Firm Risk and Leverage-Based Business Cycles

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First Draft: October 2009
This Draft: August 11, 2015

Abstract

I characterize cyclical fluctuations in the cross-sectional dispersion of firm-level productivity. Using the micro-estimated dispersion, or "risk," stochastic process as an input to a baseline small-scale financial accelerator model, I assess how well the model reproduces cyclical movements in both real and financial conditions of the economy. In the model, risk shocks calibrated to micro data lead to empirically-relevant steady-state leverage, a financial measure typically thought to be closely associated with real activity. In terms of aggregate quantities, pure risk shocks in the small-scale general equilibrium model account for a notable share of GDP fluctuations — roughly 5%. The volatility of the risk process I measure using micro data is, remarkably, not very different compared to recent estimates of risk shocks based on medium- or large-scale models using macroeconomic data. These seemingly contrasting starting points for measuring risk shocks do not imply any dichotomy at the core of a popular class of DSGE financial frictions models. Rather, it is the particular transmission channels in financial-frictions models — whether small scale or medium scale — that are critical for aggregate quantity fluctuations to arise based on risk shocks.

Keywords: second-moment shocks, time-varying volatility, credit frictions, financial accelerator
JEL Classification: E10, E20, E32, E44

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Contents

1 Introduction

There are two distinct components of this paper that contribute to the literature on macro-financial accelerator models. The first is an estimation of time-series volatility of "risk shocks" using *microeconomic* data, and the second is an application of the empirical results to a well-known *macroeconomic* framework. The notion of "risk" studied is the standard deviation in any given time period of firm-level idiosyncratic productivity. "Risk shocks" are thus exogenous time variations in this cross-sectional dispersion. On the estimation front, the time-series volatility of idiosyncratic productivity risk is consistent with or a bit larger than other existing *microeconomic* estimates, and is on the same order of magnitude compared to recent *macroeconomic* estimates. Moreover, average productivity and the cross-sectional standard deviation of idiosyncratic productivity are highly countercyclical with respect to each other.

The estimation result leads to the second component of the study, which is a quantitative application to a small-scale general equilibrium financial accelerator framework. Two main results emerge here. First, the endogenous steady-state leverage ratio in the model nearly matches that in the data (0.67 vs. 0.56, respectively), even though the calibration was not designed to do so. Second, using the estimated time-series volatility of cross-sectional risk as a driving force of the model, risk shocks alone generate volatility in standard macro aggregates, such as GDP. Variance decomposition shows that nearly 5% of GDP volatility is accounted for by risk shocks, and the rest is driven by total factor productivity. Given that these are the only two shocks in a small-scale model that, by changing just two parameters, nests the simple real business cycle model, 5% of GDP volatility accounted for by risk shocks is remarkable. Risk shocks also generate fluctuations in financial aggregates such as leverage and bankruptcy — about one-third of the variance in financial aggregates in the model is accounted for by risk shocks.

The empirical side of this paper itself consists of two different parts. First, I characterize fluctuations in firm-level dispersion using U.S. micro data for the period 1973-1988. Specifically, based on data constructed by Cooper and Haltiwanger (2006), I estimate the mean, the persistence, and the time variation in the cross-sectional dispersion of firm-level productivity. The estimates for the mean and persistence parameters are broadly similar to those in existing literature.

The estimated time variation, which is identified in this paper as risk fluctuations, can be compared to both existing *microeconomic* evidence and to recent *macroeconomic* estimates. The measure of firm risk I base on Cooper and Haltiwanger (2006) is strongly countercyclical with respect to GDP, consistent with the micro-level evidence of Bloom, Floetotto, Jaimovich, Saportka-Ekstein, and Terry (2012) — henceforth, BFJST (2012) — and Bachmann and Bayer (2013). In a microeconomic sense, firm risk is quite volatile over the business cycle: measured by the ratio of the standard deviation of *innovations* in risk to average risk, the volatility of annual firm risk is 17 percent. By this metric, volatility of firm risk is similar to that measured by BFJST (2012), but is substantially larger than that measured by Bachmann and Bayer (2013). Comparisons must be made with caution, because the U.S. micro data I examine are different from the U.S. micro data examined by BFJST (2012), which in turn are different from the German micro data examined by Bachmann and Bayer (2013). Nonetheless, the evidence I present complements these and other emerging empirical measures of firm-level risk. The estimated risk shock process is used as an input to the theoretical model.

Before proceeding to theory, though, the second empirical aspect is an extension of the leverage measure provided in Masulis (1988) to cover the time period 1973-1988, which has two advantages.[1] One advantage is to allow for clean comparability with the period over which the risk shock process is estimated. The other is that the Masulis-based leverage series also permits some comparability with a component of the Christiano, Motto, and Rostagno (2014) *macroeconomic* estimation, about which more is described soon.[2]

In terms of theory, I deploy the estimated risk shock process in the Carlstrom and Fuerst (1997) agency-cost "investment model." A closely-related existing study is Dorofeenko, Lee, and Salyer (2008) — henceforth, DLS — which also analyzes the importance of risk shocks in the Carlstrom and Fuerst (1997) model. The marginal contribution relative to DLS is the *estimation* of risk shock parameters rather than assumptions about them. The crucial risk-shock volatility parameter I *estimate* is an order of magnitude larger than their *assumed* parameter. Not surprisingly, the quantitative importance of risk shocks alone in generating fluctuations turns out be much larger in my results compared with the results in DLS.

Comparing my results to those of the prominent recent study by Christiano, Motto, and Rostagno (2014) — henceforth, CMR — my estimated volatility of risk is of the same order of magnitude. In a medium-scale model, CMR estimate, among a host of other parameters, a risk shock process based on *macroeconomic* data.[4] In contrast, my estimated risk shock process is based on *microeconomic* data. The details of my estimation are described in Section 2 and Section 5; but, to

[1] I thank Ana Lariau for her tenacity in locating the data in the IRS Corporate Income Tax Returns database in order to be able to extend the Masulis (1988) series. The leverage data provided in Masulis (1998) ended in 1984.

[2] Leverage, defined as the debt-to-asset ratio, is often thought to play a central role in connecting financial and real activity. In principle, model-based leverage fluctuations have the potential to drive, or at least be associated with, real fluctuations. Such "leverage-based business cycles" could arise through fluctuations in firms' balance sheet conditions that are induced by risk shocks. The transmission channel that the model emphasizes and tests is thus explicitly financial: if there were no agency costs in financial markets, there is no channel by which risk shocks could affect real fluctuations at all.[3] This aspect of the model is similar to the qualitative business cycle model of Williamson (1987) and the quantitative models of Dorofeenko, Lee, and Salyer (2008), Christiano, Motto, and Rostagno (2014), and others.

[4] The CMR framework's starting point is the sticky-price financial accelerator model of Bernanke, Gertler, and Gilchrist (1999).

compare the results in a macro setting, my estimated volatility of the crucial parameter for cross-sectional risk is 50% of the most comparable estimate in CMR. This result is remarkable given the completely different approaches CMR and I use in estimating this crucial parameter.

Taken together, my results (which impose microeconomic discipline), the results of DLS, and the results of CMR (which impose macroeconomic discipline) may raise an issue about the "proper" way to parameterize risk shocks in agency-cost accelerator models — use of micro data vs. macro data. That is, the contrasting starting points for empirically measuring risk shocks seem to raise a tension between a micro-calibration approach and a macro-calibration approach. This tension does not have to be portrayed in a negative light. Rather, it indicates that further research is required.

Finally, two points regarding modeling approach are in order. First, as should be clear from the discussion so far, the idea of "risk shocks" in this paper is variations over time in the cross-sectional standard deviation of firm-level productivity, holding constant average productivity. This is the same notion of idiosyncratic "second-moment shocks" that BFJST (2012), Bachmann and Bayer (2013), CMR, and DLS study. However, it is distinct from an aggregate notion of "second-moment shocks" emphasized by Justiniano and Primiceri (2008), Fernández-Villaverde and Rubio-Ramirez (2007), Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramirez, and Uribe (2011), Gourio (2013), Basu and Bundick (2011), and others, in which the standard deviation of the innovations affecting aggregate driving processes such as productivity, real interest rates, and monetary disturbances vary over time. Crucial in this latter group of studies is that they are all representative-agent economies, so there is no meaningful concept of cross-sectional dispersion. Focusing on the cross section is the main idea in BFJST (2012), Bachmann and Bayer (2013), CMR, DLS, and this paper. Second, echoing the well-articulated argument in Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramirez, and Uribe (2011), I treat cross-sectional risk fluctuations as exogenous. How to endogenize such fluctuations is an interesting question, but because the empirical evidence in this area is so new and fast moving, I adopt the view that this is not the first natural question to consider. Instead, I focus on the consequences of such fluctuations as mediated through agency-cost frictions, which focuses attention on the transmission mechanism of risk shocks. Regarding terminology, I use the terms "risk shocks," "firm-level risk," "second-moment shocks," and "dispersion shocks" interchangeably.

The rest of the paper is organized as follows. Section 2 presents new empirical evidence on firm-level risk and its business cycle properties. Section 3 briefly presents the aggregate leverage data used as a financial point of comparison for the model's results. Section 4 presents the baseline model, in which shocks to average productivity and risk shocks are independent exogenous processes. Section 5 describes (based on the results from Section 2) how the crucial risk-shock volatility parameter is determined and then presents quantitative results, which are supplemented by further analysis in the Appendix. Section 6 concludes.

2 Risk Fluctuations

The main goal of this section is to document the properties of business cycle fluctuations in firm-level dispersion. The analysis is based on a balanced panel, constructed by Cooper and Haltiwanger (2006), from the Longitudinal Research Database (LRD). The data are annual observations of plant-level measures such as revenue, materials and labor costs, and investment at approximately 7,000 large U.S. manufacturing plants over the period 1974-1988. The starting point for the analysis is Cooper and Haltiwanger's (2006) measures of plant-level profitability residuals from this panel.[5]

Briefly, Cooper and Haltiwanger (2006) compute for each plant i in year t a residual A_{it} that reconciles exactly the observations of plant i's profits and capital stock in year t when described by a profit function that depends only on the capital stock.[6] The year-specific aggregate residual z_t is computed as the mean of A_{it} across firms in year t. Plant i's profit function in year t is viewed as being shifted by both the aggregate shock z_t and an idiosyncratic shock $\omega_{it} \equiv A_{it}/z_t$. In each year, there is thus a cross-sectional distribution of ω_{it}. Denote by σ_t^ω the cross-sectional standard deviation in year t of the idiosyncratic component of profitability ω_{it}. I make three identifying assumptions regarding ω_{it} and thus the interpretation of its cross-sectional dispersion σ_t^ω. These assumptions align the analysis of the data with the model into which they will be an input.

First, although σ_t^ω measures cross-firm *dispersion*, I treat it as measuring true cross-firm *risk*.[7] The two concepts are identical only if each firm's idiosyncratic component ω_{it} has zero persistence. Cooper and Haltiwanger (2006, p. 622-623) estimate an AR(1) coefficient of the idiosyncratic component of 0.885, hence ω_{it} is actually quite persistent (recall the data are annual). However, it is computationally difficult to handle persistent idiosyncratic shocks in the theoretical model developed below, so the model assumes *iid* idiosyncratic shocks.[8] To align the empirical analysis of σ_t^ω with its role in the model, I thus proceed by assuming zero idiosyncratic persistence.

There are both advantages and potential drawbacks of this approach. An advantage is that the dispersion of firm-level *outcomes* in the model are thus calibrated to the data. A potential drawback is that σ^ω is thus an overestimate of firm-level risk, which, when input as an exogenous process to the model, in principle gives risk shocks the largest possible role in driving the model's fluctuations. As the quantitative results in Section 5 show, risk shocks alone turn out to drive about 5% of aggregate fluctuations.

The second identifying assumption is that firm-level *profitability* shocks are true *productivity* shocks. Because plant-level price deflators are unavailable in the dataset, it is impossible to dis-

[5] I thank John Haltiwanger for providing their aggregative data on profitability residuals.

[6] The Appendix in Cooper and Haltiwanger (2006) describes in detail the construction of the data and the residuals.

[7] Which is the basis for my interchangeable references to firm-level "dispersion" and firm-level "risk."

[8] To my knowledge, no DSGE models based on the agency-cost framework have been solved assuming persistent idiosyncratic shocks.

tinguish cost shocks from revenue shocks, so the ω_{it} residuals mix both supply and demand shifts (hence the term "profitability" shocks).[9] As an identifying assumption for the theoretical model, I simply interpret these profitability shocks as true productivity shocks. Thus, one can think of this aspect of the data analysis as also being conducted strictly through the lens of the model.

