

Optimal Fiscal and Monetary Policy with Sticky Wages and Sticky Prices

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Abstract

We determine the optimal degree of price inflation volatility when nominal wages are sticky and the government uses state-contingent inflation to finance government spending. We address this question in a well-understood Ramsey model of fiscal and monetary policy, in which the benevolent planner has access to labor income taxes, nominally risk-free debt, and money creation. Our main result is that sticky wages alone make price stability optimal in the face of shocks to the government budget, to a degree quantitatively similar as sticky prices alone. Key for our results is an equilibrium restriction between nominal price inflation and nominal wage inflation that holds trivially in a Ramsey model featuring only sticky prices. Our results thus show that when nominal wages are sticky, setting real wages as close as possible to their efficient path is a more important goal of optimal monetary policy than is financing innovations in the government budget via state-contingent inflation. A second important result is that the nominal interest rate can be used to indirectly tax the rents of monopolistic labor suppliers. Taken together, our results uncover features of Ramsey fiscal and monetary policy in the presence of a type of labor market imperfection that is widely-believed to be important.

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1 Introduction

In a recent strand of the Ramsey literature on optimal fiscal and monetary policy, Schmitt-Grohe and Uribe (2004b) and Siu (2004) have found that sticky product prices makes the volatility of Ramsey inflation quite small. This result contrasts with the strikingly high inflation volatility discovered by Chari, Christiano, and Kehoe (1991) in an environment with flexible prices. Given recent renewed attention to the importance of stickiness in nominal wages, a natural question is to what degree does low volatility of Ramsey inflation arise in a model featuring sticky wages, either instead of or in addition to sticky prices. In this paper, we address this question in a well-understood Ramsey environment featuring only a few key distortions. Our main result is that sticky wages alone dampen inflation volatility to a similar quantitative degree as sticky prices alone in the face of shocks to the government budget. That is, consumer price stability characterizes optimal policy if wages are sticky even if product prices are fully flexible.

Inflation volatility is high in the baseline model of Chari, Christiano, and Kehoe (1991) because surprise movements in the price level allow the government to synthesize real state-contingent debt payments from nominally risk-free government bonds. Surprise inflation thus serves as a non-distortionary instrument to finance innovations in the government budget, and so is preferred by the Ramsey planner to changes in distorting proportional taxes. As a prescriptive matter for central bankers, however, the optimality of highly volatile inflation seems peculiar. This prediction turns out to depend crucially on the absence of allocative effects of surprise inflation in Chari, Christiano, and Kehoe's (1991) environment, due to the assumption of fully-flexible nominal prices and nominal wages. In contrast, central bankers typically think of the economy as featuring nominal rigidities, which entail costs of surprise movements in the price level.

Recent work by Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2005), Levin et al (2005), and others shows that sticky nominal wages may be more important than sticky nominal prices in explaining macroeconomic dynamics. These results suggest more generally that labor market frictions are of first-order concern in the formulation of policy advice and have sparked a resurgence in studying the implications of sticky wages and other labor market imperfections for the design of optimal policy. Because the Ramsey approach to designing macroeconomic policy is an attractive one that has received increasing attention, it is of interest to investigate the impact of sticky wages on Ramsey fiscal and monetary policy. This investigation is the purpose of this paper.

Our central finding is that even when it is only nominal wages that are sticky, the Ramsey planner does not engineer volatile nominal prices in order to finance innovations to the government budget. Thus, sticky nominal wages, similar to sticky nominal prices, impose an efficiency cost an order of magnitude larger than the insurance benefit for the government of surprise inflation. The basic reason for this result is that when nominal wages are sticky, setting real wages as close as

possible to their efficient path is a much more important goal of policy than financing innovations to the government budget via state-contingent inflation.

Key for our result is a law of motion for real wages, which amounts to a restriction relating real wage growth, nominal wage inflation, and nominal price inflation. The condition itself is an identity, but is one that is a non-trivial part of the definition of equilibrium in a model featuring sticky nominal wages. Thus, this law of motion must be imposed as a constraint on the Ramsey problem. The main idea behind this restriction is that wage-setting behavior on the part of sellers of labor constrains the path of nominal wages in such a way as to make the law of motion non-trivial. This restriction that drives price inflation dynamics to try to mimic the efficient path of *real* wages when nominal wages are sticky. Quantitatively, this motive dominates the motive to use inflation to finance shocks to the government budget. In contrast, with flexible nominal wages, as in Chari, Christiano, and Kehoe (1991), Schmitt-Grohe and Uribe (2004b), and Siu (2004), the nominal wage adjusts residually to ensure consistency between the time paths of nominal prices and real wages. We develop further economic intuition for this condition when we discuss the equilibrium of our model.

Low inflation volatility in response to shocks is a statement about the dynamic properties of optimal policy in our model. Separately, our steady-state results also uncover a novel motive for the use of policy due to monopolistic labor markets. Labor market power represents a fixed factor of production. As such, the Ramsey planner would like to tax it heavily because doing so raises revenue in a non-distortionary way. Absent an instrument that directly taxes this monopoly power without distorting other margins, it can be indirectly taxed through a positive nominal interest rate. Interestingly, we find this use of the nominal interest rate is optimal only if the Friedman Rule of a zero net nominal interest rate has already been abandoned due to positive producer profits — labor market power by itself does not induce a departure from the Friedman Rule. A broader note this result sounds is that in ever-richer Ramsey models, there are likely to be untapped rents the Ramsey planner would like to access through indirect instruments. This cautionary note adds to Fernández-Villaverde’s (2005) remarks about interpreting policy advice from ever-larger models and is related to Kocherlakota’s (2005) recommendation to consider a complete set of policy instruments when studying optimal policy.

Our results complement recent work by Schmitt-Grohe and Uribe (2005), who study Ramsey policy in the Christiano, Eichenbaum, and Evans (2005) model of the business cycle. The Christiano, Eichenbaum, and Evans (2005) model features a host of frictions, including sticky nominal wages, sticky nominal prices, and various real rigidities. In this model, Schmitt-Grohe and Uribe (2005) find that sticky wages in concert with flexible prices do not reduce price inflation volatility relative to the fully-flexible benchmark when their model economy is driven by both government

spending and productivity shocks. This result contrasts with our finding that inflation volatility falls by an order of magnitude in the face of both government spending and productivity shocks. Because our calibration of the exogenous shocks is quite close to their calibration, it seems the other frictions present in their model may be masking the direct effect of sticky wages on inflation volatility. Our study instead zooms in on the consequences of sticky wages for optimal policy. Our work also builds on Erceg, Henderson, and Levin (2000), who, in a model that abstracts from fiscal considerations, study optimal monetary policy in an economy with sticky prices and sticky wages. In a model driven by productivity shocks, they find that nominal prices are volatile when prices are flexible and wages are sticky. We show that with a government financing concern present, this effect is dampened in a quantitatively important way.

The rest of the paper is organized as follows. Section 2 outlines the structure of the economy we study, which is a cash-good/credit-good environment featuring monopolistic suppliers of labor and goods and stickiness in both nominal wages and nominal prices. Section 3 presents the Ramsey problem, including an intuitive discussion of the key equilibrium restriction between price inflation and wage inflation. Section 4 presents our quantitative results, both in steady-state and dynamically. Section 5 examines how the effect of sticky nominal wages on optimal policy is altered when nominal wages are indexed to nominal prices, a phenomenon for which there is a good deal of empirical support. Section 6 considers our results from a different perspective, that of the dynamics of real government debt, which from the work of Aiyagari et al (2002), Schmitt-Grohe and Uribe (2004b), and Siu (2004) is known to be dramatically altered when the government either cannot or is restricted in its ability to make its debt payments state-contingent. Section 7 concludes.

2 The Economy

Our model economy closely resembles typical Ramsey models of fiscal and monetary policy, making our results comparable to existing studies. In particular, we abstract from capital formation, as do Chari, Christiano, and Kehoe (1991), Schmitt-Grohe and Uribe (2004a and 2004b), and Siu (2004), as well as the host of real frictions present in Schmitt-Grohe and Uribe (2005). We motivate money demand through a cash-in-advance constraint on a subset of goods purchased by consumers. We describe, in turn, the production structure of the economy, the consumer's problem, the government, the resource frontier, and the definition of a competitive monetary equilibrium in our model. The sources of uncertainty in the environment are government spending and aggregate productivity.

2.1 Production

The production side of the economy features three sectors: an intermediate goods sector that produces differentiated goods using only labor, a competitive employment sector that hires differentiated labor from consumers for resale to the intermediate goods sector, and a final goods sector that uses intermediate goods to produce the consumption good. The separation into intermediate goods and final goods is a standard convention in New Keynesian models, and the employment sector is a convenient way of introducing differentiated labor inputs, as in Erceg, Henderson, and Levin (2000).

2.1.1 Final Goods Producers

Government consumption goods and private consumption goods are physically indistinguishable. Furthermore, cash goods and credit goods are also indistinguishable. The only difference between them is that cash goods require fiat money for purchase, while credit goods do not. All final goods y_t are produced in a perfectly-competitive sector operating the CES technology

$$y_t = \left[\int_0^1 y_{jt}^{1/\varepsilon^p} dj \right]^{\varepsilon^p}, \quad (1)$$

where $\varepsilon^p/(\varepsilon^p - 1)$ is the elasticity of substitution between any two intermediate goods y_{jt} and y_{kt} , $j \neq k$. For the monopolistic intermediate producer's problem to be well-defined, $\varepsilon^p \geq 1$. Differentiated intermediate goods are indexed by j , and final goods production requires only the differentiated intermediate goods as inputs.

Final goods firms thus maximize

$$P_t y_t - \int_0^1 P_{jt} y_{jt} dj \quad (2)$$

on a period-by-period basis by choosing y_{jt} , and P_{jt} denotes the nominal price of intermediate good j . Profit maximization gives rise to demand functions for each intermediate good j ,

$$y_{jt} = \left[\frac{P_t}{P_{jt}} \right]^{\frac{\varepsilon^p}{\varepsilon^p - 1}} y_t. \quad (3)$$

2.1.2 Intermediate Goods Producers

Intermediate firm j hires labor services to produce its output using a linear technology,

$$y_{jt} = z_t N_{jt}. \quad (4)$$

Labor services N_{jt} hired by the j -th intermediate firm are an aggregate of the differentiated types of labor supplied by households. This aggregate labor is purchased from a competitive employment

agency, to be described below. Each intermediate firm is subject to the aggregate productivity realization z_t .

We now describe the profit-maximization problem of intermediate firm j . Due to the constant-returns technology and because we assume zero fixed costs of production, the marginal cost of production, denoted by mc_t , coincides with average cost. Intermediate firms face a nominal rigidity, modelled using a quadratic cost of price-adjustment. The firm incurs a real cost

$$\frac{\psi^p}{2} \left(\frac{P_{jt}}{P_{jt-1}} - 1 \right)^2 \quad (5)$$

in period t of changing its nominal price P_{jt-1} to P_{jt} , which makes its profit-maximization problem a dynamic one. Specifically, in period t , intermediate firm j 's problem is to choose P_{jt} to maximize discounted nominal profits in t and $t + 1$,

$$(P_{jt} - P_t mc_t) y_{jt} - \frac{\psi^p}{2} \left(\frac{P_{jt}}{P_{jt-1}} - 1 \right)^2 P_t + E_t \left[\frac{q_{t+1}}{\pi_{t+1}^p} \left[(P_{jt+1} - P_{t+1} mc_{t+1}) y_{jt+1} - \frac{\psi^p}{2} \left(\frac{P_{jt+1}}{P_{jt}} - 1 \right)^2 P_{t+1} \right] \right], \quad (6)$$

where q_{t+1} denotes the consumer's stochastic discount factor, derived below, for real risk-free assets and π_{t+1}^p is gross price inflation between period t and $t + 1$, $\pi_{t+1}^p \equiv P_{t+1}/P_t$. Because consumers are assumed to own intermediate firms and thus receive their profits, the consumers' nominal discount factor q_{t+1}/π_{t+1}^p is used to discount period- $t + 1$ profits. The intermediate firm takes its demand function (3), aggregate demand y_t , and the aggregate price level P_t as given.

