frac{\partial N_M}{\partial \Gamma_E}=0$.