Third, when deploying the evidence documented here in the model, I identify "plants" as "firms," abstracting from the fact that a non-negligible share of plant-level output in the LRD represents output of multi-plant firms. With these three identifying assumptions, I characterize the business cycle behavior of both z_t and of σ_t^ω, aspects of the data not studied by Cooper and Haltiwanger (2006).

2.1 Productivity Risk

I first compute the cross-sectional coefficient of variation of productivity (profitability) for each of the 15 years of the sample. Cross-sectional coefficients of variation are used because the residually-computed aggregate mean level of productivity (z_t) is not unity in the data, but it is normalized to unity in the model below. The time-averaged mean of the cross-sectional coefficient of variation is 0.156, hence I normalize long-run dispersion in the model to $\bar{\sigma}^\omega = 0.156$. Given the discussion above, true long-run "risk" is smaller than $\bar{\sigma}^\omega = 0.156$. Specifically, taking a stationary AR(1) view of idiosyncratic productivity and using the Cooper and Haltiwanger (2006, p. 622-623) estimate of idiosyncratic persistence of 0.885, true long-run firm-level risk is $\sqrt{1-0.885^2}\bar{\sigma}^\omega = 0.0726$. Aligning the empirical analysis with the model thus overstates firm-level risk by roughly a factor of two.

Figure 1 plots the time series σ_t^ω, which suggests a modest upward trend in dispersion. Figure 2 displays the HP-filtered components of σ_t^ω and GDP over the period 1974-1988. A clear negative cyclical correlation between the two series is apparent — the contemporaneous correlation between the two series is -0.83, hence expansions are associated with a decrease in dispersion of firms' idiosyncratic productivity, and recessions are associated with an increase in dispersion of firms' idiosyncratic productivity. Strongly countercyclical firm-level risk is also a robust finding in the micro evidence of Bachmann and Bayer (2013) — hereafter, BB (2013) — and BFJST (2012). In terms of volatility, the standard deviation of the cyclical component of σ_t^ω is 3.15 percent over the sample period. With an innocuous abuse of notation, I hereafter use σ_t^ω to denote the cyclical component of cross-sectional dispersion.

In the model presented in Section 4, I suppose that σ_t^ω follows the exogenous AR(1)

$$\ln \sigma_{t+1}^\omega = (1-\rho_{\sigma^\omega}) \ln \bar{\sigma}^\omega + \rho_{\sigma^\omega} \ln \sigma_t^\omega + \epsilon_{t+1}^{\sigma^\omega}, \tag{1}$$

with $\epsilon^{\sigma^\omega} \sim N(0, \sigma_{\sigma^\omega})$. Given $\bar{\sigma}^\omega = 0.156$, the point estimate (using OLS) of the AR(1) parameter

[9] More precisely, they are available only at five-year intervals, too low a frequency for business cycle analysis.

is $\rho_{\sigma^\omega} = 0.48$, with a t-statistic of 1.93. With this estimate of ρ_{σ^ω} and the standard deviation of σ_t^ω of 3.15 percent, the standard deviation of the (annual) *innovations* to the cross-firm dispersion process can be computed to be 0.0276. This implies a coefficient of variation (with respect to the mean dispersion $\bar{\sigma}^\omega = 0.156$) of 17.7 percent, which can be directly compared to the empirical evidence reported by BB (2013) and BFJST (2012). Computed in a variety of ways, BB (2013) find a coefficient of variation of innovations to firm-level productivity for their entire sample of German firms between two and three percent. However, because the Cooper and Haltiwanger (2006) analysis is of large manufacturing plants, the most comparable result in BB (2013) is their finding for the largest (ranked by employment) five percent of firms in their sample. For this sample, BB (2013) find a coefficient of variation of firm-level innovations of 5.5 percent (see their Table 8). The 17.7 percent coefficient of variation of plant-level innovations in the Cooper and Haltiwanger (2006) sample is thus substantially larger than the largest firms in BB (2013)'s sample. However, this degree of volatility of firm risk lines up much better with the evidence of BFJST (2012), who document using a variety of cross-sectional measures that dispersion of firm outcomes rises very sharply during recessions.

2.2 Average Productivity

For further consistency in the way the firm-level data are used as an input to the model, I also characterize the time-series behavior of z_t, the average productivity (profitability) residual. In the model, this measure will correspond conceptually to the standard notion of aggregate productivity (i.e., the first moment of the productivity distribution). Figures 3 and 4 display the actual series, its HP trend, and the cyclical component of average productivity.[10]

The cyclical component of z_t is highly correlated with the cyclical component of GDP, as Figure 4 shows — the contemporaneous correlation between the two is 0.87. The volatility of the cyclical component of z_t is 1.26 percent (at an annual horizon). Again with an innocuous abuse of notation, I hereafter use z_t to denote the cyclical component of average productivity.

In the model presented below, I suppose that z_t follows the exogenous AR(1)

$$\ln z_{t+1} = \rho_z \ln z_t + \epsilon_{t+1}^z, \tag{2}$$

with $\epsilon^z \sim N(0, \sigma_z)$. Estimation gives a point estimate $\rho_z = 0.48$, with a t-statistic of 1.84.[11] With this estimate of ρ_z and the standard deviation of z_t of 1.26 percent, the standard deviation of the (annual) *innovations* to the average productivity process can be computed to be 0.0111. Finally,

[10] As noted above, long-run average productivity is normalized to unity in the model, so the vertical scale in Figure 3 is arbitrary. In the empirical analysis of Cooper and Haltiwanger (2006), mean productivity was not normalized.

[11] This differs from Cooper and Haltiwanger's (2006, p. 623) estimate of the persistence of mean productivity because they do not detrend; the AR(1) coefficient of the unfiltered z_t series is 0.76.

the cyclical correlation between average productivity and the dispersion of productivity (i.e., the concept of firm risk) is -0.97; this extremely strong negative correlation is part of the motivation of the "bundled-shock" model extension considered in Appendix B.

In the model developed in Section 4, I pursue annualizations of a quarterly calibration because the leverage evidence documented in Section 3 is annual. Because the evidence presented in this section is from annual data, I use persistence parameters of $\rho_{\sigma^\omega} = 0.48^{0.25} = 0.83$ and $\rho_z = 0.48^{0.25} = 0.83$, which assumes smoothness in the processes during the year. How this inference of quarterly persistence from annual estimates affects the model calibration of the innovation parameters σ_{σ^ω} and σ_z is deferred to Section 5.2.

One final note is helpful: a concern may be the slight downward trend in "productivity" in the manufacturing sector during 1974-1988. Keep in mind, however, that (apart from the sharp recessions during this period) what is being measured is actually profitability residuals. If the relative prices of the inputs, capital and labor, trended during this period, this would show up as a trend in profitability.[12] As the results above show, the final AR(1) stochastic process that describes average profitability/productivity is very similar to a simple RBC model's average productivity process. So, if one prefers, one can think of the AR(1) process as an illustrative, off-the-shelf RBC-style process, the precise parameter settings for which are not crucial to the main conclusions of the paper.

3 Leverage Data

Table 1 displays aggregate leverage data for the manufacturing sector. The data through 1984 were obtained from Masulis (1988) and extended to include the years 1985, 1986, 1987, and 1988 so that it covers the time period used in estimating the risk shock process. As noted in the Introduction, CMR's estimation is partly based on the Masulis (1988) data.

As also noted in the Introduction, the risk-shock calibration of Section 2 turns out to endogenously match quite well the time-series average of empirical leverage as provided in Masulis (1988) and extended to include the years 1985 — 1988 so that it covers the time period used in estimating the risk shock process. The average leverage ratio during 1973-1988 was 0.56, which is quite close to the 0.67 in the calibrated model of Section 5, even though leverage was not a calibration target.

[12] I thank Larry Ball and Chris Carroll for raising these points.

4 Model

As described in the Introduction, the model is based on the well-known agency-cost frameworks used by Carlstrom and Fuerst (1997, 1998).[13] The model is most directly based on the "investment model" of Carlstrom and Fuerst (1997), in which it is only capital-goods producers that are subject to financing constraints, all prices are flexible, and there are no other rigidities or frictions whatsoever. This provides the cleanest model to evaluate the role of empirically-relevant shocks to firm risk, so I refer to the Carlstrom and Fuerst (1997) — henceforth, CF — investment model as "the" underlying model, recognizing that it is meant to capture an entire literature of work. In a study with a very similar motivation, DLS also study the role of risk shocks in the CF model; DLS parameterize the risk process in an illustrative way, rather than calibrating it to micro data as I do.[14]

As an aid to the ensuing description of the model, Figure 5 illustrates the timing of events in the model. Because the model is virtually identical to the CF investment model, with only a couple of modifications made to align the model with the data analysis in Sections 2 and 3, readers familiar with the CF model may prefer to skip to the analysis beginning in Section 5.

4.1 Households

A representative household maximizes expected lifetime discounted utility over streams of consumption c_t and labor n_t,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(n_t) \right], \tag{3}$$

subject to the sequence of flow budget constraints

$$c_t + q_t k_{ht+1} = w_t n_t + k_{ht} \left[r_t + q_t (1-\delta) \right] + \Pi_t. \tag{4}$$

The functions $u(.)$ and $v(.)$ are standard strictly-increasing and strictly-concave subutility functions over consumption and labor, respectively. The rest of the notation is as follows. The household's subjective discount factor is $\beta \in (0,1)$, k_{ht} denotes the household's capital holdings at the start of period t, w_t is the real wage that is taken as given, r_t is the market rental rate on capital that is also taken as given, δ is the per-period depreciation rate of capital, and q_t denotes the price (which is endogenous in equilibrium) of one unit of the capital good in terms of consumption goods. The price

[13] Bernanke, Gertler, and Gilchrist (1999) recasts the Carlstrom and Fuerst (1997) framework to allow for nominal rigidities and monetary policy.

[14] In a previous version of this paper, I used the risk shock estimates from Section 2 in the Carlstrom and Fuerst (1998) output model. However, because virtually all of the agency-cost-based financial frictions literature uses as its starting point the investment model, the results of deploying the micro-estimated risk shocks in the investment model are easier to compare to existing literature and are more informative for future research.

q_t is the model's notion of Tobin's q. The household also receives aggregate dividend payments Π_t from capital-producing firms as lump-sum income, the determination of which is described below.[15]

Emerging from household optimization is a completely standard labor supply condition

$$-\frac{v'(n_t)}{u'(c_t)} = w_t, \tag{5}$$

and a capital Euler condition

$$q_t u'(c_t) = \beta E_t \left\{ u'(c_{t+1}) \left[r_{t+1} + q_{t+1}(1-\delta) \right] \right\}, \tag{6}$$

which follow as usual from the household's period-t first-order conditions with respect to c_t, n_t, and k_{ht+1}. The one-period-ahead stochastic discount factor is defined as $\Xi_{t+1|t} = \beta u'(c_{t+1})/u'(c_t)$, with which both capital-producing entrepreneurs and consumption-producing firms, in equilibrium, discount profit flows.

4.2 Production

There is a mass of identical representative firms. Within a representative firm, there is a mass η of "entrepreneurs" that produce capital goods and a unit mass of Walrasian consumption good producers. Only the entrepreneurial side of the representative firm can operate the investment-production (capital-production) technology. We will refer to the "entrepreneurs" interchangeably as "capital producers," or, on occasion (because there is a large mass of them), "capital-producing firms." The capital producers are heterogenous in their productivity and operate semi-autonomously, with the objectives of the representative firm (and therefore, ultimately, the representative household) in mind. An entrepreneur i produces capital using a linear technology, the details of which are described next.

4.2.1 "Entrepreneurial" Capital-Producing Firms

Each period, entrepreneur i's idiosyncratic productivity is a draw from a distribution with cumulative distribution function $\Phi(\omega)$, which has a constant mean $E(\omega_t) = 1$, a time-varying standard deviation σ_t^ω, and associated density function $\phi(\omega)$, all of which are identical across entrepreneurs. As in DLS, the time-varying volatility σ_t^ω is the key innovation in the model compared to CF. Given the constant first and time-varying second moments $E(\omega_t) = 1$ and σ_t^ω common across

[15] I could also introduce two other assets in the budget constraint. First, shares could be included in order to directly price streams of dividends paid by capital-producing firms to households. Second, an intra-period bond could be explicitly introduced, which would be the source of funding for the capital-producing firms — in the terminology used by CF, this source of funds is "capital mutual funds" (which is shown in the acronym CMR in the timeline in Figure 5); anticipating equilibrium, the gross return on the intra-period bond would be $R_t = 1$. Thus, because neither of these extra details are necessary for the main results, they are omitted from the budget constraint.

capital-producing firms, idiosyncratic productivity for a given firm is i.i.d. over time, an assumption made for tractability.[16]

As noted above, in aggregate, capital-producing firms are owned by households, and the objective of capital producers is to maximize the expected present discounted value of dividends remitted to households. Denote by Π_{it} the dividend payment made by capital-producer i to households. For descriptive convenience, I decompose Π_{it} into a "non-retained earnings" component Π^e_{it} and a retained earnings — or, in terminology used in Section 4.2.3 below, an asset-evolution — component.