The first-order-condition of this problem with respect to P_{jt} gives rise to a standard New Keynesian Phillips Curve,

$$\left[1 - \frac{\varepsilon^p}{\varepsilon^p - 1} + \frac{\varepsilon^p}{\varepsilon^p - 1} mc_t \right] y_t - \psi^p (\pi_t^p - 1) \pi_t^p + \psi^p E_t [q_{t+1} (\pi_{t+1}^p - 1) \pi_{t+1}^p] = 0, \quad (7)$$

which we refer to as the price Phillips Curve to distinguish it from an analogous expression for the evolution of nominal wages that arises from the consumer sector of our model, described below. The static gross markup in intermediate goods market j is ε^p .

We restrict attention to symmetric equilibria in which all intermediate producers charge the same nominal price and produce the same quantity. Thus, in equilibrium $P_{jt} = P_t$ and $y_{jt} = y_t$ for all j , and there is no price dispersion. In a symmetric equilibrium, real profits of the representative intermediate goods producer in period t , which we assume are distributed to consumers lump-sum at the start of period $t + 1$, are given by

$$pr_t = (1 - mc_t) y_t - \frac{\psi^p}{2} (\pi_t^p - 1)^2. \quad (8)$$

2.1.3 Employment Agencies

Perfectly-competitive employment agencies hire the differentiated labor of consumers and aggregate them using the CES technology

$$N_t = \left[\int_0^1 n_{it}^{1/\varepsilon^w} di \right]^{\varepsilon^w}, \quad (9)$$

where $\varepsilon^w/(\varepsilon^w - 1)$ measures the elasticity of substitution between the different types of labor, and $\varepsilon^w \geq 1$ is the static gross markup of consumer i in the type- i labor market. The final labor input N_t is then sold to intermediate goods producers. Nominal profits of the representative employment agency are given by

$$W_t N_t - \int_0^1 W_{it} n_{it} di, \quad (10)$$

where W_t denotes the aggregate nominal wage rate (the nominal price of N_t) and W_{it} is the nominal wage of type- i labor. Maximizing (10) with respect to (9) yields the demand function for the i -th consumer's labor,

$$n_{it} = \left[\frac{W_t}{W_{it}} \right]^{\frac{\varepsilon^w}{\varepsilon^w - 1}} N_t. \quad (11)$$

As with intermediate producers, we consider only symmetric equilibria in labor markets, in which $W_{it} = W_t$ and $n_{it} = N_t$ for all i .

2.2 Consumers

There is a continuum of consumers, each of whom supplies to employment agencies his differentiated labor n_{it} according to the demand function (11) after choosing his nominal wage. Each consumer also chooses consumption of cash goods c_{1t} and credit goods c_{2t} , as well as holdings of money M_t and nominal government bonds B_t . We follow the timing described by Chari, Christiano, and Kehoe (1991), in which securities market trading precedes goods and factor market trading. As we make clear below, all consumers make the same decisions, so we do not need to index allocations and asset holdings by i , except for during optimization with respect to the nominal wage. Thus, each consumer i maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, n_{it}) \quad (12)$$

subject to the flow budget constraint

$$M_t - M_{t-1} + B_t - R_{t-1} B_{t-1} = (1 - \tau_{t-1}^n) W_{i,t-1} n_{i,t-1} + P_{t-1} p r_{t-1} - P_{t-1} c_{1t-1} - P_{t-1} c_{2t-1} - \frac{\psi^w}{2} \left(\frac{W_{it-1}}{W_{it-2}} - 1 \right)^2 P_{t-1}, \quad (13)$$

the cash-in-advance constraint

$$P_t c_{1t} \leq M_t, \quad (14)$$

and the demand function for his labor (11). In (13), the term $\frac{\psi^w}{2} \left(\frac{W_{it}}{W_{it-1}} - 1 \right)^2$ measures the real cost of wage adjustment. Thus, we model sticky wages analogously to how we model sticky prices,

using a quadratic adjustment cost. Note that in (13), the consumer pays the wage adjustment cost incurred in period $t-1$ at the beginning of period t , in keeping with the timing of Chari, Christiano, and Kehoe (1991). Wage income is subject to the proportional tax rate τ_t^n . The nominal price of final goods is P_t , and nominal profits of the intermediate goods sector, $P_{t-1}pr_{t-1}$, are received lump-sum with a one-period lag.

Consumers have access to two government-issued assets: fiat money and one-period nominally risk-free government bonds. The consumer's asset holdings at the close of the period- t asset markets consist of M_t and B_t . Nominal bonds B_t pay a gross nominal return R_t in the securities market in $t+1$, and the nominal return is known when bonds are purchased. Constraint (14) motivates money demand. Because the bonds are government bonds, they are not in zero net supply.

Because we assume a quadratic cost of wage adjustment and focus on a symmetric equilibrium, each consumer is identical to every other in all his choices, but takes the nominal wages of all others as given when choosing his optimal nominal wage. After substituting (11) into (12) and (13), each consumer chooses sequences for c_{1t} , c_{2t} , W_{it} , M_t , and B_t to maximize (12) subject to (13) and (14). Given optimal choices for these sequences, the household's labor supply n_{it} is given by the demand function (11).

Associate the Lagrange multipliers ϕ_t/P_{t-1} with the sequence of budget constraints and λ_t/P_{t-1} with the sequence of cash-in-advance constraints. The consumer's first-order conditions with respect to cash good consumption, credit good consumption, money holdings, and bond holdings are thus:

$$u_{1t} - \lambda_t - \beta E_t \phi_{t+1} = 0, \quad (15)$$

$$u_{2t} - \beta E_t \phi_{t+1} = 0, \quad (16)$$

$$-\frac{\phi_t}{P_{t-1}} + \frac{\lambda_t}{P_t} + \beta E_t \left(\frac{\phi_{t+1}}{P_t} \right) = 0, \quad (17)$$

$$-\frac{\phi_t}{P_{t-1}} + \beta E_t \left(R_t \frac{\phi_{t+1}}{P_t} \right) = 0, \quad (18)$$

where the notation u_{1t} denotes the value of marginal utility of cash goods in period t , and similarly for u_{2t} . The notation ∂u_t does not mean that the function is time-varying, just that its value is. The optimality condition surrounding labor supply is described in Section 2.2.1.

From (18), we get a usual Fisher relation,

$$1 = R_t E_t \left[\frac{\beta \phi_{t+1}}{\phi_t} \frac{1}{\pi_t^p} \right], \quad (19)$$

where $\pi_t^p \equiv P_t/P_{t-1}$ is the gross rate of price inflation between period $t-1$ and period t . The stochastic discount factor $E_t[(\beta \phi_{t+1}/\phi_t)(1/\pi_t^p)]$ prices a nominally risk-free one-period asset. We can express the Fisher relation alternatively in terms of marginal utilities. Combining (15) and (17), we get

$$\phi_t = \frac{u_{1t}}{\pi_t^p}. \quad (20)$$

Substituting this expression into (19) gives us the pricing formula for a one-period risk-free nominal bond,

$$1 = R_t E_t \left[\frac{\beta u_{1t+1}}{u_{1t}} \frac{1}{\pi_{t+1}^p} \right], \quad (21)$$

the form of the Fisher equation we use in constructing the Ramsey problem in Section 3. As alluded to in the description of intermediate firms above, define the nominal pricing kernel between t and $t + 1$ as

$$Q_{t+1} \equiv \left(\frac{\beta u_{1t+1}}{u_{1t}} \frac{1}{\pi_{t+1}^p} \right), \quad (22)$$

and thus the real pricing kernel as

$$q_{t+1} \equiv Q_{t+1} \pi_{t+1}^p. \quad (23)$$

The consumer first-order conditions also imply that the gross nominal interest rate equals the marginal rate of substitution between cash and credit goods. Specifically,

$$R_t = \frac{u_{1t}}{u_{2t}}, \quad (24)$$

a standard condition in cash-good/credit-good models. Note that in a monetary equilibrium, $R_t \geq 1$, otherwise consumers could earn unbounded profits by buying money and selling bonds.

2.2.1 Wage Phillips Curve

The consumer's first-order-condition with respect to his nominal wage W_{it} can be expressed as the wage Phillips Curve,

$$\begin{aligned} & - [\varepsilon^w u_{3t} + (1 - \tau_t^n) u_{2t} m c_t m p n_t] n_t \\ & - \psi^w (\varepsilon^w - 1) u_{2t} (\pi_t^w - 1) \pi_t^w + \beta \psi^w (\varepsilon^w - 1) E_t [u_{2t+1} (\pi_{t+1}^w - 1) \pi_{t+1}^w] = 0, \end{aligned} \quad (25)$$

in which π_t^w denotes gross nominal wage inflation between time $t - 1$ and t , π_t^p denotes gross nominal price inflation between $t - 1$ and t , and $m p n_t$ denotes the marginal product of labor (which is simply equal to z_t). In deriving the wage Phillips Curve, we use the identity

$$m c_t = \frac{w_t}{m p n_t} \quad (26)$$

to substitute out the real wage. We impose symmetric equilibrium, in which $W_{it} = W_t$ and $n_{it} = N_t$ for all i . The wage Phillips Curve is forward-looking in that when setting W_{it} , labor suppliers consider the future nominal wage W_{it+1} they expect to charge. This forward-looking aspect of wage-setting is analogous to that embodied in the standard New Keynesian price Phillips Curve (7). With nominal wage inflation determined by the forward-looking wage Phillips Curve, hours worked are determined residually from the demand curve.

If nominal wages are flexible, $\psi^w = 0$, so (25) reduces to

$$-\varepsilon^w \frac{u_{3t}}{u_{2t}} = (1 - \tau_t^n) mc_t mpn_t, \quad (27)$$

the familiar condition that the consumer's marginal rate of substitution between consumption and leisure (modified to account for static labor market power) equals the after-tax real wage. The presence of nominal wage rigidities alters this optimality condition.

2.3 Government

The government's flow budget constraint is

$$M_t + B_t + P_{t-1} \tau_{t-1}^n w_{t-1} n_{t-1} = M_{t-1} + R_{t-1} B_{t-1} + P_{t-1} g_{t-1}. \quad (28)$$

Thus, the government finances government spending through labor income taxation, issuance of nominal debt, and money creation. Note that government consumption is a credit good, following Chari, Christiano, and Kehoe (1991), because g_{t-1} is not paid for until period t . As in Schmitt-Grohe and Uribe (2004a and 2004b), we assume the government does not have the ability to directly tax profits of the intermediate goods producers.¹ Furthermore, the government does not have instruments that directly tax the monopoly surplus accruing to labor suppliers.

Using the first-order conditions of the consumer, we can express the government budget constraint as

$$c_{1t} \pi_t^p + b_t \pi_t^p + \tau_{t-1}^n mc_{t-1} mpn_{t-1} n_{t-1} = c_{1t-1} + \frac{u_{1t-1}}{u_{2t-1}} b_{t-1} + g_{t-1}, \quad (29)$$

which we use in our formulation of the Ramsey problem, described in Section 3. In (29), $b_t \equiv B_t/P_t$ denotes the time- t real value of nominal government debt payable in period $t + 1$.

2.4 Resource Constraint

Summing the time- t consumer budget constraint and the time- t government budget constraint gives the economy-wide resource constraint,

$$c_{1t-1} + c_{2t-1} + g_{t-1} + \frac{\psi^p}{2} (\pi_{t-1}^p - 1)^2 + \frac{\psi^w}{2} (\pi_{t-1}^w - 1)^2 = y_{t-1}, \quad (30)$$

in which the resource costs associated with price and wage adjustment appear. Note that this resource frontier describes production possibilities for period $t - 1$ because of the timing of markets in our model — specifically, because (all) goods are paid for with a one period lag, summing the time- t consumer and government budget constraints gives rise to the time- $t - 1$ resource frontier.²

¹As Schmitt-Grohe and Uribe (2004a) show, this assumption is not as restrictive as it seems. All that is needed is that the profit tax rate be bounded below 100 percent because complete confiscation of profits would be optimal from the Ramsey point of view.

²We note a technical issue this timing imposes on the formulation of the Ramsey problem in Section 3.

2.5 Monetary Equilibrium

A symmetric monetary equilibrium in our model is a set of endogenous processes $\{c_{1t}, c_{2t}, n_t, M_t, B_t, mc_t, \pi_t^p, \pi_t^w\}$ satisfying (7), (15) - (18), (25), (28), and (30), along with bond-market and money-market clearing, for given policy $\{\tau_t^n, R_t\}$ and exogenous processes $\{g_t, z_t\}$. The restriction $R_t \geq 1$ must also be satisfied. Finally, the evolution of real wages is governed by

$$\frac{w_t}{w_{t-1}} = \frac{\pi_t^w}{\pi_t^p}, \quad (31)$$

which states that growth in real wages depends on by how much growth in nominal wages exceeds growth in nominal prices. This expression is of course an identity, but with both a price-setting equation (the price Phillips Curve) and a wage-setting equation (the wage Phillips Curve) in our model, the real wage becomes a state variable, requiring a law of motion to complete the description of equilibrium. In the next section, we discuss the economic intuition for why the law of motion for real wages is a necessary component of the definition of equilibrium in the presence of sticky wages.

3 Ramsey Problem

We now describe our formulation of the Ramsey problem. An important constraint on the Ramsey problem that arises with sticky wages is the law of motion for real wages (31), which does not hold trivially in our model. Indeed, Erceg, Henderson, and Levin (2000) and Schmitt-Grohe and Uribe (2005) also impose such a constraint in their models of optimal policy with sticky wages and sticky prices. Because this restriction is important for the results we obtain and because we find it illuminating for the determination of nominal wages in monetary models generally, we describe more fully the economic intuition behind it.

Consider a model with both flexible prices and flexible wages, but with a nontrivial demand for money. In such an environment, the only economic force acting on either price inflation or wage inflation is the money demand function, which influences price inflation. Supposing the real wage, and hence real wage growth, is pinned down by real features of the economy, nominal wage inflation is free to adjust as a residual to satisfy the law of motion (31). Instead, if the environment features sticky prices and flexible wages, as in Schmitt-Grohe and Uribe (2004b) and Siu (2004), there are two economic forces acting on price inflation: the money demand function and price-setting behavior on the part of firms (what we call the price Phillips curve). These two forces are balanced against each other somehow, and some rate of price inflation is determined. Suppose real wage growth continues to be pinned down by real features of the economy. Wage inflation in this case is again free to adjust as a residual to make condition (31) hold.

However, suppose the environment features sticky wages, with either flexible or sticky prices, and real wage growth is still pinned down by real factors. Now there is an independent economic force exerting influence on wage inflation, namely wage-setting behavior on the part of labor suppliers (what we call the wage Phillips curve). With wage-setting behavior influencing nominal wage inflation coupled with the forces discussed above influencing nominal price inflation (money demand either alone or in combination with price-setting behavior), in general these two rates of inflation will not be consistent with condition (31). Thus, (31) is a non-trivial condition describing equilibrium and so must be imposed as a constraint on the optimal policy problem. To express condition (31) in a more convenient form for the Ramsey problem, we use (26) to substitute for the real wage, so we have

$$\frac{mc_t m p n_t}{mc_{t-1} m p n_{t-1}} = \frac{\pi_t^w}{\pi_t^p}. \quad (32)$$

The complexity of our model does not allow us to write the Ramsey problem in pure primal form, in which all prices and policy instruments are substituted out and it is only allocations that are directly chosen, or pure dual form, in which allocations are specified as functions of only policy instruments and it is policy variables that are directly chosen. Rather, we resort to a hybrid setup in which policy instruments and allocations are chosen simultaneously. The set of constraints summarizing the dynamic equilibrium of the economy are the law of motion for real wages, the resource constraint, the Fisher equation, the wage Phillips Curve, the price Phillips Curve, and the government budget constraint. The Ramsey problem is thus to choose sequences $\{c_{1t}, c_{2t}, n_t, mc_t, \tau_t^n, \pi_t^p, \pi_t^w, b_t\}$ to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, n_t) \quad (33)$$

subject to (7), (21), (25), (29), (30), and (32), and given exogenous sequences $\{g_t, z_t\}$. In (21), we substitute for R_t using (24); recalling that (21) arises from combining (15), (17), and (18), the consumer's optimal choice rules for c_{1t} , c_{2t} , M_t , and B_t are therefore all respected. (The consumer's optimality condition with regard to labor supply is the wage Phillips curve (25).) In principle, we must also impose the constraint $R_t \geq 1$, which ensures the chosen allocation can be supported as a monetary equilibrium. However, because in our models $R > 1$ in steady-state and the volatilities of the exogenous processes are sufficiently small, the lower bound $R_t = 1$ was never reached in any of our simulations of the model omitting this constraint. We therefore proceed from here on dropping the constraint $R_t \geq 1$ from the Ramsey problem.

A technical issue that arises in the formulation of our Ramsey problem is the dating of the Lagrange multiplier associated with the resource constraint. Recall from Section 2.4 that it is the time- $t - 1$ resource frontier that is implied by the time- t consumer and government flow budget constraints. Because the assumed timing of our model is that the Ramsey planner observes g_t and

z_t before choosing time- t allocations and policies, the multiplier associated with constraint (30) is dated $t - 1$ – in other words, terms in the Ramsey first-order-conditions arising from the time- t resource constraint carry a multiplier dated t . This formulation of course sounds natural, but we simply mean to point out that because of the assumed timing of markets (namely, asset markets preceding goods and labor markets), care must be taken in writing the Ramsey problem.³

4 Quantitative Results

We briefly describe the functional forms and parameter values we use, which are fairly standard in dynamic macro models, to obtain our main results. We then analyze optimal policy in the Ramsey steady-state before presenting our simulation-based dynamic results.

4.1 Parameterization

The time unit in our model is one quarter, so we set the subjective time discount factor to $\beta = 0.99$. We use a period utility function

$$u(c_t, n_t) = \frac{(c_t(1 - n_t)\zeta)^{1-\sigma} - 1}{1 - \sigma}, \quad (34)$$

with the consumption index a CES aggregate of cash goods and credit goods,

$$c_t = \left((1 - \gamma)c_{1t}^\phi + \gamma c_{2t}^\phi \right)^{1/\phi}. \quad (35)$$

We set $\sigma = 1$ so that preferences are separable in leisure and consumption. In the consumption aggregator, we use $\phi = 0.79$ and $\gamma = 0.62$, as estimated by Siu (2004). The parameter ζ is calibrated so that in the model with perfect competition in both product and labor markets, consumers spend a fraction $n = 0.30$ of their time working in the deterministic steady-state of the Ramsey allocation. The value that we need turns out to be $\zeta = 2.27$, and we hold this value constant as we move to the imperfectly competitive environments with various combinations of sticky prices and sticky wages.

The exogenous productivity and government spending shocks follow AR(1) processes in logs,

$$\ln(z_t) = \rho_z \ln(z_{t-1}) + \epsilon_t^z, \quad (36)$$

$$\ln(g_t) = (1 - \rho_g) \ln(\bar{g}) + \rho_g \ln(g_{t-1}) + \epsilon_t^g, \quad (37)$$

³With flexible prices and wages, it is customary to use the household budget constraint (equivalently, the government budget constraint) in present-value form in conjunction with the flow resource constraint. This present-value constraint is what Lucas and Stokey (1983) and Chari, Christiano, and Kehoe (1991) term the implementability constraint. Imposing the implementability constraint in conjunction with the sequence of resource constraints makes, in a flexible-price/flexible-wage environment, the timing issue we need to be careful about disappear.

where \bar{g} denotes the steady-state level of government spending, which we calibrate in the model with perfect competition in both product and labor markets to constitute 20 percent of steady-state output in the Ramsey allocation. The resulting value is $\bar{g} = 0.06$, which we hold constant as we vary the degrees of imperfect competition and nominal rigidities in goods and labor markets. The innovations ϵ_t^z and ϵ_t^g are distributed $N(0, \sigma_{\epsilon^z}^2)$ and $N(0, \sigma_{\epsilon^g}^2)$, respectively, and are independent of each other. We choose parameters $\rho_z = 0.95$, $\rho_g = 0.97$, $\sigma_{\epsilon^z} = 0.007$, and $\sigma_{\epsilon^g} = 0.03$ in keeping with the RBC literature and Chari, Christiano, and Kehoe (1991). Also regarding policy, we assume that the steady-state government debt-to-GDP ratio (at an annual frequency) is 0.5, in line with evidence for the U.S. economy and with the calibrations of Schmitt-Grohe and Uribe (2004b) and Siu (2004).

Finally, it remains to choose parameter values governing the degree of imperfect competition and nominal rigidity in product markets and labor markets. For our main results, we set the gross markup in labor markets to $\varepsilon^w = 1.05$ and the gross markup in intermediate goods markets to $\varepsilon^p = 1.1$, in line with the findings of Christiano, Eichenbaum, and Evans (2005). Also in line with their findings, we calibrate ψ^w and ψ^p so that on average nominal wages and nominal prices are fixed for three quarters.⁴ As we report below, we conduct experiments in which one or the other, prices or wages, is flexible, in which case we leave the average length of stickiness of the other at three quarters.

4.2 Ramsey Steady-State

Using the nonlinear Ramsey first-order conditions, we numerically compute the non-stochastic Ramsey steady-state allocation and policy. We first discuss how the optimal policy varies with market power in goods and labor markets, and then show how nominal rigidities influence optimal policy.