Because they are ultimately owned by households, capital producers apply the representative household's stochastic discount factor (the one-period-ahead discount factor is $\Xi_{t+1|t}$, as defined above) to their intertemporal optimization problem. However, capital producers are also assumed to be "more impatient" than households by the factor $\gamma < 1$, which can be thought of as a principal-agent problem, or, more broadly, as semi-autonomy, that prevents perfect alignment of the capital producers' managerial objectives with the overall large firms' (in turn, households') intertemporal preferences.[17] At a technical level, $\gamma < 1$ ensures that capital producers cannot accumulate enough assets to become self-financing, which would render irrelevant the financial frictions described below.[18] This device for avoiding self-financing outcomes is common in models of financial frictions.

[16] The assumption of zero persistence of the idiosyncratic component of a firm's productivity was noted in Section 2, and it greatly simplifies the computation of the model because the firm sector essentially can be analyzed as a representative agent. This point is discussed further below when I come to the aggregation of the model. This simplification still allows me to illustrate the main point of the model, which is that variations in cross-sectional productivity dispersion can lead to fluctuations in aggregate financial measures and economic activity. In addition to greatly reducing the computational burden, the assumption of zero persistence in idiosyncratic shocks also retains the simplicity of the CF and Bernanke and Gertler (1989) contracting specifications. If persistent shocks were allowed, it is not clear that the simple debt contracts of these models could not be improved upon by the contracting parties by, say, multi-period contracts. Sidestepping this issue is yet another reason to assume no persistence in realized idiosyncratic productivity. Note, however, that assuming persistence in shocks to σ^ω_t, as the empirical results in Section 2 indicate, does not pose any of these problems; indeed, shocks to σ^ω_t really are aggregate shocks.

[17] A prime example is that when Disney acquired Pixar in 2006, one of the conditions was that Pixar would operate autonomously of Disney and Disney Animation. It's turned out magically since then — Pixar has produced smash hit after smash hit, Pixar continues to maintain its own email system, and, to date, not one employee has been "encouraged" by Disney to shift to Walt Disney World in Florida or end a telephone call with "Have a magical day." (Source: http://www.slideshare.net/AdwitiyaTiwari/disney-pixar-ma). Yet another, more recent, example is Apple's 2014 acquisition of Beats Music and Beats Electronics — Beats operates autonomously of Apple.

[18] The quantitative results of this paper are virtually the same if either the "dying-on-the-corner" assumption (in which capital producers would simply discount by $\gamma\beta\ \forall t$, rather than by $\gamma\Xi_{t+1|t}$) or the *equilibrium* risk-neutrality (in which each capital producer has its own "linear utility function") assumption were instead used. For the sake of parsimony, I have not included the results from these variants of the model.

4.2.2 Capital-Producers Financing and Contractual Arrangement

This section describes the financial arrangements of the model. To facilitate comparison of general equilibrium with optimization by capital producers, the following intuitive description of state variables is helpful. From a general equilibrium perspective, financial outcomes are contingent on the exogenous aggregate state $(z_t, E(\omega_t), \sigma_t^{\omega})$ of the economy.[19]

From capital-producing firm i's partial equilibrium perspective, financial outcomes also take as given net worth nw_{it} and Tobin's q_t, each of which is determined in other markets (which in turn are contingent on the aggregate state $(z_t, E(\omega_t), \sigma_t^{\omega})$). As in CF and as shown in Figure 5, every capital producer is assumed to commit to using all of its inputs for production after observing the aggregate exogenous state $(z_t, E(\omega_t), \sigma_t^{\omega})$, but before observing its idiosyncratic realization ω_{it}.

Part of the financing of the capital producer's costs comes from its own accumulated net worth, which is held primarily in the form of capital. The capital that each capital producer accumulates is rented on spot markets to Walrasian consumption-goods producing firms, just like households rent their capital on spot markets. Entrepreneur i's capital holdings at the start of period t are k_{it}^e.

However, the entrepreneur's internal funds (which I refer to interchangeably as its net worth or its equity) are insufficient to cover all input costs. To finance the remainder, a capital producer must borrow short-term — formally, intraperiod. A capital producer requires external financing because of the "semi-autonomy" assumption that it is more impatient than households, as described above.[20] By acquiring external funds, the capital producer is able to leverage its net worth in period t,

$$nw_{it} = k_{it}^e \left[r_t + q_t(1-\delta)\right] + e_t, \tag{7}$$

into production of investment goods i_{it}. Total borrowing by the generic capital-producing firm is thus $i_{it} - nw_{it}$. The component e_t of net worth in expression (7) is a small amount of "endowment income" that each capital producer receives to ensure its continued operations in the event that it was unable to repay its debt and thus had to undergo costly reorganization in the previous period. In closing the model, this endowment is absorbed into the payout Π_{it} the capital producer pays to its owners, which is the representative household. The payout Π_{it} is thus interpreted as net of the endowment e_t.[21]

[19] Although $E(\omega_t) = 1\ \forall t$, it is maintained as a state variable for the sake of generality.

[20] As noted above, this is a standard assumption in this class of models and avoids the self-financing outcome. See, for example, Carlstrom and Fuerst (1997, 1998) and Bernanke, Gertler, and Gilchrist (1999).

[21] Thus, e_t can loosely be interpreted as a lump-sum transfer of "startup funds" provided by households to capital-producing firms, as in Gertler and Karadi (2011). By allowing a "firm's" operations to continue in the event of bankruptcy, the assumption of a startup fund brings great analytical tractability to the model. Thus, the "costs of bankruptcy" in the model are more properly interpreted as "costs of reorganization" without any disruption of its output-producing activities (i.e., bringing in new management to oversee ongoing operations).

I describe only briefly the outcome of the contracting arrangement between borrowers (capital-producing firms) and lenders (households) because it is standard in this class of models.[22] The financial contract is a debt contract, which is fully characterized by a reorganization threshold $\bar{\omega}_t$ and a loan size $i_{it} - nw_{it}$. A firm must be "reorganized" if its realized productivity ω_{it} is below the contractually specified threshold $\bar{\omega}_t$. Below this endogenous threshold, capital-producer i does not have enough resources to fully repay its loan. In that case, the capital producer must undergo reorganization and receives nothing, while the lender must pay reorganization costs that are proportional to the total output of the firm and receives, net of these reorganization costs, all of the output of the entrepreneur. Note again that all capital producers, regardless of whether or not they end up requiring reorganization, do produce capital up to their full (idiosyncratic) capacity.

Define by $f(\bar{\omega}_t)$ the expected share of idiosyncratic output $\omega_{it} i_{it}$ the borrower (the capital producer) keeps after repaying the loan, and by $g(\bar{\omega}_t)$ the expected share received by the lender.[23] These expectations are conditional on the realization of the time-t exogenous aggregate state $(z_t, E(\omega_t), \sigma_t^{\omega})$, but before revelation of a firm's idiosyncratic productivity ω_{it}. The contractually-specified project size is characterized by a zero-profit condition on the part of lenders,

$$i_{it} = \frac{nw_{it}}{1 - q_t g(\bar{\omega}_t)}, \tag{8}$$

and the contractually-specified reorganization threshold is characterized by

$$\frac{q_t f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} = -\frac{f'(\bar{\omega}_t)}{g'(\bar{\omega}_t)}, \tag{9}$$

in which Tobin's $q_t > 1$ arises solely from the external financing needs of the entrepeneur.[24]

The loan size $i_{it} - nw_{it}$ is firm-specific. However, the reorganization threshold $\bar{\omega}_t$ is not because idiosyncratic productivity has zero persistence. Condition (9) thus implies that the Tobin q_t is also identical across firms, which is the key result that makes aggregation in the model simple, which

[22] In the context of general equilibrium settings, familiar expositions appear in Carlstrom and Fuerst (1997, 1998), Bernanke, Gertler, and Gilchrist (1999), and, in an application to labor search and matching, Chugh (2013). In partial-equilibrium settings, analysis of this type of contractual arrangement traces back to Townsend (1979), Gale and Hellwig (1985), and Williamson (1987).

[23] Formally, $f(\bar{\omega}_t) \equiv \int_{\bar{\omega}_t}^{\infty} (\omega_i - \bar{\omega}_t)\, \phi(\omega_i) d\omega_i = \int_{\bar{\omega}_t}^{\infty} \omega_i \phi(\omega_i) d\omega_i - [1 - \Phi(\bar{\omega}_t)]\bar{\omega}_t$ is the share received by the firm, and $g(\bar{\omega}_t) \equiv \int_0^{\bar{\omega}_t} (\omega_i - \mu)\, \phi(\omega_i) d\omega_i + \int_{\bar{\omega}_t}^{\infty} \bar{\omega}_t \phi(\omega_i) d\omega_i = \int_0^{\bar{\omega}_t} \omega_i \phi(\omega_i) d\omega_i + [1 - \Phi(\bar{\omega}_t)]\bar{\omega}_t - \mu\Phi(\bar{\omega}_t)$ is the share received by the lender.

[24] The background assumptions of the zero profit condition are that lending is a perfectly competitive activity and entry into lending is costless (which is simply a summary of CF's "capital mutual funds" which they, and thus Figure 5, denote as CMR.) Formally, the two conditions characterizing the optimal contract result from maximizing (the entrepreneur's share of) the return on the financial contract (because the entrepreneur, if it remains solvent, is the residual claimant on output), $q_t f(\bar{\omega}_t) i_{it}$, subject to the zero profit condition of the lender, $q_t g(\bar{\omega}_t) i_{it} = i_{it} - nw_{it}$. Define λ_t as the shadow value on the zero-profit constraint.

in turn justifies omission of entrepreneur-i indexes for the variables q and $\bar{\omega}$. Finally, the contract multiplier λ_t that is associated with conditions (8) and (9) is

$$\lambda_t = \frac{q_t f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)}. \tag{10}$$

An interesting point regarding λ_t is quantified in Section 5.

4.2.3 Asset Evolution of Capital Producers

Capital-producers take as given contractual outcomes when maximizing profits. Regarding their dynamic aspect, capital producer i begins period t with assets k^e_{it}, whose beginning-of-period-t market value determines its net worth nw_{it}, as shown in (7). The entrepreneur borrows $i_{it} - nw_{it}$ against the value of these assets, and it expects to keep $q_t f(\omega_t) i_{it}$ after repaying its loan.[25] Of these "excess" resources, the entrepreneur can either accumulate assets or make payments to households. That is,

$$\Pi^e_{it} + q_t k^e_{it+1} = q_t f(\bar{\omega}_t) i_{it}, \tag{11}$$

which highlights that $q_t k^e_{it+1}$ can be thought of as retained earnings, as mentioned in Section 4.2.1. Substituting the contractually-specified project size, $i = \frac{nw}{1-qg(\bar{\omega})}$, this can be re-written as

$$\Pi^e_{it} + q_t k^e_{it+1} = \frac{q_t f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} nw_{it}. \tag{12}$$

Further substituting the definition of net worth from (7), the entrepreneur's asset evolution is described by

$$\Pi^e_{it} + q_t k^e_{it+1} = \frac{q_t f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} \left(k^e_{it} \left[r_t + q_t (1-\delta) \right] + e_t \right). \tag{13}$$

4.2.4 Walrasian Producers...

The representative Walrasian firm hires on spot markets existing capital goods k_{it} and labor n_{it} to produce the final consumption good via the constant-returns-to-scale aggregate technology $z_t F(k_{it}, n_{it})$. The total cost of Walrasian production is $w_t n_{it} + r_t k_{it}$, hence a Walrasian firm's period-t profits are $z_t F(k_{it}, n_{it}) - w_t n_{it} - r_t k_{it}$.

[25] This is because, as noted in footnote 24, the entrepreneur keeps the entire (expected) surplus from the contractual arrangement. Hence, in expectation, the entrepreneur is left with $q_t f(\omega_t) i_{it}$ after the sequence of borrowing, renting factors of production, producing capital-goods output, and repaying its loan.

4.2.5 ...and Profit Maximization

Finally, the dynamic profit function of the "large" representative firm ("large" because of the semi-autonomous measure of capital-good entrepreneurs within the representative firm) is

$$E_0 \sum_{t=0}^{\infty} \gamma^t \Xi_{t|0} \left\{ \eta \left(\frac{q_t f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} \left(k_{it}^e \left[r_t + q_t(1-\delta) \right] + e_t \right) - q_t k_{it+1}^e \right) + z_t F(k_{it}, n_{it}) - w_t n_{it} - r_t k_{it} \right\}. \tag{14}$$

Maximization of (14) with respect to capital rental k_{it} and labor hiring n_{it} gives rise to the capital demand condition

$$r_t = z_t F_k(k_{it}, n_{it}) \tag{15}$$

and the labor demand condition

$$w_t = z_t F_n(k_{it}, n_{it}). \tag{16}$$

Maximization of (14) with respect to entrepreneurial asset accumulation k_{it+1}^e yields the capital Euler equation for the semi-autonomous entrepreneurs,

$$q_t = \gamma E_t \left\{ \Xi_{t+1|t} \left(\frac{q_{t+1} f(\bar{\omega}_{t+1})}{1 - q_{t+1} g(\bar{\omega}_{t+1})} \right) \left[r_{t+1} + q_{t+1}(1-\delta) \right] \right\}, \tag{17}$$

which, note, is independent of entrepreneur-i conditions.