4.2.1 Steady-State Policy as Function of Goods Market Power

Table 1 presents the steady-state labor tax rate, price inflation rate, wage inflation rate, and nominal interest rate for various degrees of imperfect competition in our model with both flexible wages and flexible prices. For a given markup in the labor market, our results resemble Schmitt-Grohe and Uribe's (2004a) results for imperfectly competitive product markets. As they prove in their

⁴With our cost-of-price-adjustment assumption, nominal prices and wages in our model are actually free to be reset every period, in contrast to the Taylor or Calvo formulations of price stickiness. The exact finding of Christiano, Eichenbaum, and Evans (2005), who assume Calvo-pricing, is that nominal wages are fixed for 2.8 quarters on average and nominal prices are fixed for 2.5 quarters on average. For convenience, we simply set three quarters as the average duration for each type of contract and use a mapping between the Calvo price-rigidity parameter and our parameters ψ^p and ψ^w . This mapping is derived in Appendix A.

model and as we prove for our model in Appendix D, goods market power leads to a deviation from the Friedman Rule of a zero net nominal interest rate. For a given wage markup, the steady-state labor tax rate, nominal interest rate, price inflation rate, and wage inflation rate are all increasing in the degree of product market power. The reason that the nominal interest rate is increasing in ε^p is that in the absence of a 100 percent profit tax, the nominal interest rate indirectly taxes profit income, as first explained by Schmitt-Grohe and Uribe (2004a). Profit income represents payments to a fixed factor, producers' monopoly power, which the Ramsey planner would like to tax as heavily as possible because it would be non-distortionary. With confiscation of profits ruled out, the nominal interest rate acquires the auxiliary role of indirectly taxing profits. Thus, the Friedman Rule of a zero net nominal interest rate ceases to be optimal once product markets exhibit monopoly power. With the steady-state real interest rate pinned down by the subjective discount factor and independent of monopoly power, steady-state price inflation thus also rises above the Friedman deflation as ε^p increases. Because real wages are by construction constant in the steady-state, nominal wages must grow at the same rate as nominal prices. Finally, the labor income tax rate is also increasing in the degree of product market power, again consistent with Schmitt-Grohe and Uribe (2004a). As in their model, the reason for this is the shrinking labor tax base as ε^p rises — steady-state n (not shown in the table) falls as ε^p rises.

4.2.2 Steady-State Policy as Function of Labor Market Power

We can also analyze how the steady-state policy responds to labor market power. As Table 1 shows, the labor tax rate increases as ε^w increases regardless of the degree of competition in goods markets. This result is again due to a shrinking labor tax base, as labor hours (not shown in the table) decline as ε^w rises. In contrast, the nominal interest rate, and hence both price inflation and wage inflation, are invariant to ε^w when product markets are perfectly competitive because there are no producer profits to (indirectly) tax. This latter result is overturned, however, when product markets are imperfectly competitive. When $\varepsilon^p > 1$, the nominal interest rate, and consequently inflation rates, rise as ε^w rises. This result seems related to the Schmitt-Grohe and Uribe (2004a) finding that the Ramsey planner can tax market power — in their case, product market power, in the case here, labor market power — through monetary instruments.

To shed some light on this, we can modify our model slightly to allow for a proportional labor subsidy to undo the static distortion due to the markup. In particular, suppose the government subsidizes labor income at the rate $1 + \tau^w$, so that total nominal labor income of the consumer is given by

$$(1 + \tau^w)(1 - \tau^n)Wn. \tag{38}$$

In numerical results not reported, setting a labor subsidy $1 + \tau^w = \varepsilon^w$ to completely offset labor

Gross Labor Markup	Variable	Gross Product Markup		
		$\varepsilon^P = 1$	$\varepsilon^P = 1.1$	$\varepsilon^P = 1.2$
$\varepsilon^w = 1$	τ^n	0.2218	0.2669	0.3203
	R	0	0.9704	2.8232
	π^P	-4	-3.0684	-1.2898
	π^w	-4	-3.0684	-1.2898
$\varepsilon^w = 1.05$	τ^n	0.2296	0.2771	0.3338
	R	0	1.1270	3.1903
	π^P	-4	-2.9181	-0.9373
	π^w	-4	-2.9181	-0.9373
$\varepsilon^w = 1.1$	τ^n	0.2375	0.2873	0.3476
	R	0	1.2817	3.5619
	π^P	-4	-2.7696	-0.5806
	π^w	-4	-2.7696	-0.5806
$\varepsilon^w = 1.2$	τ^n	0.2532	0.3081	0.3686
	R	0	1.5891	3.9909
	π^P	-4	-2.4744	-0.1688
	π^w	-4	-2.4744	-0.1688

Table 1: Steady-state optimal policy for various values of ε^P and ε^w when both prices and wages are flexible. Nominal interest rate, price inflation rate, and wage inflation rate reported in annualized percentage points.

	τ^n	R	π^p	π^w
Sticky price, flexible wage	0.2758	4.0505	-0.1115	-0.1115
Flexible price, sticky wage	0.2758	4.0502	-0.1118	-0.1118
Sticky price, sticky wage	0.2758	4.1075	-0.0568	-0.0568

Table 2: Steady-state optimal policy in sticky-price and/or sticky-wage economies, for $\varepsilon^p = 1.1$ and $\varepsilon^w = 1.05$. Nominal adjustment parameters set so that the respective nominal rigidity lasts for three quarters on average. Nominal interest rate, price inflation rate, and wage inflation rate reported in annualized percentage points.

market power and hence eliminate labor rents renders the labor tax rate τ^n and the interest rate R invariant to ε^w . Thus, the rise in R absent the labor subsidy seems to be due to the Ramsey planner taxing labor market power indirectly through the nominal interest rate. Once the monopolistic labor distortion is removed with a labor subsidy, there no longer are labor rents to tax, hence no motive for the Ramsey planner to increase R beyond that needed to indirectly tax *product* market power. Interestingly, positive producer profits are necessary for R to be able to tax labor power because with perfect competition in product markets R does not vary with ε^w , as Table 1 shows.⁵ That is, labor market power alone does not induce deviations from the Friedman Rule, but once the Friedman Rule has already been abandoned, increasing labor power induces larger deviations from the Friedman Rule.

4.2.3 Steady-State Policy with Nominal Rigidities

Finally, Table 2 displays the steady-state Ramsey policy when either or both prices and wages are sticky, with the static markups at our baseline values of $\varepsilon^w = 1.05$ and $\varepsilon^p = 1.1$. With either sticky prices or sticky wages, or both, near-perfect nominal price stability and nominal wage stability are optimal. That the steady-state price and wage inflation rates are identical follows immediately from a constant real wage in condition (31), which explains how a motive to stabilize only nominal wages translates into stabilization of nominal prices. This link is also crucial for understanding our dynamic results, to which we turn next.

4.3 Optimal State-Contingent Inflation

We now turn to the dynamics of the Ramsey policy. To study dynamics, we approximate our model by linearizing in levels the Ramsey first-order conditions for time $t > 0$ around the non-stochastic

⁵As we show in the Appendix, deviation from the Friedman Rule depends only on monopoly power in the goods market, not on monopoly power in the labor market.

steady-state of these conditions.⁶ Thus, we ignore any transition from an arbitrary initial policy to the long-run policy.⁷ Our numerical method is our own implementation of the perturbation algorithm described by Schmitt-Grohe and Uribe (2004c). As in Khan, King, and Wolman (2003), we assume that the initial state of the economy is the asymptotic Ramsey steady-state. Throughout, we assume that the first-order conditions of the Ramsey problem are necessary and sufficient and that all allocations are interior. We also point out that because we assume full commitment on the part of the Ramsey planner, the use of state-contingent inflation is not a manifestation of time-inconsistent policy. The “surprise” in surprise inflation is due solely to the unpredictable components of government spending and technology, and not due to a retreat on past promises.

For each version of our model, we conduct 5000 simulations of 500 periods each and discard the first 100 periods. For each simulation, we then compute first and second moments and report the averages of these moments over the 5000 simulations. To make the comparisons meaningful, the same realizations for the government spending shocks and productivity shocks are used across versions of our model.

4.3.1 Both Government Spending and Productivity Shocks

Table 3 presents simulation-based moments for the key policy variables along with the real wage for several parameterizations of our model when the driving forces of the economy are both technology shocks and government spending shocks.⁸ We start with this case because it is the case often focused on in the existing Ramsey literature. In the next subsection, we focus on the case in which the economy is driven by only government spending shocks.

The top panel of Table 3 presents results for the model with perfect competition in both product and labor markets. As originally found by Chari, Christiano, and Kehoe (1991), price inflation is quite volatile and the labor tax rate very smooth over time in this environment. The reason for this, as described in the introduction and well-known in the Ramsey literature, is that the Ramsey planner uses surprise variations in the price level to render nominally risk-free debt state-contingent in real terms, thus financing a large portion of innovations to the government budget in a non-distortionary way and facilitating tax-smoothing. The new result we display in the top panel is that nominal wage inflation, because it is highly correlated with price inflation through condition (32), is also quite volatile. These results, high volatility of both price inflation and wage inflation, carry over to the economy with monopoly power but flexible prices in both goods and

⁶From the results of Schmitt-Grohe and Uribe (2004b, p. 219), we have some reason to believe that the results from a second-order approximation would be close to those from a first-order approximation.

⁷As we show in Section 6, when wages are sticky, government debt becomes highly persistent, which may make it quite interesting on its own to *not* ignore such transitions.

⁸Appendix E also presents simulated moments for output and hours.

labor markets, the second panel in Table 3.

The third panel in Table 3 displays results for the model with sticky prices and flexible wages. Consistent with the findings of Schmitt-Grohe and Uribe (2004b) and Siu (2004), the volatility of price inflation falls by an order of magnitude, and the mean inflation rate rises from close to the Friedman deflation to near zero. The reason for this is that with costs of nominal price adjustment, the Ramsey planner keeps price changes to a minimum in levying the inflation tax on nominal wealth. The value to the Ramsey planner of state-contingent levies on nominal assets is largely dominated by the resource cost of non-zero inflation. Nominal wage inflation volatility also falls, although not as dramatically — its volatility of 1.67 percent is about one-fourth the volatility under perfect competition. The tradeoff facing the Ramsey planner here seems to be the one described by Erceg, Henderson, and Levin (2000): in engineering the optimal real wage, fluctuations in nominal prices are undesirable because of the cost of price adjustment. The Ramsey planner thus resorts to fluctuations in nominal wages, which are costless. With smaller variations in inflation, however, more of the surprise revenue generation falls on the labor tax rate, thus the rise in the volatility of τ^n . Note, however, that the fluctuations in inflation here are not driven solely by government revenue requirements because productivity is also fluctuating. In the next subsection, we focus on the case of only government spending shocks.

The fourth panel of Table 3 shows results for the model with flexible prices and sticky wages. A natural conjecture for this case is that price inflation volatility is intermediate between the fully-flexible case and the sticky-price/flexible-wage case. Our numerical results confirm this. This result contrasts with that found by Schmitt-Grohe and Uribe (2005). They find that with sticky wages and flexible prices, price inflation volatility is virtually the same as that under full flexibility — namely, highly volatile price inflation — when the economy is driven by both government spending and productivity shocks. Because our model does not consider the other frictions their model considers, our results suggest that stickiness in nominal wages alone does dampen Ramsey inflation volatility, but this effect can be overridden by other frictions present in the environment. We find that wage inflation volatility for this case is small, as would be expected due to the assumed wage rigidity. This is the reverse of the results with sticky prices and flexible wages and seems to be driven by the same concern of the Ramsey planner for implementing a real wage as close as possible to the efficient real wage: with stickiness in only wages, the Ramsey planner generates some volatility in prices, but not as much as predicted by the Schmitt-Grohe and Uribe (2005) model. We further disentangle the effects of the exogenous shocks on price inflation and wage inflation volatility in this flexible-price/sticky-wage case in the next subsection. Note also that the volatility of the labor tax rate in this case is about the same as in the sticky-price/flexible-wage case.