4.2.6 Aggregation

Capital-producing firms are heterogenous with respect to their net worth and differ (only) in size — a firm with a larger net worth receives a proportionately larger loan and so produces more capital. However, the size distribution of firms is irrelevant for computing prices and hence aggregates in the economy, which makes the agency-cost framework tractable in a DSGE setting. The capital-production side of the economy can thus be analyzed as if there were a representative entrepreneur that held the average quantity of net worth. The specific assumptions and results behind this aggregation result are: the constant-returns nature of the production function $F(.)$; the linearity of the monitoring technology (in the quantity monitored); and, crucially, the result that the price r_t and Tobin's q_t are identical for all production units.[26]

The stand-in "large" representative firm has a profit function identical to (14) (with firm indices dropped), which clearly gives rise to the same optimality conditions (15), (16), and (17). The (aggregate) profits that get transferred to households are thus

$$\Pi_t \quad = \Pi_t^e \quad = \frac{q_t f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} \left(k_t^e \left[r_t + q_t(1-\delta) \right] + e_t \right) - k_{t+1}^e + z_t F(k_t, n_t) - w_t n_t - r_t k_t$$

[26]The result that q is identical for all firms is an implication of zero persistence of firms' idiosyncratic productivity, which, as described above, makes it impossible to condition the contractually-specified liquidation threshold $\bar{\omega}$ on production-unit-specific variables. See also CF (1997, 1998) for further discussion.

$$
\begin{aligned}
&= \frac{q_t f(\bar{\omega}_t)}{1-q_t g(\bar{\omega}_t)} \left(k_t^e\left[1+r_t-\delta\right]+e_t\right) - k_{t+1}^e + z_t F(k_t,n_t) - z_t F_n(k_t,n_t)n_t - z_t F_k(k_t,n_t)k_t \\
&= \frac{q_t f(\bar{\omega}_t)}{1-q_t g(\bar{\omega}_t)} \left(k_t^e\left[1+r_t-\delta\right]+e_t\right) - k_{t+1}^e. \qquad (18)
\end{aligned}
$$

The second line makes use of the factor price conditions (15) and (16), and the third line follows because $F(.)$ is constant-returns. The capital Euler equation that arises from maximizing this representative-firm profit function with respect to aggregate entrepreneurial capital holdings k_{t+1}^e is clearly identical to (17).

Finally, the transformation function of the economy is described jointly by the consumption-goods resource frontier

$$c_t + \eta i_t = z_t F(k_t, n_t), \tag{19}$$

the investment-goods resource constraint

$$\eta i_t\left[1-\mu\Phi(\omega_t)\right] = k_{t+1} - (1-\delta)k_t, \tag{20}$$

and the aggregate quantity of capital

$$k_t = k_{ht} + \eta k_t^e. \tag{21}$$

Note that aggregate monitoring costs apply only to the capital-goods producers.

4.3 Private Sector Equilibrium

A symmetric private-sector equilibrium is made up of state-contingent endogenous processes $\{c_t, n_t, k_{ht+1}, k_{t+1}^e, k_{t+1}, i_t, \Pi_t^e, w_t, r_t, q_t, \bar{\omega}_t\}$ that satisfy the following conditions: the labor-supply condition

$$-\frac{v'(n_t)}{u'(c_t)} = w_t; \tag{22}$$

the labor-demand condition

$$w_t = z_t F_n(k_t, n_t); \tag{23}$$

the capital-demand condition

$$r_t = z_t F_k(k_t, n_t); \tag{24}$$

the representative household's Euler equation for capital holdings

$$q_t = E_t\left\{\Xi_{t+1|t}\left[r_{t+1} + q_{t+1}(1-\delta)\right]\right\}; \tag{25}$$

the (representative) entrepreneur's Euler equation for capital holdings

$$q_t = \gamma E_t\left\{\Xi_{t+1|t}\left(\frac{q_{t+1} f\left(\bar{\omega}_{t+1}\right)}{1-q_{t+1} g\left(\bar{\omega}_{t+1}\right)}\right)\left[r_{t+1} + q_{t+1}(1-\delta)\right]\right\}; \tag{26}$$

aggregate capital market clearing

$$k_t = k_{ht} + \eta k_t^e; \tag{27}$$

the law of motion for the aggregate capital stock

$$k_{t+1} = (1-\delta)k_t + \eta i_t \left[1 - \mu\Phi(\bar{\omega}_t)\right]; \tag{28}$$

the aggregate consumption-goods resource constraint

$$c_t + \eta i_t = z_t F(k_t, n_t); \tag{29}$$

the contractually-specified project size

$$i_t = \frac{nw_t}{1 - q_t g\left(\bar{\omega}_t\right)}, \tag{30}$$

in which expression (7) for nw_t is substituted in; the contractually-specified liquidation threshold

$$\frac{q_t f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} = -\frac{f'(\bar{\omega}_t)}{g'(\bar{\omega}_t)}; \tag{31}$$

and the evolution of the average assets of firms (equivalently, the assets of the representative firm)

$$\Pi_t^e + q_t k_{t+1}^e = \frac{q_t f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} \left(k_t^e \left[r_t + q_t(1-\delta)\right] + e_t\right). \tag{32}$$

The private sector takes as given the stochastic process for $\{z_t, E(\omega_t), \sigma_t^\omega\}_{t=0}^\infty$, k_0, and $E(\omega_t) = 1$ $\forall t$. To emphasize, and as noted above, conditions (30) and (31) characterize the partial-equilibrium financial outcomes, and hence can be viewed (in partial equilibrium) as taking q_t and net worth as given.

5 Quantitative Analysis

5.1 Computational Strategy

Changes in cross-sectional risk are aggregate, not idiosyncratic, shocks in the model economy. Because I track only aggregate outcomes and do not track any idiosyncratic outcomes, there is no reason to think that local approximation methods will misrepresent the model's aggregate dynamics.[27] To study the dynamics of the model, I thus compute a first-order approximation of the equilibrium.[28] Because the main interest is in business cycle fluctuations, such methods are likely to accurately portray the model's dynamic behavior. This also reinforces the point made

[27]Recall the discussion above that, given the maintained assumptions of the model, aggregates in the model do not depend on distributions of outcomes at the firm level.

[28]The numerical algorithm is my own implementation of the perturbation method described by Schmitt-Grohe and Uribe (2004).

by DLS (2008, p. 386) that linearization does not impose certainty equivalence on this type of second-moment (a cross-sectional variance) shock. The quantitative results reported below are thus fundamentally driven by the model's mechanism — changes in cross-sectional risk, which then potentially are transmitted to the real economy — rather than choice of approximation method.

Before presenting the dynamic results, I complete the description of the calibration of the model (which was begun in Section 2) and briefly describe some of its long-run predictions.

5.2 Calibration

The novel aspect of the model calibration is the risk shock process using micro data. As described in Section 2, long-run dispersion of firm productivity is $\bar{\sigma}^{\omega} = 0.156$. This is about half the value used by CF (1998, p. 590) and Bernanke, Gertler, and Gilchrist (1999, p. 1368), which are calibrated to aggregate financial data, not firm-level data: the former set $\bar{\sigma}^{\omega} = 0.37$, and the latter set $\bar{\sigma}^{\omega} = 0.28$. CMR's estimate is $\bar{\sigma}^{\omega} = 0.26$. Thus, direct micro evidence indicates less cross-sectional dispersion than standard macro calibrations of agency-cost models.

As also discussed in Section 2, I assume sufficient smoothness in the average productivity and risk processes so that I can set quarterly persistence parameters $\rho_z = 0.83$ and $\rho_{\sigma^{\omega}} = 0.83$, even though the data on which the estimation is based are annual. This mismatch between (desired) model frequency and empirical frequency raises the question of the appropriate calibration of the standard errors of the quarterly innovations in the productivity and risk processes.[29]

Given the quarterly frequency of the model and the annual frequency of the data examined in Section 2, I simply time aggregate the simulated data from the model, and set parameters σ_z and $\sigma_{\sigma^{\omega}}$ so that the annualized volatilities of average productivity and dispersion of productivity in the model match their annual empirical counterparts. As documented in Section 2, the empirical volatilities are, respectively, 1.26 percent and 3.15 percent.

This calibration procedure leads to $\sigma_z = 0.002$ and $\sigma_{\sigma^{\omega}} = 0.0374$. The quarterly innovation in the aggregate productivity process, $\sigma_z = 0.002$, is about three times smaller than that "typically" used in a baseline RBC model, in which a benchmark value is 0.007. Here, of course, $\sigma_z = 0.002$ is computed directly from micro data. For the sake of comparison with DLS and CMR, their respective pairs of parameters are: $\sigma_z = 0.007$ and $\sigma_{\sigma^{\omega}} = 0.007$ (see DLS Table 4) and $\sigma_z = 0.0046$ and $\sigma_{\sigma^{\omega}} = 0.07$ (see CMR Table 2). As noted in the Introduction, the value $\sigma_{\sigma^{\omega}} = 0.07$ CMR obtain and the value $\sigma_{\sigma^{\omega}} = 0.0374$ I obtain are highly similar given the completely different approaches used to estimating this crucial parameter.

[29] Recall from Section 2 that the point estimates for annual persistence are $\rho_z = 0.47$ and $\rho_{\sigma^{\omega}} = 0.48$, and the standard deviation of the annual innovations in the average productivity and risk processes are, respectively, 0.0111 and 0.0276.

Besides the calibration of the exogenous processes, Table 3 lists all functional forms used in the quantitative experiments, and Table 4 lists all baseline parameter settings. The preference and production parameters are standard in business cycle models. One important point of comparison to note is that households' preferences are quasi-linear in labor just like in CF and DLS.

The agency cost parameter is set to $\mu = 0.15$, which is the same as the calibrated value in Covas and den Haan (2011) and in line with the estimate $\mu = 0.12$ by Levin, Natalucci, and Zakrajsek (2004). The value for entrepreneurs' "additional" discount factor is set to $\gamma = 0.96$, which allows the model to match a long-run annualized external finance premium of two percent. This value of γ is quite close to CMR's calibration $\gamma = 0.985$, and is a bit larger than the calibrations of CF and BGG.

5.3 Long-Run Dispersion and Long-Run Equilibrium

The long-run deterministic (steady-state) equilibrium is computed numerically using a standard nonlinear equation solver. The main comparative static exercise is presented in Figure 7, which plots long-run equilibria as a function of long-run cross-sectional dispersion $\bar{\sigma}^{\omega}$. The range of $\bar{\sigma}^{\omega}$ in Figure 7 covers all of the parameter values used in CF, DLS, Bernanke, Gertler, and Gilchrist (1999), and CMR. My estimate of $\bar{\sigma}^{\omega}$ lies in the low end of the interval in Figure 7.[30]

Figure 7 shows the long-run response of the economy to changes in $\bar{\sigma}^{\omega}$. Focusing first on the solid lines in Figure 7, aggregate quantities, such as GDP, gross investment, and consumption (for brevity, the latter two are not shown in Figure 7), monotonically decline as long-run risk dispersion $\bar{\sigma}^{\omega}$ increases. Regarding financial variables, long-run leverage (upper right panel) declines as $\bar{\sigma}^{\omega}$ increases, and both the bankruptcy (aka reorganization) rate (middle left panel) and the long-run gross external premium (middle right panel) increase as $\bar{\sigma}^{\omega}$ increases.

All of the other structural parameters are held fixed at their baseline values (see Table 4) in the impulse response just described. There are many parameter changes with which we could experiment, but, if we had to zoom in on just one parameter, the most important is the long-run reorganization cost μ. If we instead chose to hold the (endogenous) reorganization rate fixed — perhaps because it's easier to empirically observe — then the long-run reorganization cost μ would be larger. The consequences of larger values of μ as $\bar{\sigma}^{\omega}$ increases appear as the dashed-dotted lines in Figure 7.[31]

Regardless of which of the two comparative static exercises displayed in Figure 7 is preferred, a broad interpretation is that long-run aggregate quantities are relatively insensitive over a large

[30] As discussed in Section 2, the value I obtain of $\bar{\sigma}^{\omega} = 0.156$ could be viewed as overstating long-run risk by a factor of two, which is the reason that the lowest point of the interval of $\bar{\sigma}^{\omega}$ in Figure 7 is $\bar{\sigma}^{\omega} = 0.07$.

[31] The solid and dashed-dotted lines in the bottom panel of Figure 7 are identical to each other, hence difficult to distinguish.

region of long-run dispersion. The insensitivity seems to be due to the *zero* steady-state elasticity of the endogenous contract multiplier λ with respect to $\bar{\sigma}^{\omega}$, which is quantitatively shown in the bottom panel of Figure 7. This insensitivity of the steady-state elasticity of the contract multiplier arises for any parameter value μ. To the best of my knowledge, the steady-state invariance of the contract multiplier with respect to the dispersion of productivity has not previously been shown, or, if it has been shown, has not been prominent in discussions of the CF (1997, 1998) models. The steady-state invariance of the contract multiplier in the CF framework is an informative economic summary of the "linearity" of the model.

For the baseline calibration of $\bar{\sigma}^{\omega} = 0.156$, the model's long-run leverage ratio is 0.67, which is remarkably close to the average leverage ratio of 0.56 during the period 1973-1988, as Section 3 showed. The calibration of the model was not designed to match average leverage. Furthermore, given the Masulis (1988) definition of leverage in the first line below, the subsequent lines characterize leverage in model notation

$$
\begin{aligned}
\ell(\bar{\omega};\sigma^{\omega}) &\equiv \frac{\text{debt}}{\text{assets}} \\
&= \frac{i - nw}{i - nw + nw} \\
&= 1 - \frac{nw}{i}.
\end{aligned}
$$

As long-run dispersion $\bar{\sigma}^{\omega}$ shrinks to zero, lenders face no risk whatsoever on their loans, which in turn implies entrepreneurs have to accumalate zero net worth. Leverage thus approaches its upper limit of unity as $\bar{\sigma}^{\omega} \to 0$.[32]

It is useful to also highlight that the long-run values implied by the model (using the baseline calibration in Table 4) of the (annualized) finance premium is in line with most of the measures of premia presented in DeGraeve (2008), and the bankruptcy/reorganization rate is lower than in the Dun & Bradstreet evidence cited by CF (1998, p. 590).[33]

5.4 Business Cycle Dynamics

Now I turn to the model's cyclical fluctuations.

[32] The extreme case of $\bar{\sigma}^{\omega} = 0$ is simply the textbook RBC model.

[33] As discussed extensively by DeGraeve (2008), it is not clear what is the most relevant empirical counterpart to the model's external finance premium. Many natural alternatives suggest themselves, such as the difference between the prime borrowing rate and the short-term T-bill rate, the interest spread between AAA-rated commercial paper and T-bills, the spread between BB (2013) B-commercial paper and T-bills, and so on. DeGraeve (2008) documents that these various empirical measures of "the external finance premium" behave differently enough over the business cycle that it remains an open question what the natural empirical counterpart of the model's external finance premium is.

5.4.1 Risk Shocks

The first set of experiments conducted is dynamics driven by risk shocks alone. Figure 8 presents impulse responses to a one-time, one-standard-deviation positive shock to the cross-sectional dispersion of entrepreneurs' productivity, holding constant average productivity. The distribution of entrepreneurial productivity thus has larger idiosyncratic risk. Complementing this impulse-response analysis are the simulated business cycle statistics reported in Table 6. There are two main results from these experiments from a purely risk-shock driven economy.

As Figure 8 highlights, a pure risk shock induces hump-shaped downturns in both GDP and investment. These downturns are quantitatively large: at the peak of their respective declines, GDP falls by 0.5 percent compared to its steady state, and investment falls by over 1.5 percent. Aggregate consumption (in the middle-left panel in Figure 8), however, is countercyclical with respect to GDP upon a risk shock. The degree of countercyclicality (-0.44, in annualized terms) is shown in Table 6, which is in line with the results in DLS (p. 391, second row of Table 4).

Despite the challenge of understanding the economic intuition behind the countercyclicality of consumption — one candidate explanation is that the tractable, but small-scale, CF model was simply not designed to capture this moment in response to a risk shock — the economic intuition of the impulse responses of financial variables are clear. Upon a temporary increase in entrepreneurial risk, leverage declines, Tobin's q declines, and the external premium rises, all of which induce entrepreneurs to accumulate internal funds.

5.4.2 Both Risk Shocks and Average Productivity Shocks...

Next, consider the model economy's dynamics when hit by independent shocks to both average productivity and cross-firm dispersion. Important to keep in mind here is that both of these shock processes, as described in Section 2, were measured using *micro* data, with the remaining parameters calibrated as in a standard macro model. Whether or not these micro-based processes together with the parsimonious set of "standard macro parameters" portray well empirically-relevant aggregate business cycles during the years 1973-1988 is an open question.

Table 7 shows that it portrays quantity fluctuations and labor fluctuations remarkably well, especially given the small-scale nature of the model and the limited number of shocks. Variance decomposition shows that 4.6% of GDP volatility is account for by risk shocks. The main exception, as noted above, is the countercyclicality of consumption with GDP. Before rushing to judgement, we should keep two things in mind: 1) as stated in Section 5.4.1, perhaps the small-scale CF model was not designed to capture the procyclicality of both consumption and investment with GDP in response to risk shocks; and 2) the recessionary nature of the U.S. economy during 1973-1988 — 38 of the 64 quarters during that period are defined by the NBER as contractions.

5.4.3 ...and Micro vs. Macro Calibration of Risk Shocks

Taken together, a main take-away message from the results presented here and those of CMR is that they both convey the same quantitative result that risk shocks lead to a significant fraction of GDP volatility. One of CMR's headline results is that 20% of GDP business-cycle fluctuations are driven by risk shocks, whereas I find that nearly 5% of GDP volatility is driven by risk shocks. Fully comparing and contrasting CMR's results to the ones of this paper is challenging due to the richness of their model vs. the small-scale model considered here. But the fact that using a completely different data set and a much smaller-scale DSGE model conveys the same big-picture message that risk shocks matter for macro fluctuation is remarkable.

6 Conclusion

This paper measured the business-cycle properties of cross-sectional productivity risk based on micro-level data. Micro-disciplined risk is fairly volatile over the cycle and highly countercyclical. Using a small-scale quantitative financial accelerator model, which, by changing just two parameters, nests the frictionless RBC model, the main theoretical question was to assess the extent to which micro-disciplined cross-sectional risk can explain aggregate volatility. Empirically-relevant micro risk shocks turn out to drive nearly 5% of GDP volatility, which is remarkably large given the small-scale nature of the framework and that there are only two shocks driving the model (risk shocks and aggregate productivity shocks).

The sparse model makes clear that risk shocks can have significant impacts on aggregate measures of the economy. However, due to its sparsity, it cannot be expected to match all of the business-cycle moments that the macroeconomics profession typically looks at. Instead, it's the combination of risk shocks and the transmission mechanism embedded in any given structural model that matters. The mechanism of the small-scale financial accelerator model is at the heart of many medium-scale frameworks that have recently been emerging.

Acknowledgements

I thank the editor and a referee for many constructive comments and suggestions, and many audience members in seminar presentations at Georgetown University, the Federal Reserve Bank of Cleveland, London Business School, Boston University, Boston College, the Federal Reserve Bank of Dallas, the Federal Reserve Bank of Boston, the Kiel Institute, the IZA Institute, Drexel University, the Federal Reserve Bank of Philadelphia, the University of Virginia, the IMF Institute, Emory University, Johns Hopkins University, and the Fall 2010 NBER Workshop on Methods and Applications for DSGE Models at the Federal Reserve Bank of Atlanta. I thank John Haltiwanger for sharing data, Charles Carlstrom and Timothy Fuerst for in-depth conversations, and, for helpful discussions at various stages of the project, David Arseneau, S. Boragan Aruoba, Rudi Bachmann, Larry Ball, Susanto Basu, Christian Bayer, Chris Carroll, Ryan Chahrour, Pablo D'Erasmo, Simon Gilchrist, Francois Gourio, John Haltiwanger, Cosmin Ilut, Andre Kurmann, and Stephanie Schmitt-Grohe. Ana Lariau has provided brilliant research assistance.

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A Leverage: Empirics and Theory

As noted in the Introduction, the risk-shock calibration of Section 2 turns out to endogenously match quite well the time-series average of empirical leverage as provided in Masulis (1988) and extended to include the years 1987 and 1988 so that it covers the time period used in estimating the risk shock process. Table 1 displays the leverage data, which is for the manufacturing sector. The average leverage ratio during 1973-1988 was 0.56, which is quite close to the 0.67 in the calibrated model of Section 5, even though leverage was not a calibration target.

Formally, given the nature of the CF investment model, in which both the ex-ante signing of contracts and ex-post resolutions of contracts occurs intraperiod, we only need to zoom in on the equilibrium contractual conditions (30) and (31). To emphasize that the contractual terms depend on σ^{ω}, for this Appendix I write $f(\bar{\omega}, \sigma^{\omega})$ and $g(\bar{\omega}, \sigma^{\omega})$, omitting time subscripts because the contract is intraperiod.

Let $\ell(\bar{\omega}; \sigma^{\omega})$ denote leverage (and recall that i denotes investment and nw denotes net worth). Defining leverage as in Masulis (1988) in the first line, the subsequent lines characterize leverage in model notation

$$\begin{aligned} \ell(\bar{\omega}; \sigma^{\omega}) &\equiv \frac{\text{debt}}{\text{assets}} \\ &= \frac{i - nw}{i - nw + nw} \\ &= 1 - \frac{nw}{i} \\ &= 1 - (1 - qg(\bar{\omega}; \sigma^{\omega})) \\ &= qg(\bar{\omega}; \sigma^{\omega}), \end{aligned} \tag{33}$$

in which the next-to-last line uses the contractual version of the (binding) zero-profit condition (30) of the lender.

Inserting this expression for $\ell(\bar{\omega}; \sigma^{\omega})$ into condition (31), which characterizes the terms of the financial contract, allows for yet another way of expressing leverage,

$$\ell(\bar{\omega}; \sigma^{\omega}) = 1 + \frac{qf(\bar{\omega}; \sigma^{\omega}) g_{\bar{\omega}}(\bar{\omega}; \sigma^{\omega})}{f_{\bar{\omega}}(\bar{\omega}; \sigma^{\omega})}. \tag{34}$$

The expected share functions $f(.)$ and $g(.)$ and their derivatives depend on the cross-sectional dispersion σ^{ω} of firm productivity, hence the leverage ratio also depends on σ^{ω}.

The contract maximization problem was briefly mentioned in Section 4, but not fully fleshed out. Formally, the lender and capital producer i maximize

$$qf(\bar{\omega}, \sigma^{\omega})\, i_i \tag{35}$$

subject to

$$qg(\bar{\omega}, \sigma^{\omega})\, i_i \geq i_i - nw_i \tag{36}$$

with respect to i_i and $\bar{\omega}$. Letting λ denote the multiplier on the lending constraint, the first-order conditions with respect to i_i and $\bar{\omega}$ are

$$qf\left(\bar{\omega}, \sigma^{\omega}\right) + \lambda\left[qg\left(\bar{\omega}, \sigma^{\omega}\right) - 1\right] = 0 \tag{37}$$

and

$$qf_{\bar{\omega}}\left(\bar{\omega}, \sigma^{\omega}\right) i_i + \lambda q g_{\bar{\omega}}\left(\bar{\omega}, \sigma^{\omega}\right) i_i = 0. \tag{38}$$

The contract multiplier can thus be stated as either

$$\lambda = -\frac{f_{\bar{\omega}}(\bar{\omega}, \sigma^{\omega})}{g_{\bar{\omega}}(\bar{\omega}, \sigma^{\omega})} \tag{39}$$

or

$$\lambda = \frac{qf(\bar{\omega}, \sigma^{\omega})}{1 - qg(\bar{\omega}, \sigma^{\omega})}. \tag{40}$$

Equalizing these two expressions for λ gives exactly the equilibrium condition (31). (Important to emphasize is that conditions (30) and (31) are purely static (within-period) conditions. Thus, the financial contract is of one-period duration.)

Figure 6 sketches why changes in the cross-sectional dispersion of firms' TFP would be expected to cause changes in leverage. Suppose the solid black curve in Figure 6 is the pdf $\phi(\omega)$ before a risk shock occurs. The liquidation threshold $\bar{\omega}$ shown is for this initial distribution. Suppose there is an exogenous reduction in dispersion. If the liquidation threshold $\bar{\omega}$ were to remain unchanged, fewer firms would draw an idiosyncratic $\omega < \bar{\omega}$, which lenders understand because the density $\phi(\omega)$ is common knowledge. This in turn means that fewer firms are expected to be unable to repay their loans, which reduces lenders' risk. Conditional on a value for $\bar{\omega}$, lenders would be willing to extend more credit, which implies higher leverage ratios for firms (borrowers). In equilibrium, $\bar{\omega}$ will of course also change, which can only be determined quantitatively given the log-linear distribution of idiosyncratic productivity used in Section 5.