Finally, the bottom panel of Table 3 presents results for the model with both sticky prices and

Variable	Mean	Std. Dev.	Auto corr.	Corr(x, y)	Corr(x, g)	Corr(x, z)
<u>Perfect Competition</u>						
τ^n	0.2218	0.0044	0.8420	-0.1275	0.9254	-0.2910
π^p	-4.0113	6.9631	0.0776	-0.1022	-0.1632	-0.1407
π^w	-4.0113	6.5833	0.0957	-0.0817	-0.1725	-0.1154
R	0	—	—	—	—	—
w	0.9999	0.0118	0.8051	0.9681	0.0074	1
<u>Flexible prices, flexible wages</u>						
τ^n	0.2771	0.0072	0.7382	-0.4648	0.8169	-0.4802
π^p	-2.9160	4.6032	-0.0091	-0.1631	0.3279	-0.1053
π^w	-2.9163	4.2308	0.0009	-0.1081	0.3658	-0.0486
R	1.1262	0.3780	-0.2910	0.2515	0.0299	0.2376
w	0.9090	0.0108	0.8051	0.9721	0.0074	1
<u>Sticky prices, flexible wages</u>						
τ^n	0.2759	0.0240	0.6336	-0.8702	0.5811	-0.3957
π^p	-0.1144	0.4986	-0.1055	-0.0119	-0.2161	0.2198
π^w	-0.1134	1.6712	-0.3770	-0.0936	-0.01565	0.1077
R	4.0653	0.9008	-0.1527	0.2505	0.1058	-0.0818
w	0.9090	0.0166	0.1720	-0.0322	0.1856	0.4844
<u>Flexible prices, sticky wages</u>						
τ^n	0.2759	0.0203	0.5961	-0.8758	0.6469	-0.5344
π^p	-0.1131	0.9113	-0.0189	-0.2481	-0.0438	-0.1144
π^w	-0.1133	0.5218	0.1966	0.0159	-0.0928	0.1947
R	4.0660	1.1331	-0.0480	0.0633	0.0132	-0.0535
w	0.9090	0.0107	0.8051	0.8375	0.0074	1
<u>Sticky prices, sticky wages</u>						
τ^n	0.2759	0.0221	0.5966	-0.7613	0.7041	-0.4590
π^p	-0.0584	0.4832	0.0705	-0.1543	-0.1854	-0.1551
π^w	-0.0594	0.5294	0.1784	0.4667	-0.0721	0.4614
R	4.1094	0.9411	-0.0766	0.1390	-0.0433	0.1035
w	0.9090	0.0129	0.8916	0.7654	0.0713	0.9380

Table 3: Simulation-based moments with g_t and z_t as the driving processes. π^p , π^w , and R reported in annualized percentage points. Markup parameters are $\varepsilon^w = 1.05$ and $\varepsilon^p = 1.1$. For either sticky wages or sticky prices, ψ^w and/or ψ^p are set so that the respective nominal rigidity lasts three quarters on average.

sticky wages. Here, we find that wage inflation volatility and price inflation volatility are roughly equal to each other, and roughly equal to the volatility of either nominal wages or nominal prices when either alone is sticky.

Our numerical results also confirm the intuition discussed earlier about how the restriction (32) affects the Ramsey plan. In our flexible-wage models, the Lagrange multiplier (not shown in the tables) associated with this constraint in the Ramsey problem turns out to be zero both in steady-state and dynamically, meaning this constraint is redundant. The nominal wage thus is adjusting as it needs to satisfy the law of motion. In contrast, with sticky wages, the Lagrange multiplier on this constraint is strictly positive both in steady-state and dynamically, meaning this constraint binds. The constraint (32) binds because the path of nominal wages is restricted by the wage Phillips curve.

4.3.2 Disentangling the Effects of Shocks

Table 3 is comparable to Table 4A in Schmitt-Grohe and Uribe (2005) in that it considers the responses of the Ramsey allocation to both technology and government spending shocks. The core motive of the use of surprise changes in the price level in a Ramsey model, however, is to finance government spending shocks in a non-distortionary way. Indeed, Siu (2004) considers only government spending shocks in his study. Thus, we find it instructive to examine the effect of sticky wages when the economy is buffeted by only government spending shocks or only technology shocks. To this end, Tables 4 and 5 present simulation-based moments when there is only one driving process in the model, only government spending shocks in Table 4 and only technology shocks in Table 5.

First consider the fully-flexible cases, the top two panels of Tables 4 and 5. When the economy is hit with only government spending shocks, the dynamics of price inflation are identical to the dynamics of wage inflation. This is to be expected because by assumption the marginal product of labor is constant and the marginal cost of production is constant because prices are flexible.⁹ The real wage is thus constant, and condition (31) then reveals that price inflation and wage inflation track each other perfectly. The dynamics of wage and price inflation differ, of course, when the real wage is fluctuating, as is the case in Table 5 with technology shocks — note that both price inflation and wage inflation are quite volatile, as expected with flexible prices and wages because the Ramsey planner faces little cost of state-contingent inflation, but are not perfectly-correlated.

Turning to the third panel of Table 4, we find that sticky prices dampen price inflation volatility by an order of magnitude, also as expected. Because the marginal cost of production mc_t fluctuates over time due to sticky prices, the dynamics of wage inflation are no longer identical to those of

⁹Real marginal cost with flexible prices is simply the inverse of the gross markup in the product market.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x, y)	Corr(x, g)	Corr(x, z)
<u>Perfect Competition</u>						
τ^n	0.2218	0.0042	0.8515	0.7494	0.9788	—
π^p	-4.0107	5.6969	0.1431	0.0081	-0.1992	—
π^w	-4.0107	5.6969	0.1431	0.0081	-0.1992	—
R	0	—	—	—	—	—
w	1	—	—	—	—	—
<u>Flexible prices, flexible wages</u>						
τ^n	0.2771	0.0061	0.7627	0.1604	0.9586	—
π^p	-2.9149	2.8764	-0.0116	-0.2468	0.4105	—
π^w	-2.9149	2.8764	-0.0116	-0.2468	0.4105	—
R	1.1279	0.2056	-0.2725	0.1921	0.0424	—
w	0.9090	—	—	—	—	—
<u>Sticky prices, flexible wages</u>						
τ^n	0.2759	0.0188	0.3285	-0.9554	0.7442	—
π^p	-0.1128	0.7809	-0.1362	-0.1760	-0.2912	—
π^w	-0.1122	2.2164	-0.4172	-0.3245	-0.0180	—
R	4.0572	0.6653	-0.1570	0.3938	0.1428	—
w	0.9091	0.0111	-0.2449	-0.6991	0.2753	—
<u>Flexible prices, sticky wages</u>						
τ^n	0.2758	0.0153	0.6444	-0.9392	0.8623	—
π^p	-0.1123	0.3720	0.1209	-0.2036	-0.1323	—
π^w	-0.1123	0.3720	0.1209	-0.2036	-0.1323	—
R	4.0551	0.7886	-0.0772	0.1743	0.0190	—
w	0.9090	—	—	—	—	—
<u>Sticky prices, sticky wages</u>						
τ^n	0.2759	0.0132	0.6394	-0.8664	0.8565	—
π^p	-0.0582	0.2131	0.3398	0.1546	-0.4202	—
π^w	-0.0580	0.4362	-0.0613	0.0388	-0.0898	—
R	4.1094	0.7441	-0.0630	0.1218	-0.0557	—
w	0.9091	0.0021	0.1040	-0.5000	0.3866	—

Table 4: Simulation-based moments with g_t as the only driving process. π^p , π^w , and R reported in annualized percentage points. Markup parameters are $\varepsilon^w = 1.05$ and $\varepsilon^p = 1.1$. For either sticky wages or sticky prices, ψ^w and/or ψ^p are set so that the respective nominal rigidity lasts three quarters on average.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x, y)	Corr(x, g)	Corr(x, z)
<u>Perfect Competition</u>						
τ^n	0.2218	0.0014	0.7692	-0.9515	—	-0.9227
π^P	-4.0001	4.0031	-0.0691	-0.1788	—	-0.2385
π^w	-4.0001	3.2989	-0.0625	-0.1630	—	-0.2227
R	0	—	—	—	—	—
w	0.9999	0.0119	0.8051	0.9957	—	1
<u>Flexible prices, flexible wages</u>						
τ^n	0.2771	0.0037	0.6769	-0.9732	—	-0.9361
π^P	-2.9192	2.1684	-0.0177	-0.2120	—	-0.1758
π^w	-2.9195	1.4692	0.0299	-0.1477	—	-0.1079
R	1.1253	0.4641	-0.3171	0.3097	—	0.3186
w	0.9090	0.0108	0.8051	0.9906	—	1
<u>Sticky prices, flexible wages</u>						
τ^n	0.2759	0.0149	0.1554	-0.9004	—	-0.6424
π^P	-0.1132	0.7033	-0.0743	0.1018	—	0.3301
π^w	-0.1127	1.4991	-0.2979	0.0995	—	0.1901
R	4.0585	0.6079	-0.1563	0.1669	—	-0.1231
w	0.9090	0.0125	0.4932	0.3363	—	0.6451
<u>Flexible prices, sticky wages</u>						
τ^n	0.2759	0.0133	0.5353	-0.9578	—	-0.8192
π^P	-0.1126	0.9479	-0.0390	-0.2561	—	-0.1199
π^w	-0.1129	0.3657	0.2716	0.1324	—	0.2822
R	4.0616	0.8133	-0.0281	0.0124	—	-0.0766
w	0.9090	0.0108	0.8051	0.9344	—	1
<u>Sticky prices, sticky wages</u>						
τ^n	0.2759	0.0091	0.5077	-0.9023	—	-0.8194
π^P	-0.0579	0.4332	0.0019	-0.2220	—	-0.1699
π^w	-0.0590	0.3011	0.6860	0.8828	—	0.8152
R	4.1067	0.5755	-0.1117	0.1741	—	0.1667
w	0.9090	0.0128	0.9136	0.8958	—	0.9512

Table 5: Simulation-based moments with z_t as the only driving process. π^P , π^w , and R reported in annualized percentage points. Markup parameters are $\varepsilon^w = 1.05$ and $\varepsilon^P = 1.1$. For either sticky wages or sticky prices, ψ^w and/or ψ^P are set so that the respective nominal rigidity lasts three quarters on average.

price inflation. Nonetheless, the volatility of wage inflation also falls substantially, although not by as much as the volatility of price inflation. Comparing the results here with those in the third panel of Table 5, we see that neither price inflation nor wage inflation is dampened as much with technology shocks as the sole driving force (relative to the perfectly-competitive benchmark), but they are both lower than in the fully-flexible cases.

Examining the fourth panel of Table 4 reveals the source of our main result. With stickiness in only nominal wages, the dynamics of price inflation and wage inflation are identical because the marginal cost of production is constant and equal to the inverse of the gross product markup ε^p . The marginal product of labor is constant by assumption because we have shut down productivity shocks and technology is linear in labor. A natural conjecture may have been that even with technology shocks shut down, the real wage would fluctuate because of innovations to government spending. However, government spending shocks are fundamentally non-distortionary shocks in our environment. They become potentially distortionary only if lump-sum taxes are ruled out, as they are in the canonical Ramsey framework.¹⁰ But in our environment the Ramsey planner has back-door ways of generating lump-sum revenues — the inflation tax on nominal assets and the departure from the Friedman Rule that indirectly taxes monopoly power in both product and labor markets. Thus, with only government spending shocks and access to ways to generate lump-sum revenues, the Ramsey planner would like to keep the real wage from fluctuating, which requires that the dynamics of price inflation and wage inflation be identical. With a cost of adjusting nominal wages, however, the Ramsey planner is reluctant to generate wage inflation, and thus also reluctant to generate price inflation because of the restriction (31). Finally, the fifth panels of Tables 4 and 5 show that with both sticky prices and sticky wages, both price inflation and wage inflation are stabilized even closer to zero.