B Bundled Shocks: Productivity-Induced Risk Fluctuations

Countercyclicality of risk can be modeled by linking time-variation in average TFP directly to fluctuations in the cross-section of risk. Specifically, the cross-sectional dispersion of productivity across entrepreneurs is now assumed to decline when average TFP improves. Second-moment shocks are thus assumed to be bundled with first-moment shocks, and I refer to the entire bundle as an "aggregate shock." The two processes are assumed to be linked according to

$$\ln \sigma_t^{\omega} = \ln \bar{\sigma}^{\omega} + \varphi \ln z_t. \tag{41}$$

This condition replaces the exogenous law of motion (1) for $\ln \sigma_t^{\omega}$, and the evolution of $\ln z_t$ is still characterized by (2). The rest of the model is exactly the same. The elasticity φ is clearly the key parameter of the bundled-shock version of the model, with $\varphi < 0$ implying countercyclicality of firm-level risk.[34] In terms of correlation between average TFP and dispersion of entrepreneurial productivity, $\varphi < 0$ obviously implies a perfect negative correlation between the two, but this portrayal is not counterfactually stark compared to the data; recall from Section 2 that the contemporaneous cyclical correlation between average TFP and dispersion of TFP is -0.98.

Figure 10 illustrates why $\varphi < 0$ leads to countercyclical risk. A positive shift in average TFP will, all else equal, increase GDP. If at the same time cross-sectional dispersion declines due to $\varphi < 0$, and supposing initially that the bankruptcy threshold $\bar{\omega}$ were fixed, fewer firms would be expected to go bankrupt. This in turn would induce lenders to extend more credit, hence leverage rises for given net worth. Indeed, the second part of the intuitive argument is exactly the same as that underlying Figure 6. What is different from the baseline model is the event that now induces the change in dispersion. In the baseline model, the change in dispersion itself was the exogenous event, whereas here the source is a positive shock to average TFP.

This bundled aggregate shock is of course a reduced-form construct. However, I bring the same empirical evidence presented in Section 5.2 to bear on the calibration of the crucial elasticity parameter φ. The calibration approach is to choose φ so that the model matches the observed time-series variation in cross-sectional dispersion. Section 5.2 documented that the time-series volatility in annual cross-sectional dispersion is 3.15 percent. Given this target and holding fixed all parameters in Table 4, this calibration procedure (with average TFP fluctuations now as the sole truly exogenous driving process) leads to $\varphi = -2.5$.

Table 9 presents simulation-based business cycle statistics. Aggregate quantity volatility is smaller compared to Table 7. But, by construction of the bundled-shock model, the results are consistent with the empirically-observed countercyclicality of cross-sectional firm risk (see the last

[34] Clearly, $\varphi > 0$ would deliver procyclical entrepreneurial risk, and $\varphi = 0$ would recover the baseline CF model in which there are never any changes in firm risk.

two rows of the lower panel of Table 9). Although I do not take up this extension here, a conjecture is that a combination of bundled shocks along with independent, exogenous, shocks to firm risk may help in capturing all these dimensions of the data.[35]

[35] Of course, there are a host of other model features and/or shocks one could consider introducing to the model. Such analysis is left to future work.

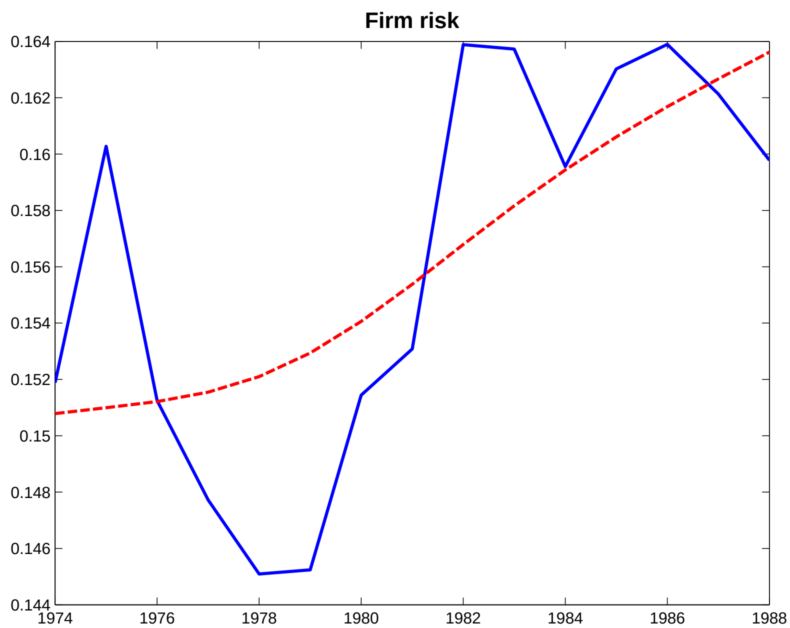

Figure 1: **Cross-sectional dispersion of profitability.** Cross-sectional coefficient of variation of firm-level profitability over the period 1974-1988. Data are annual. Trend component constructed with HP filter (smoothing parameter 100). Based on profitability series from Cooper and Haltiwanger (2006).

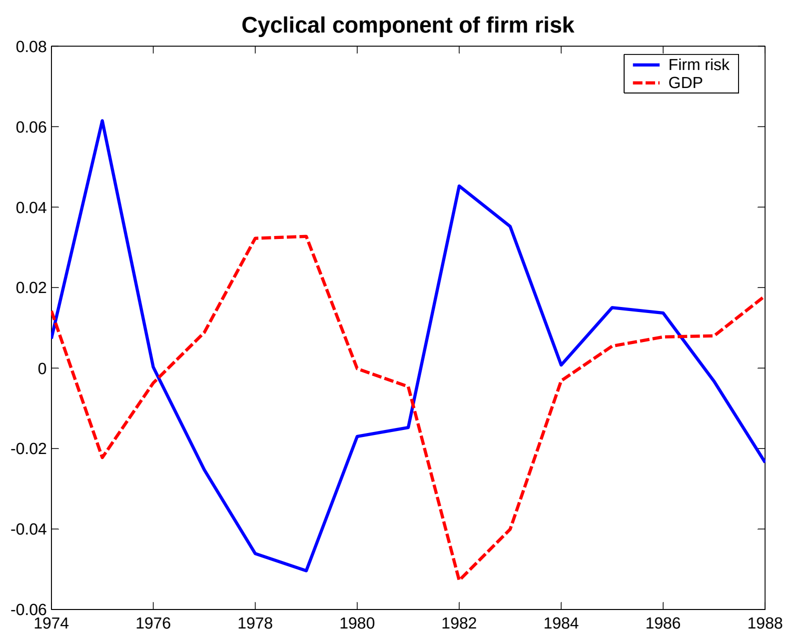

Figure 2: **Cyclical component of cross-sectional dispersion of profitability.** Cyclical component of cross-sectional coefficient of variation of firm-level profitability over the period 1974-1988. Vertical axis is percentage deviation from HP trend. Computed from profitability residuals constructed by Cooper and Haltiwanger (2006).

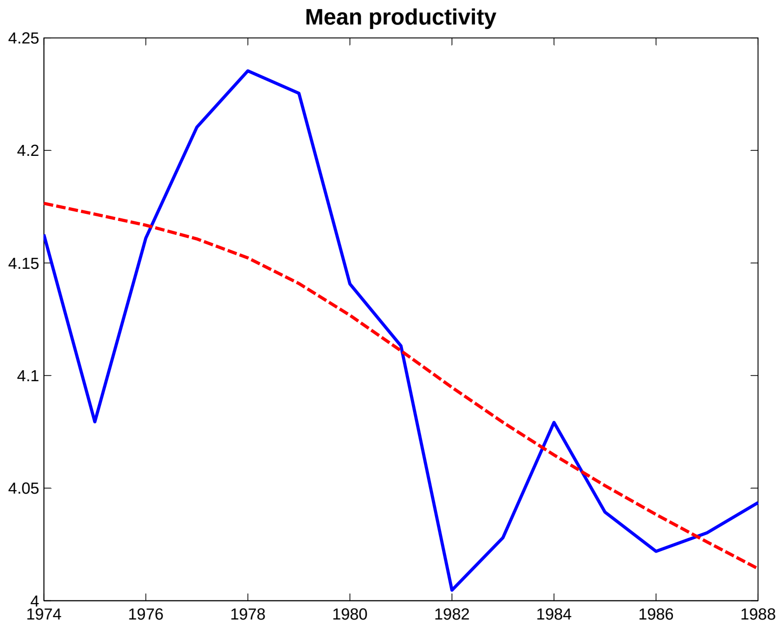

Figure 3: **Mean profitability.** Mean level of firm-level profitability residuals over the period 1974-1988. Data are annual. Trend component constructed with HP filter (smoothing parameter 100). Based on profitability series from Cooper and Haltiwanger (2006).

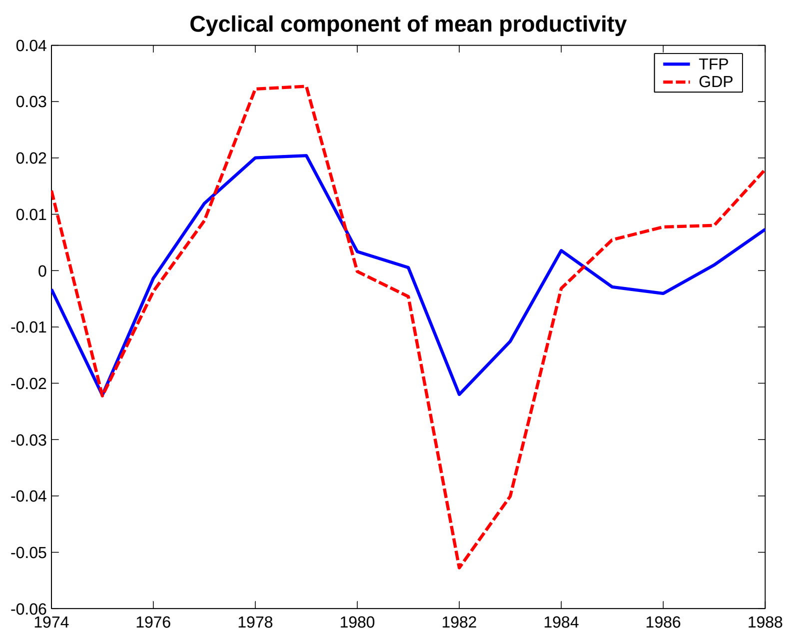

Figure 4: **Cyclical profitability residuals.** Cyclical component of mean of firm-level profitability residuals over the period 1974-1988. Vertical axis is percentage deviation from HP trend. Computed from profitability residuals constructed by Cooper and Haltiwanger (2006).

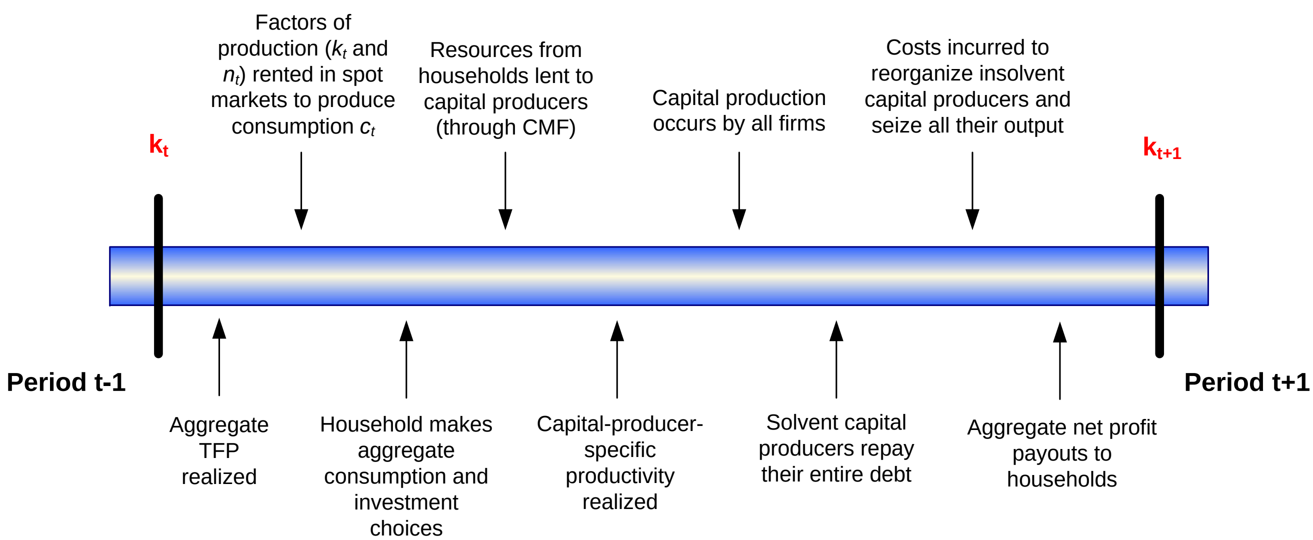

Figure 5: **Timing of events.** The term "CMF" refers to capital mutual funds, as described in Carlstrom and Fuerst (1997, Table 1).