To relate our findings to those of Schmitt-Grohe and Uribe (2005), we point out that in our model, whether the economy is hit by just technology shocks or just government purchase shocks or both, the dampening effect on inflation volatility is quite similar, as comparison of the fourth panels across Tables 3, 4, and 5 shows. In contrast, in Schmitt-Grohe and Uribe (2005), the inflation volatility result is quite sensitive to the fundamental driving shocks, as they note in their footnote 5. In particular, when their model is driven by both technology and government purchase shocks, inflation volatility is not dampened when nominal wages are sticky but prices are flexible. In contrast, when it is only government purchase shocks driving their model, inflation volatility is dramatically reduced, indeed quantitatively quite similar to our finding. Thus, the other frictions present in their model seem to be interacting with the technology shock in a way that makes volatile

¹⁰Albanesi (2004) provides a good discussion of the crucial differences between “Ramsey optimal policy models” and their fundamentally non-distortionary government spending shocks and “New Keynesian optimal policy models” and their fundamentally distortionary cost-push shocks.

inflation not too costly. This issue may be worth further study.

The central conclusion from all of our results, then, is that sticky nominal wages alone dramatically reduce the Ramsey planner’s use of surprise changes in the price level to finance innovations in government spending. Through the equilibrium restriction (31), an apriori concern for stabilizing nominal wages, because of the resource cost wage adjustment entails, translates into a concern for stabilizing nominal prices if the efficient real wage does not fluctuate much. Price inflation dynamics are thus driven by the dynamics of the efficient real wage, rather than by a concern for buffering shocks to the government budget. Deviating too much from this path of real wages is more costly for the Ramsey planner than giving up the use of price inflation variability as a shock absorber. This driving force behind price inflation is quite different from that in a baseline Ramsey monetary model, such as that of Chari, Christiano, and Kehoe (1991). Another way to put the argument is that if the Ramsey planner were to use state-contingent inflation as a capital levy on nominal assets, nominal price variability would translate into nominal wage variability through condition (31). But such wage variability imposes costs, which apparently are too high for the Ramsey planner to tolerate for realistic degrees of wage stickiness. Viewing the logic either way, the Ramsey planner largely gives up state-contingent inflation as a fiscal instrument. Thus, a concern for stabilizing only nominal wages leads to stabilization of nominal prices, as well.

5 Wage Indexation

Based on empirical evidence, models with sticky nominal wages often assume some degree of indexation of nominal wages to nominal prices.¹¹ In this section, we briefly explore the consequences of wage indexation for our main results. We assume here that nominal wage inflation between period $t - 1$ and t is indexed to nominal price inflation between period t and $t - 1$, but the results are quite similar if we instead assume indexation to lagged price inflation.

The cost of nominal wage adjustment is now given by

$$\frac{\psi^w}{2} \left(\frac{W_t}{W_{t-1} (\pi_t^p)^\chi} - 1 \right)^2, \quad (39)$$

with the parameter χ measuring the degree of indexation. Nominal wages are not indexed at all to nominal prices if $\chi = 0$, are fully indexed if $\chi = 1$, and are partially indexed if $\chi \in (0, 1)$. With indexation, it is deviations of the rate of wage inflation from the rate of price inflation, rather than deviations of the level of nominal wages from lagged levels of nominal wages, that impose a resource cost. As we show in Appendix B, with indexation to contemporaneous price inflation, the wage Phillips curve becomes

$$- [\varepsilon^w u_{3t} + (1 - \tau_t^n) u_{2t} m c_t m p n_t] n_t \quad (40)$$

¹¹For example, Levin et al (2005) estimate a high degree of indexation in nominal wages.

	$\chi = 0$ (no indexation)			$\chi = 0.5$ (partial indexation)			$\chi = 1$ (full indexation)		
	Mean	SD	Auto corr.	Mean	SD	Auto corr.	Mean	SD	Auto corr.
<u>Flexible prices</u>									
τ^n	0.2759	0.0203	0.5961	0.2760	0.0188	0.6258	0.2771	0.0088	0.7940
π^P	-0.1131	0.9113	-0.0189	-0.4085	2.3518	0.0310	-2.9201	4.1280	0.0077
π^w	-0.1133	0.5218	0.1966	-0.4087	1.7153	0.1089	-2.9204	3.6847	0.0330
<u>Sticky prices</u>									
τ^n	0.2759	0.0221	0.5966	0.2759	0.0223	0.4719	0.2759	0.0224	0.4517
π^P	-0.0584	0.4832	0.0705	-0.0929	0.6479	0.3440	-0.1159	0.7977	0.4984
π^w	-0.0594	0.5294	0.1784	-0.0940	0.9746	0.3076	-0.1169	1.3187	0.3261

Table 6: Dynamics of optimal policy under different degrees of nominal wage indexation. π^P and π^w reported in annualized percentage points.

$$-\psi^w(\varepsilon^w - 1)u_{2t} \left(\frac{\pi_t^w}{(\pi_t^P)^\chi} - 1 \right) \frac{\pi_t^w}{(\pi_t^P)^\chi} + \beta\psi^w(\varepsilon^w - 1)E_t \left[u_{2t+1} \left(\frac{\pi_{t+1}^w}{(\pi_{t+1}^P)^\chi} - 1 \right) \frac{\pi_{t+1}^w}{(\pi_{t+1}^P)^\chi} \right] = 0,$$

which generalizes (25). The resource constraint also modifies in the presence of wage indexation to

$$c_{1t} + c_{2t} + g_t + \frac{\psi^P}{2} (\pi_t^P - 1)^2 + \frac{\psi^w}{2} \left(\frac{\pi_t^w}{(\pi_t^P)^\chi} - 1 \right)^2 = y_t. \quad (41)$$

Table 6 shows simulation-based moments for the labor tax rate, price inflation, and wage inflation when wages are fixed on average for three quarters and prices are either flexible or also fixed on average for three quarters. The other parameter values are held fixed at their values in Section 4. In these experiments, the economy is driven by both technology and government spending shocks.

The first three columns of Table 6, the case of no indexation of wages to prices, reproduce the results presented in the fourth and fifth panels of Table 3. The second three columns consider a partial indexation scheme, parameterized by $\chi = 0.5$. Partial indexation reduces somewhat the costs associated with wage variability and thus partially re-instates the Ramsey planner's use of state-contingent inflation as a fiscal instrument. Under either flexible or sticky prices, inflation volatility rises compared to the no-indexation benchmark. The last three columns of Table 6 consider the full-indexation scheme, $\chi = 1$. Under full indexation, it is only deviations of the wage inflation rate from the price inflation rate, rather than non-zero wage inflation itself, that entail resource costs. The rigidity in the level of nominal wages is thus virtually removed. Comparisons of the upper right panel of Table 6 with the second panel of Table 3 and of the lower right panel of Table 6 with the third panel of Table 3 show that full indexation recovers the flexible-wage use of state-contingent inflation as a fiscal tool.

No indexation ($\chi = 0$)		
Quarters wage stickiness	Auto corr of b_t	SD(π_t^p)
1	0.0356	2.5713
2	0.8877	1.0829
3	0.9612	0.7113
4	0.9808	0.7081
5	0.9863	0.6972
6	0.9819	0.6933
Full indexation ($\chi = 1$)		
Quarters wage stickiness	Auto corr of b_t	SD(π_t^p)
1	0.0352	2.5713
2	0.0986	2.7118
3	0.0841	2.3477
4	0.0809	2.2131
5	0.0710	2.4066
6	0.0702	2.8111

Table 7: Serial correlation of real government liabilities and standard deviation of price inflation volatility as functions of the degree of wage stickiness. Upper panel assumes no indexation of nominal wages to nominal price inflation. Lower panel assumes full indexation of nominal wages to contemporaneous nominal price inflation. π^p reported in annualized percentage points.

6 Government Debt Dynamics

Another way of viewing our results is by considering the dynamics of real government liabilities as the degree of wage-stickiness varies. As Schmitt-Grohe and Uribe (2004b) and Siu (2004) have shown, the presence of nominal rigidities in prices imparts near-random-walk behavior in real government liabilities. This occurs because when the Ramsey planner refrains from using inflation to make nominally-risk-free debt state-contingent in real terms, it is as if the government is restricted to issue only real risk-free debt. When the government is restricted to issue only real risk-free debt, Aiyagari et al (2002) have shown that government liabilities become a near-random-walk regardless of the serial correlation of the shocks hitting the economy. In this section, we explore whether sticky wages have a similar effect on real government liabilities.

To investigate this issue, we again analyze simulations of our model, this time focusing on the dynamics of government debt. In these experiments, we assume fully-flexible prices ($\psi^p = 0$), and we assume the shocks to technology and government spending have zero persistence — that is, we set $\rho_z = 0$ and $\rho_g = 0$. We assume the shocks have no persistence because it is known in the Ramsey literature that with fully-state-contingent government debt, real debt payments under the Ramsey plan have zero persistence.¹² If sticky wages impart a random walk to debt in our model, the result will thus be most stark with uncorrelated driving shocks.

¹²See, for example, Aiyagari et al (2002) and Chari and Kehoe (1999) for more discussion.

Table 7 shows the persistence of real realized government debt payments (i.e., after state-contingent inflation has occurred) when wage-stickiness ranges from one quarter (fully-flexible wages, given our quarterly model) to six quarters. As the top panel shows, when nominal wages are not indexed to nominal prices, the volatility of price inflation declines and the serial correlation of real government debt rises as nominal wages become increasingly rigid. With just two-quarter stickiness in nominal wages, government debt becomes highly serially-correlated, and the effect becomes more pronounced as the degree of stickiness increases. We note that the results in Table 7 do not quantitatively match those in Table 3 because the shocks here are assumed to have no persistence.

Finally, we showed in the previous section that when wages are indexed to prices, price inflation volatility is quite close to that under flexible wages. The lower panel of Table 7 shows that under full indexation of nominal wages to nominal prices, government debt continues to exhibit its correlation under the flexible wages, in this case zero. This is another way of seeing that in our model full indexation of nominal wages to nominal prices essentially removes the sticky-wage distortion.

7 Conclusion

In this paper we studied the effects of sticky nominal wages on the incentive to use surprise nominal debt deflation to finance shocks to the government budget in a well-understood Ramsey model of fiscal and monetary policy. Our central finding is that, due to a condition relating real wage growth, nominal wage inflation, and nominal price inflation that is non-trivial in the presence of sticky wages, price stability characterizes optimal policy in the face of fiscal shocks even if nominal prices are flexible. The resource cost of nominal wage adjustment essentially forces nominal price inflation to adjust in order to bring about the efficient path of real wages rather than in order to finance shocks to the government budget. Loosely speaking, with sticky nominal wages, getting the path of real wages right is more important than financing innovations in the government budget via state-contingent inflation.

We also uncover the ability of the nominal interest rate to indirectly tax monopolistic labor suppliers' rents. This result hinges on the absence of an instrument that directly taxes labor market rents. We cannot think of a real-world instrument that accomplishes this. If this missing-tax problem describes well the real world, then using the nominal interest rate for this purpose may well be sound policy advice. Rather than drawing such a conclusion, we only wish to remind that in a Ramsey framework, the set of instruments assumed available is very important for the policy implications. This point is well-understood in the Ramsey literature, but the necessary instruments to include in the complete set may not even be obvious a priori. We would add this to the list of "caveats" Fernández-Villaverde (2005) suggests keeping in mind when thinking about the policy

implications of Ramsey models. Interestingly, this channel of taxing labor rents is available in our model only if monopoly power exists in product markets as well — that is, positive producer profits give the Ramsey planner the leverage to tax labor rents. It would be interesting to explore further why this is Ramsey-optimal.

Our study makes a contribution to the broader investigation of the consequences of labor market failures for optimal macroeconomic policy. Sticky nominal wages are but one labor market friction that one may be interested in considering. Also of interest may be studying the effects of labor market search and matching frictions or efficiency wages on optimal policy. More generally, the consequences of labor market frictions for the conduct of policy is an issue that has been of interest for a long time but seems to not have received proportionate attention in the DSGE optimal policy literature. Recent developments in DSGE model-building incorporating labor market frictions seem to warrant renewed attention to this issue.

A Calvo-to-Rotemberg Mapping

Here we show our mapping between the probability q of receiving a signal to change prices in the Calvo formulation of price-stickiness and the parameter ψ in the Rotemberg quadratic-cost formulation of price stickiness. We proceed by considering a slightly simpler representation of the pricing problem firms face in our full model. Specifically, we consider the problem of a firm minimizing the discounted costs of setting a price different from some flexible-price target P^* as well as different from its most-recently set price. For simplicity, we take the process for $\{P_t^*\}$ as given.

Under Rotemberg-pricing, the simplified problem we consider is thus

$$\min_{P_{it+s}} E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{2} (P_{it+s} - P_{it+s}^*)^2 + \frac{\psi}{2} (P_{it+s} - P_{it+s-1})^2 \right]. \quad (42)$$

The first-order condition with respect to P_{it} is

$$P_{it} - P_{it}^* + \psi(P_{it} - P_{it-1}) - \beta\psi E_t(P_{it+1} - P_{it}) = 0, \quad (43)$$

or

$$\beta E_t \Delta P_{it+1} + \frac{1}{\psi} (P_{it}^* - P_{it}) = \Delta P_{it}, \quad (44)$$

where $\Delta P_{it} \equiv P_{it} - P_{it-1}$. Assuming symmetry across all price-setters i , we have

$$\Delta P_t = \beta E_t \Delta P_{t+1} + \frac{1}{\psi} (P_t^* - P_t). \quad (45)$$

We now derive an analogous expression under the Calvo formulation of price stickiness. First note that with the usual assumption that price-change signals in the Calvo formulation are Poisson, $1/q$ is the time interval between successive signals. The fraction q of firms is allowed to set a new price in a period, and the fraction $1 - q$ must keep its old price. We again consider the simplified problem of a firm minimizing the loss from not being at the flexible-price optimum P^* . The relevant loss function from expecting to deviate from the flex-price optimum for $1/(1 - q)$ periods is

$$E_t \sum_{s=0}^{\infty} \beta^s (1 - q)^s (P_{it} - P_{it+s}^*)^2, \quad (46)$$

where we again suppose the loss to be quadratic in the deviations, and P_{it} is the price set in t and still in effect in $t + s$ with probability $(1 - q)^s$. Minimizing this loss with respect to P_{it} ,

$$\sum_{s=0}^{\infty} \beta^s (1 - q)^s (P_{it} - E_t P_{it+s}^*) = 0. \quad (47)$$

Because $\sum_{s=0}^{\infty} \beta^s (1 - q)^s = 1/[1 - \beta(1 - q)]$, the above becomes

$$\begin{aligned} P_{it} &= [1 - \beta(1 - q)] \sum_{s=0}^{\infty} \beta^s (1 - q)^s E_t P_{it+s}^* \\ &= [1 - \beta(1 - q)] P_{it}^* + [1 - \beta(1 - q)] \beta(1 - q) \sum_{s=0}^{\infty} \beta^s (1 - q)^s E_t P_{it+1+s}^*. \end{aligned} \quad (48)$$

Using the first line in the above, the last term in the second line equals $\beta(1 - q)E_t P_{it+1}$. Thus the second line can be written more compactly as

$$P_{it} = [1 - \beta(1 - q)]P_{it}^* + \beta(1 - q)E_t P_{it+1}. \quad (49)$$

Again assuming symmetry across all price-setters, $P_{it} = P_t$ and $P_{it}^* = P_t^*$. The aggregate price level is the weighted average of those who get to change price and those who do not. The latter have, on average, a price equal to the price level in $t - 1$ (because price-change signals are independent over time). The average price level is therefore

$$P_t = qP_{it} + (1 - q)P_{t-1}, \quad (50)$$

so

$$P_{it} = \frac{1}{q}P_t - \frac{1 - q}{q}P_{t-1}. \quad (51)$$

Use this to substitute for P_{it} and P_{it+1} in (49), to get

$$\left(\frac{1}{q}P_t - \frac{1 - q}{q}P_{t-1}\right) = [1 - \beta(1 - q)]P_t^* + \beta(1 - q)E_t \left(\frac{1}{q}P_{it+1} - \frac{1 - q}{q}P_t\right), \quad (52)$$

or

$$\Delta P_t = \beta E_t \Delta P_{t+1} + \frac{q}{1 - q}[1 - \beta(1 - q)](P_t^* - P_t). \quad (53)$$

Comparing (53) and (45), we obtain the mapping from q to ψ ,

$$\psi = \frac{1 - q}{q} \frac{1}{1 - \beta(1 - q)}. \quad (54)$$

Thus, for example, in our quarterly model with $\beta = 0.99$, if we wish to impose an average duration of sticky prices of three quarters, we set $q = 1/3$, which means that on average 1/3 of firms set price in a given period (and hence the average duration of a price is 3 periods). The corresponding Rotemberg parameter is thus $\psi = 5.88$. We use this mapping to calibrate both price stickiness and wage stickiness.

B Wage Phillips Curve

We derive the wage Phillips curve allowing for indexation of nominal wages to nominal price inflation. Consumer i 's problem is

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, n_{it}) \quad (55)$$

subject to the flow budget constraint

$$M_t - M_{t-1} + B_t - R_{t-1}B_{t-1} = (1 - \tau_{t-1}^n)W_{it-1}n_{it-1} + P_{t-1}pr_{t-1} - P_{t-1}c_{1t-1} - P_{t-1}c_{2t-1} - \frac{\psi^w}{2} \left(\frac{W_{it-1}}{W_{it-2}(\pi_{t-1}^p)^{\chi}} - 1 \right)^2 P_{t-1}, \quad (56)$$

the cash-in-advance constraint

$$P_t c_{1t} \leq M_t, \quad (57)$$

and taking as given the demand function for type- i labor,

$$n_{it} = \left[\frac{W_t}{W_{it}} \right]^{\frac{\varepsilon^w}{\varepsilon^w - 1}} n_t. \quad (58)$$

Substitute for n_{it} in the utility function and the budget constraint using the demand function, and let ϕ_t/P_{t-1} be the Lagrange multiplier on the flow budget constraint (the cash-in-advance constraint is not directly relevant for the derivation of the wage Phillips curve). The first-order condition with respect to W_{it} is

$$\begin{aligned} & -\frac{\varepsilon^w}{\varepsilon^w - 1} u_{3t} n_t \left[\frac{W_t}{W_{it}} \right]^{\frac{\varepsilon^w}{\varepsilon^w - 1}} \frac{1}{W_{it}} - \frac{\beta}{\varepsilon^w - 1} (1 - \tau_t^n) \left[\frac{W_t}{W_{it}} \right]^{\frac{\varepsilon^w}{\varepsilon^w - 1}} \frac{n_t}{P_t} E_t \phi_{t+1} \\ & - \beta \psi^w \left(\frac{W_{it}}{W_{it-1} (\pi_t^p)^\chi} - 1 \right) \frac{1}{W_{it-1} (\pi_t^p)^\chi} E_t \phi_{t+1} - \beta \psi^w E_t \left[\beta \phi_{t+2} \left(\frac{W_{it+1}}{W_{it} (\pi_{t+1}^p)^\chi} - 1 \right) \left(\frac{-W_{it+1}}{W_{it}^2 (\pi_{t+1}^p)^\chi} \right) \right] = 0. \end{aligned} \quad (59)$$

In this expression, impose symmetric equilibrium, so that $W_{it} = W_t$. Define $\pi_t^w \equiv W_t/W_{t-1}$, and also use the consumer first-order-condition with respect to c_{2t} , $u_{2t} = \beta E_t \phi_{t+1}$, to express the first-order condition with respect to W_{it} as

$$\begin{aligned} & -\frac{\varepsilon^w}{\varepsilon^w - 1} u_{3t} n_t \frac{1}{W_t} - \frac{1}{\varepsilon^w - 1} (1 - \tau_t^n) u_{2t} n_t \frac{1}{P_t} \\ & - \psi^w u_{2t} \left(\frac{\pi_t^w}{(\pi_t^p)^\chi} - 1 \right) \frac{1}{W_{t-1} (\pi_t^p)^\chi} + \beta \psi^w E_t \left[u_{2t+1} \left(\frac{\pi_{t+1}^w}{(\pi_{t+1}^p)^\chi} - 1 \right) \frac{\pi_{t+1}^w}{(\pi_{t+1}^p)^\chi} \frac{1}{W_t} \right] = 0. \end{aligned} \quad (60)$$

Finally, multiply through by $(\varepsilon^w - 1)W_t$ to obtain

$$\begin{aligned} & -[\varepsilon^w u_{3t} + (1 - \tau_t^n) u_{2t} m c_t m p n_t] n_t \\ & - \psi^w (\varepsilon^w - 1) u_{2t} \left(\frac{\pi_t^w}{(\pi_t^p)^\chi} - 1 \right) \frac{\pi_t^w}{(\pi_t^p)^\chi} + \beta \psi^w (\varepsilon^w - 1) E_t \left[u_{2t+1} \left(\frac{\pi_{t+1}^w}{(\pi_{t+1}^p)^\chi} - 1 \right) \frac{\pi_{t+1}^w}{(\pi_{t+1}^p)^\chi} \right] = 0, \end{aligned} \quad (61)$$

in which we use the definition of the real wage $w_t \equiv W_t/P_t$ followed by the identity $w_t = m c_t m p n_t$. Setting $\chi = 0$ means nominal wages are not indexed to price inflation; setting $\chi = 1$ means nominal wages are fully indexed to price inflation; and $\chi \in (0, 1)$ means nominal wages are partially indexed to price inflation.

C Ramsey Problem

The Ramsey government chooses sequences $\{c_{1t}, c_{2t}, n_t, m c_t, \tau_t^n, \pi_t^p, \pi_t^w, b_t\}$ to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, n_t) \quad (62)$$

subject to the resource constraint

$$e^{z_t} n_t - c_{1t} - c_{2t} - g_t - \frac{\psi^p}{2} (\pi_t^p - 1)^2 - \frac{\psi^w}{2} \left(\frac{\pi_t^w}{(\pi_t^p)^\chi} - 1 \right)^2 = 0, \quad (63)$$

the wage Phillips Curve

$$x^w(c_{1t+1}, c_{2t+1}, n_{t+1}, c_{1t}, c_{2t}, n_t, mc_t, mc_{t-1}, \pi_{t+1}^w, \pi_t^w, \pi_t^p, \tau_t^n) = 0, \quad (64)$$

the price Phillips Curve

$$x^p(c_{1t+1}, c_{2t+1}, n_{t+1}, c_{1t}, c_{2t}, n_t, mc_t, \pi_{t+1}^p, \pi_t^p) = 0, \quad (65)$$

the household's first-order condition on bond accumulation (i.e., the Fisher relation)

$$F(c_{1t+1}, c_{2t+1}, n_{t+1}, c_{1t}, c_{2t}, n_t, \pi_{t+1}^p) = 0, \quad (66)$$

the time- $t + 1$ government budget constraint

$$H(c_{1t+1}, c_{2t+1}, n_{t+1}, c_{1t}, c_{2t}, n_t, mc_t, b_{t+1}, b_t, \tau_t^n, \pi_{t+1}^p) = 0, \quad (67)$$

and the equilibrium wage restriction

$$\frac{mc_t mpn_t}{mc_{t-1} mpn_{t-1}} - \frac{\pi_t^w}{\pi_t^p} = 0. \quad (68)$$

The function H is defined as

$$H_t \equiv c_{1t+1} \pi_{t+1}^p + b_{t+1} \pi_{t+1}^p + \tau_t^n mc_t mpn_t n_t - c_{1t} - \frac{u_{1t}}{u_{2t}} b_t - g_t, \quad (69)$$

and the function F is defined as

$$F_t \equiv 1 - \beta \frac{u_{1t}}{u_{2t}} E_t \left[\frac{u_{1t+1}}{u_{2t+1}} \frac{1}{\pi_{t+1}} \right]. \quad (70)$$

In the government budget constraint, the Fisher relation, and the law of motion for the real wage, we have substituted out R_t and w_t using the consumer first-order conditions and the equilibrium relationship $w_t = mc_t mpn_t$.