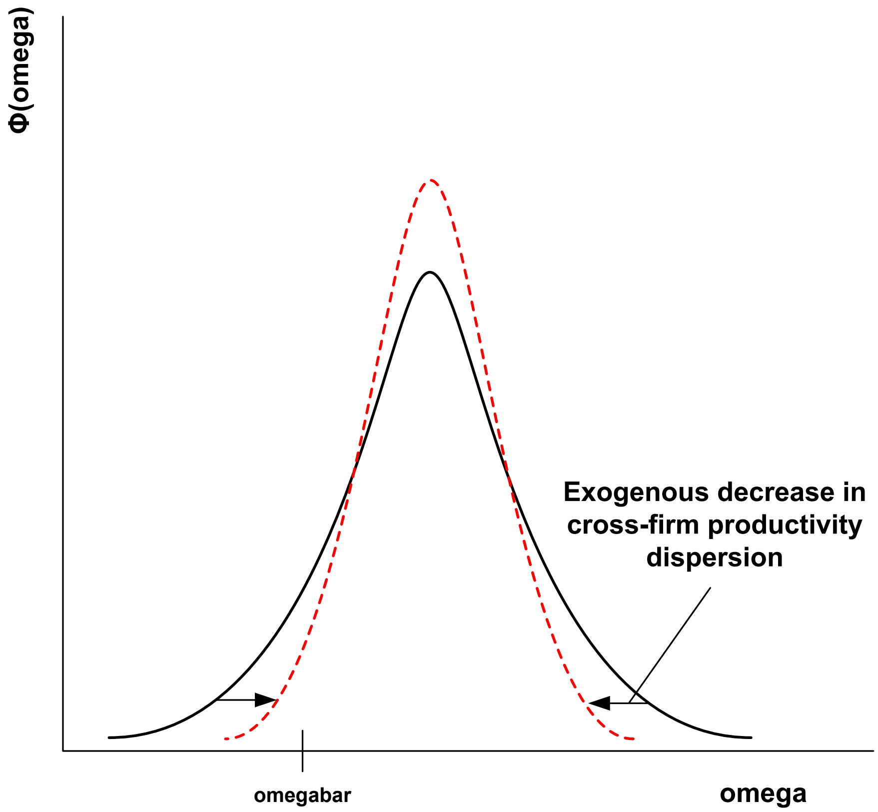

Figure 6: **Mean-preserving spread.** An exogenous decrease in dispersion of productivity across entrepreneurs. The bankruptcy threshold $\bar{\omega}$ shown is for the original distribution; if the threshold were to remain unchanged, fewer entrepreneurs would be expected to go bankrupt, which in turn would make lenders willing to allow larger leverage ratios.

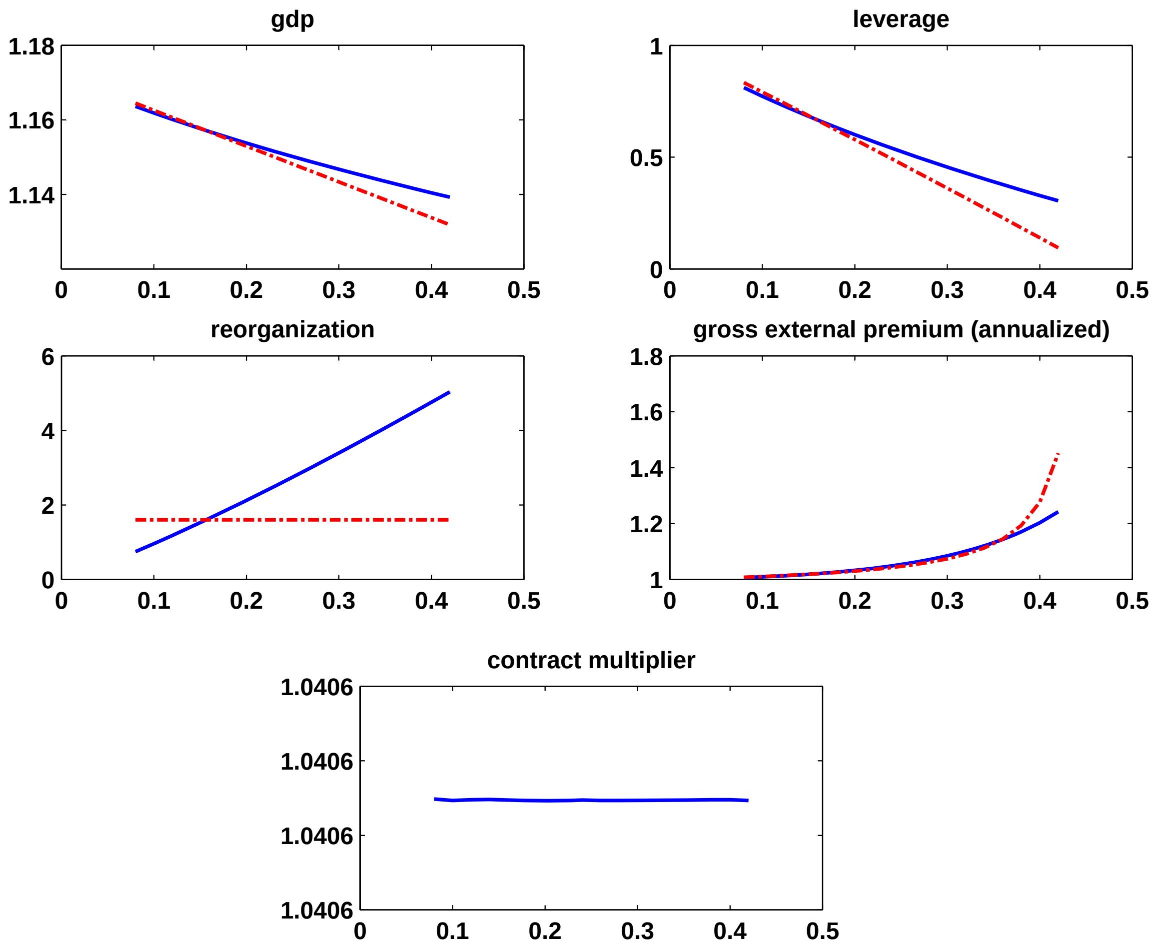

Figure 7: **Long-run equilibrium.** Steady-state equilibrium as long-run standard deviation of idiosyncratic productivity distribution, $\bar{\sigma}^{\omega}$, varies; $\bar{\sigma}^{\omega}$ plotted on horizontal axis. Solid line: all other model parameters held fixed as in Table 4. Dashed-dotted line: all other model parameters held fixed as in Table 4, except the bankruptcy cost μ, which varies with $\bar{\sigma}^{\omega}$ to maintain a long-run reorganization rate across $\bar{\sigma}^{\omega}$.

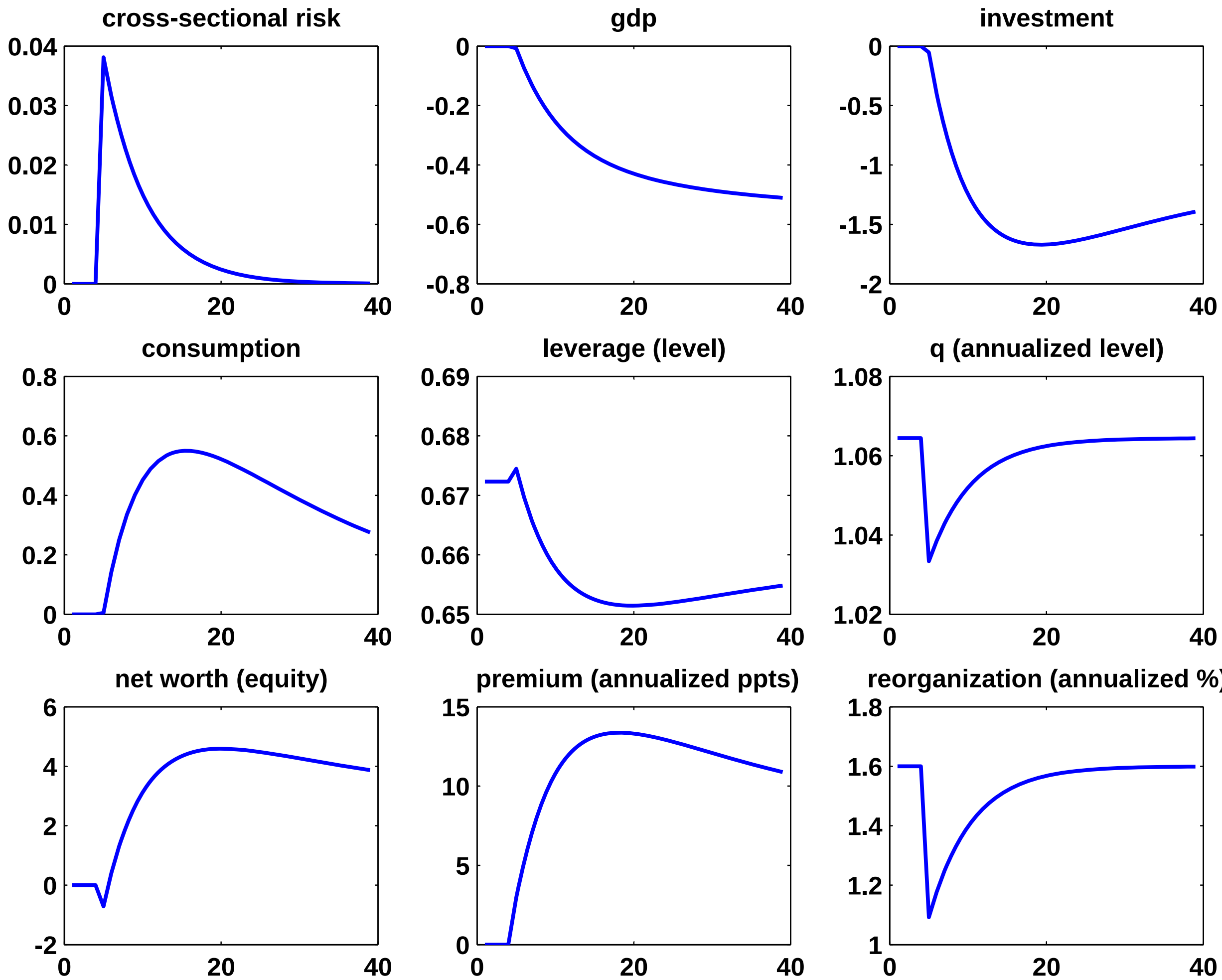

Figure 8: **Impulse response to risk shock.** Impact of a one-standard-deviation exogenous increase in the dispersion σ^{ω} of firm productivity (upper-left panel), holding constant average productivity. Unless stated otherwise, vertical scale measures percent deviation from steady state. Shock occurs in period 5.

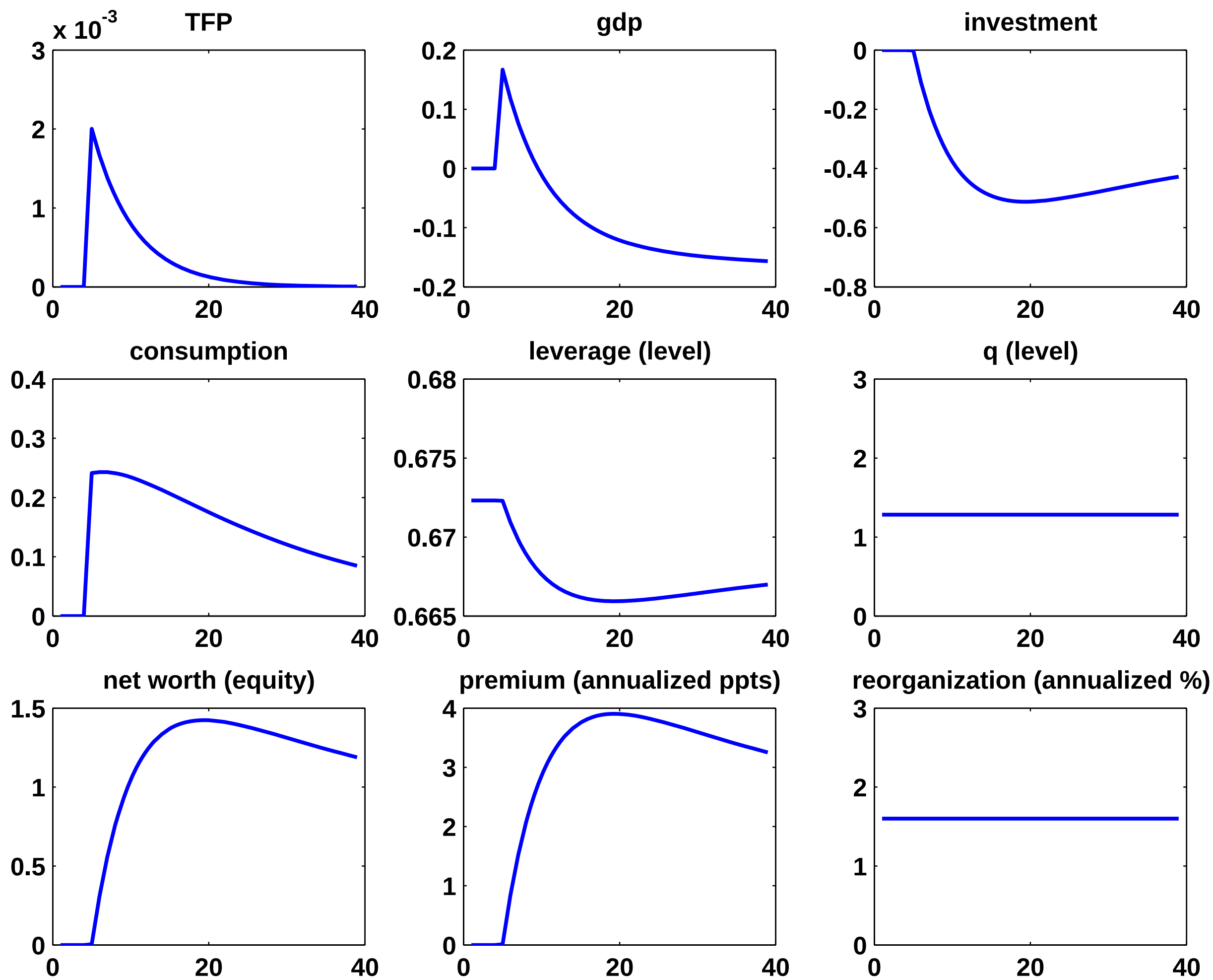

Figure 9: **Impulse response to average productivity shock.** Impact of a one-standard-deviation exogenous increase in average productivity z (upper-left panel), holding constant dispersion σ^{ω}. Unless stated otherwise, vertical scale measures percent deviation from steady state. Shock occurs in period 5.

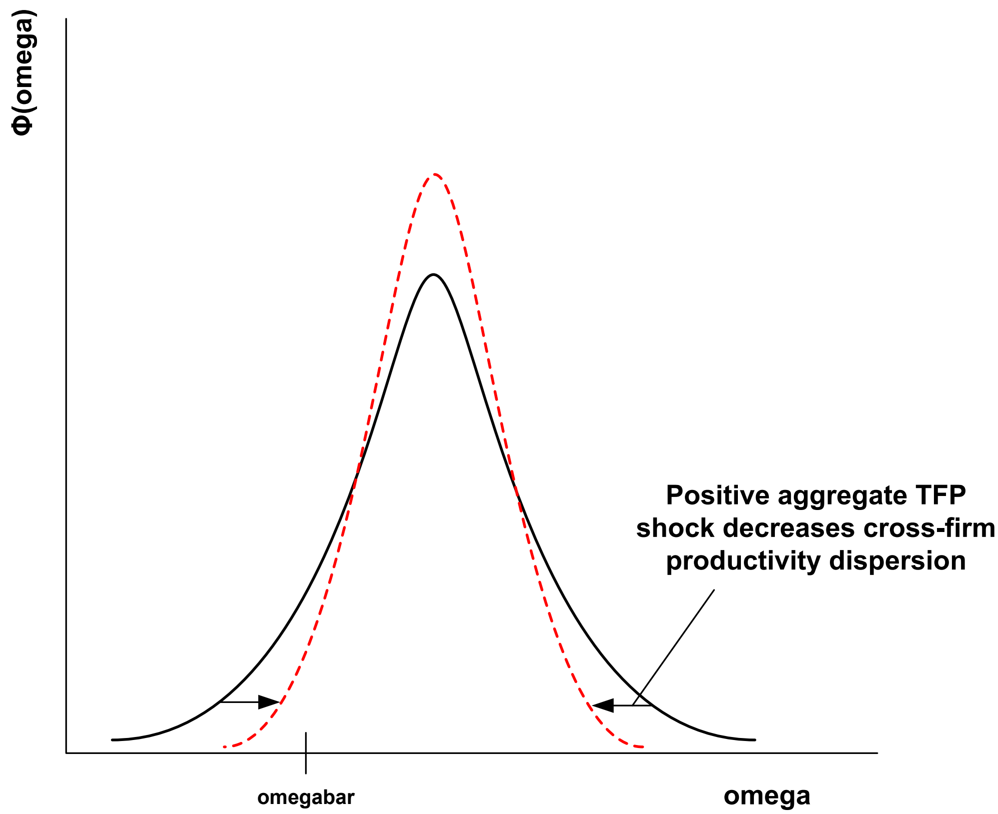

Figure 10: **Bundled shock.** An endogenous decrease in dispersion of productivity across entrepreneurs. The bankruptcy threshold $\bar{\omega}$ shown is for the original distribution; if the threshold were to remain unchanged, fewer entrepreneurs would be expected to go bankrupt, which in turn would induce lenders to allow larger leverage ratios.

Year	Leverage
1973	0.51
1974	0.53
1975	0.52
1976	0.53
1977	0.53
1978	0.54
1979	0.55
1980	0.56
1981	0.56
1982	0.56
1983	0.57
1984	0.58
1985	0.58
1986	0.62
1987	0.62
1988	0.63

Table 1: **Annual leverage ratios (≡ debt/total assets) for the manufacturing sector, 1973-1988.** Sources: for 1973-1979, Masulis (1988, Tables 1-3); for 1980-1986, Statistics of Income, Corporation Source Books, IRS (http://www.irs.gov/uac/SOI-Tax-Stats-Archive — 1929-to-1999-Corporation-Source-Book); for 1987-1988, Statistics of Income Bulletins, Corporation Income Tax Returns, IRS (http://www.irs.gov/uac/SOI-Tax-Stats-SOI-Bulletins).

	GDP	C	I	hours	leverage
Std. dev. (%)	2.31	2.10	8.38	2.08	1.36
Relative std. dev (/GDP)	1	0.90	3.62	0.89	0.58
Auto. corr.	0.51	0.72	0.18	0.55	0.01
Corr. w/ GDP	1	0.88	0.86	0.83	0.02

Table 2: **Empirical business cycle dynamics.** 1973-1988 annual business cycle comovements for standard macro aggregates: real GDP, real personal consumption expenditures, gross investment in nonresidential investment, total hours worked, leverage ratio in U.S. manufacturing sector. Sources for leverage data: for 1973-1979, Masulis (1988, Tables 1-3); for 1980-1986, Statistics of Income, Corporation Source Books, IRS (http://www.irs.gov/uac/SOI-Tax-Stats-Archive — 1929-to-1999-Corporation-Source-Book); for 1987-1988, Statistics of Income Bulletins, Corporation Income Tax Returns, IRS (http://www.irs.gov/uac/SOI-Tax-Stats-SOI-Bulletins). Based on annual HP-filtered cyclical components.

Functional Form	Description
$\ln \sigma^{\omega}_{t+1} = (1-\rho_{\sigma^{\omega}}) \ln \bar{\sigma}^{\omega} + \rho_{\sigma^{\omega}} \ln \sigma^{\omega}_{t} + \epsilon^{\sigma^{\omega}}_{t+1}$	Exogenous process for firm productivity dispersion
$\ln z_{t+1} = \rho_z \ln z_t + \epsilon^{z}_{t+1}$	Exogenous process for mean of TFP
$u(c) = \ln c$	Consumption subutility
$v(n) = -\psi n$	Labor subutility
$F(k,n) = k^{\alpha} n^{1-\alpha}$	Production technology

Table 3: **Functional forms for quantitative analysis.**

Parameter Value	Description/Notes
	Preferences
$\beta = 0.99$	Households' quarterly subjective discount factor
$\gamma = 0.96$	Autonomous capital-producers' quarterly subjective discount factor
$\psi = 2.90$	Labor calibrating parameter
	Production Technology
$\alpha = 0.36$	Capital's share in production function
$\delta = 0.02$	Depreciation rate of capital
	Financial Markets and Agency Costs
$\mu = 0.15$	Per-unit monitoring cost
$E(\omega_t) = 1$	Mean of idiosyncratic productivity
$\eta = 0.02$	Measure of capital-producing entrepreneurs
$\bar{\sigma}^{\omega} = 0.156$	Long-run standard deviation of distribution of $\ln \omega$
$\rho_{\sigma^{\omega}} = 0.83$	Quarterly persistence of log firm risk process
$\sigma_{\sigma^{\omega}} = 0.0033$	Standard deviation of innovations to log firm risk
	Exogenous Process
$\rho_z = 0.83$	Quarterly persistence of log mean-TFP process
$\sigma_z = 0.002$	Standard deviation of innovations to log mean-TFP

Table 4: **Parameter values.**

Financial Measure	Long-Run Value
Leverage ratio, $\ell(\bar{\omega})$	0.672
External premium	2.00 percent
Bankruptcy rate, $100\Phi(\bar{\omega})$	1.60 percent
Reorganization costs / GDP	0.21 percent

Table 5: **Long-run financial variables.** External premium, $\bar{\omega}/g(\bar{\omega})$, reported in annualized terms. Fourth line reports the percentage of GDP absorbed by reorganization costs, $\mu\Phi(\bar{\omega})$.

	GDP	C	I	hours	leverage
Std. dev. (%)	3.15	2.22	8.09	2.12	16.25
Relative std. dev (/GDP)	1	0.70	2.57	0.67	5.16
Auto. corr.	0.99	0.99	0.99	0.99	0.99
Corr. w/ GDP	1	-0.44	0.89	0.83	0.90

Annual std. dev (%) average productivity	—	(Data: 1.26)
Annual std. dev (%) σ^ω	3.14	(Data: 3.15)
Annual correlation (GDP, σ^ω)	0.06	(Data: -0.83)
Annual correlation (average productivity, σ^ω)	—	(Data: -0.98)

Table 6: **Risk shocks only.** Annualized simulation-based business cycle statistics.

	GDP	C	I	hours	leverage
Std. dev. (%)	3.28	2.41	8.42	2.22	17.03
Relative std. dev (/GDP)	1	0.74	2.57	0.68	5.19
Auto. corr.	0.97	0.98	0.99	0.98	0.97
Corr. w/ GDP	1	-0.39	0.88	0.80	0.88

Annual std. dev (%) average productivity	1.27	(Data: 1.26)
Annual std. dev (%) σ^ω	3.14	(Data: 3.15)
Annual correlation (GDP, σ^ω)	0.07	(Data: -0.83)
Annual correlation (average productivity, σ^ω)	-0.27	(Data: -0.98)

Table 7: **Independent average productivity shocks and risk shocks.** Annualized simulation-based business cycle statistics.

	GDP	C	I	hours	leverage
Std. dev. (%)	1.09	0.95	2.63	0.71	4.87
Relative std. dev (/GDP)	1	0.87	2.41	0.65	4.46
Auto. corr.	0.97	0.96	0.99	0.98	0.99
Corr. w/ GDP	1	-0.08	0.80	0.65	0.80

Annual std. dev (%) average productivity	1.27	(Data: 1.26)
Annual std. dev (%) σ^{ω}	—	(Data: 3.15)
Annual correlation (GDP, σ^{ω})	—	(Data: -0.83)
Annual correlation (average productivity, σ^{ω})	—	(Data: -0.98)

Table 8: **Average productivity shocks only.** Annualized simulation-based business cycle statistics.

	GDP	C	I	hours	leverage
Std. dev. (%)	0.66	0.68	1.46	0.41	2.67
Relative std. dev (/GDP)	1	1.02	2.19	0.62	4.08
Auto. corr.	0.95	0.93	0.99	0.98	0.99
Corr. w/ GDP	1	0.18	0.69	0.46	0.68

Annual std. dev (%) TFP	1.26	(Data: 1.26)
Annual std. dev (%) σ^{ω}	3.15	(Data: 3.15)
Annual correlation (GDP, σ^{ω})	-0.51	(Data: -0.83)
Annual correlation (TFP, σ^{ω})	-1	(Data: -0.98)

Table 9: **Bundled aggregate shocks.** Annualized simulation-based business cycle statistics, in which average productivity (first moment) shocks induce changes in cross-sectional dispersion. Elasticity parameter $\varphi = -2.5$.