D Sub-optimality of Friedman Rule with $\varepsilon^p > 1$

In this section, we show that product market power makes the Friedman Rule, $R_t = 1$, sub-optimal, but that labor market power by itself ($\varepsilon^w > 1$ and $\varepsilon^p = 1$) does not by itself cause a deviation from the Friedman Rule. The proof builds on Siu (2004). Throughout, suppose both labor and goods

markets feature flexible prices, so $\psi^w = 0$ and $\psi^p = 0$. In this case, the equilibrium conditions are easily captured by a single present-value implementability condition,

$$\sum_{t=0}^{\infty} \beta^t \left[u_{1t}c_{1t} + u_{2t}c_{2t} + \varepsilon^w u_{3t}n_t - u_{2t} \left(1 - \frac{1}{\varepsilon^p} \right) z_t n_t \right] = \phi_0 \left(\frac{M_{-1} + R_{-1}B_{-1}}{P_0} \right), \quad (71)$$

where ϕ_0 is the time-zero multiplier on the budget constraint from the consumer's problem, along with the resource constraint

$$c_{1t} + c_{2t} + g_t = z_t n_t. \quad (72)$$

The implementability condition, which encodes the first-order conditions of the household along with the government budget constraint, is a slight extension of that in the flexible-price model of Siu (2004) because it allows for monopoly power in labor markets ($\varepsilon^w \geq 1$).¹³ The Ramsey problem in this case is thus to maximize the representative consumer's lifetime utility subject to the resource constraint and the implementability constraint. For the rest of this section, assume that the utility function is additively-separable in c_t (which is an aggregate of c_{1t} and c_{2t}) and n_t , which is the case in our quantitative work.

To establish that deviations from the Friedman Rule are not caused by labor market power, take the Ramsey first-order-conditions with respect to c_{1t} and c_{2t} for $t > 0$. Combining them gives

$$u_{1t} + \xi \left[u_{11t}c_{1t} + u_{1t} + u_{21t}c_{2t} - \left(1 - \frac{1}{\varepsilon^p} \right) z_t n_t u_{21t} \right] = u_{2t} + \xi \left[u_{21t}c_{1t} + u_{2t} + u_{22t}c_{2t} - \left(1 - \frac{1}{\varepsilon^p} \right) z_t n_t u_{22t} \right], \quad (73)$$

where ξ is the Lagrange multiplier on the implementability constraint. Suppose that $\varepsilon^p = 1$, so that goods markets are perfectly-competitive. In this case, Chari, Christiano, and Kehoe (1991) have shown that the Friedman Rule is optimal for the class of preferences we use.

Instead, assume $\varepsilon^p > 1$. Suppose that the Friedman Rule were in this case optimal. That implies that $u_{1t} = u_{2t}$, as shown in (24). Impose $u_{1t} = u_{2t}$ in the previous expression. With $\varepsilon^p > 1$, this equation is violated. This is a contradiction, so the Friedman Rule must not be optimal.

Notice that this proof does not depend on the degree of labor market power, measured by ε^w , because preferences are assumed additively-separable between consumption and leisure. Thus, if $\varepsilon^w > 1$ but $\varepsilon^p = 1$, the Friedman Rule is optimal. Labor market power alone does not cause a deviation from the Friedman Rule. Formally, because we have just shown that $R_t = 1 \forall t$ when $\varepsilon^p = 1$, clearly $\partial R_t / \partial \varepsilon^w = 0$ when $\varepsilon^p = 1$, which our numerical results confirm.

¹³In the text, we did not employ the primal approach to the Ramsey problem because the intuitive clarity it delivers is diminished when prices and wages are sticky. Here, because we assume both flexible prices and wages, it is useful to use the primal formulation. For brevity, we omit the derivation of the implementability condition, which is by now well-known in the Ramsey literature. The derivation can be found in, among others, Lucas and Stokey (1983), Chari, Christiano, and Kehoe (1991), and Ljungqvist and Sargent (2004).

E Dynamic Results

Tables 8, 9, and 10 reproduce the dynamic results presented in Tables 3, 4, and 5 and also present the dynamics of output and hours in the models studied.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x, y)	Corr(x, g)	Corr(x, z)
<u>Perfect Competition</u>						
τ^n	0.2218	0.0044	0.8420	-0.1275	0.9254	-0.2910
π^P	-4.0113	6.9631	0.0776	-0.1022	-0.1632	-0.1407
π^w	-4.0113	6.5833	0.0957	-0.0817	-0.1725	-0.1154
R	0	—	—	—	—	—
w	0.9999	0.0118	0.8051	0.9681	0.0074	1
y	0.2995	0.0037	0.8193	1	0.2062	0.9681
n	0.2995	0.0005	0.4919	0.2936	0.8017	0.0469
<u>Flexible prices, flexible wages</u>						
τ^n	0.2771	0.0072	0.7382	-0.4648	0.8169	-0.4802
π^P	-2.9160	4.6032	-0.0091	-0.1631	0.3279	-0.1053
π^w	-2.9163	4.2308	0.0009	-0.1081	0.3658	-0.0486
R	1.1262	0.3780	-0.2910	0.2515	0.0299	0.2376
w	0.9090	0.0108	0.8051	0.9721	0.0074	1
y	0.2627	0.0019	0.0347	1	0.6487	0.7527
n	0.2627	0.0007	0.0754	0.4106	0.9428	-0.2902
<u>Sticky prices, flexible wages</u>						
τ^n	0.2759	0.0240	0.6336	-0.8702	0.5811	-0.3957
π^P	-0.1144	0.4986	-0.1055	-0.0119	-0.2161	0.2198
π^w	-0.1134	1.6712	-0.3770	-0.0936	-0.01565	0.1077
R	4.0653	0.9008	-0.1527	0.2505	0.1058	-0.0818
w	0.9090	0.0166	0.1720	-0.0322	0.1856	0.4844
y	0.2626	0.0084	0.4816	1	-0.3226	0.7545
n	0.2626	0.0032	0.3701	0.9473	-0.4287	0.5052
<u>Flexible prices, sticky wages</u>						
τ^n	0.2759	0.0203	0.5961	-0.8758	0.6469	-0.5344
π^P	-0.1131	0.9113	-0.0189	-0.2481	-0.0438	-0.1144
π^w	-0.1133	0.5218	0.1966	0.0159	-0.0928	0.1947
R	4.0660	1.1331	-0.0480	0.0633	0.0132	-0.0535
w	0.9090	0.0107	0.8051	0.8375	0.0074	1
y	0.2626	0.0084	0.6728	1	-0.2971	0.8375
n	0.2626	0.0030	0.6062	0.9597	-0.4176	0.6506
<u>Sticky prices, sticky wages</u>						
τ^n	0.2759	0.0221	0.5966	-0.7613	0.7041	-0.4590
π^P	-0.0584	0.4832	0.0705	-0.1543	-0.1854	-0.1551
π^w	-0.0594	0.5294	0.1784	0.4667	-0.0721	0.4614
R	4.1094	0.9411	-0.0766	0.1390	-0.0433	0.1035
w	0.9090	0.0129	0.8916	0.7654	0.0713	0.9380
y	0.2626	0.0084	0.6380	1	-0.3381	0.8043
n	0.2626	0.0031	0.5657	0.9540	-0.4643	0.5896

Table 8: Simulation-based moments with g_t and z_t as the driving processes. π^P , π^w , and R reported in annualized percentage points. Markup parameters are $\varepsilon^w = 1.05$ and $\varepsilon^P = 1.1$. For either sticky wages or sticky prices, ψ^w and/or ψ^P are set so that the respective nominal rigidity lasts three quarters on average.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x, y)	Corr(x, g)	Corr(x, z)
Perfect Competition						
τ^n	0.2218	0.0042	0.8515	0.7494	0.9788	—
π^P	-4.0107	5.6969	0.1431	0.0081	-0.1992	—
π^w	-4.0107	5.6969	0.1431	0.0081	-0.1992	—
R	0	—	—	—	—	—
w	1	—	—	—	—	—
y	0.2995	0.0009	0.5856	1	0.8609	—
n	0.2995	0.0004	0.5856	1	0.8609	—
Flexible prices, flexible wages						
τ^n	0.2771	0.0061	0.7627	0.1604	0.9586	—
π^P	-2.9149	2.8764	-0.0116	-0.2468	0.4105	—
π^w	-2.9149	2.8764	-0.0116	-0.2468	0.4105	—
R	1.1279	0.2056	-0.2725	0.1921	0.0424	—
w	0.9090	—	—	—	—	—
y	0.2627	0.0013	0.0834	1	0.9880	—
n	0.2627	0.0006	0.0834	1	0.9880	—
Sticky prices, flexible wages						
τ^n	0.2759	0.0188	0.3285	-0.9554	0.7442	—
π^P	-0.1128	0.7809	-0.1362	-0.1760	-0.2912	—
π^w	-0.1122	2.2164	-0.4172	-0.3245	-0.0180	—
R	4.0572	0.6653	-0.1570	0.3938	0.1428	—
w	0.9091	0.0111	-0.2449	-0.6991	0.2753	—
y	0.2626	0.0045	0.3838	1	-0.6179	—
n	0.2626	0.0022	0.3838	1	-0.6179	—
Flexible prices, sticky wages						
τ^n	0.2758	0.0153	0.6444	-0.9392	0.8623	—
π^P	-0.1123	0.3720	0.1209	-0.2036	-0.1323	—
π^w	-0.1123	0.3720	0.1209	-0.2036	-0.1323	—
R	4.0551	0.7886	-0.0772	0.1743	0.0190	—
w	0.9090	—	—	—	—	—
y	0.2626	0.0037	0.6020	1	-0.6864	—
n	0.2626	0.0019	0.6020	1	-0.6864	—
Sticky prices, sticky wages						
τ^n	0.2759	0.0132	0.6394	-0.8664	0.8565	—
π^P	-0.0582	0.2131	0.3398	0.1546	-0.4202	—
π^w	-0.0580	0.4362	-0.0613	0.0388	-0.0898	—
R	4.1094	0.7441	-0.0630	0.1218	-0.0557	—
w	0.9091	0.0021	0.1040	-0.5000	0.3866	—
y	0.2626	0.0042	0.5708	1	-0.6799	—
n	0.2626	0.0021	0.5708	1	-0.6799	—

Table 9: Simulation-based moments with g_t as the only driving process. π^P , π^w , and R reported in annualized percentage points. Markup parameters are $\varepsilon^w = 1.05$ and $\varepsilon^P = 1.1$. For either sticky wages or sticky prices, ψ^w and/or ψ^P are set so that the respective nominal rigidity lasts three quarters on average.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x, y)	Corr(x, g)	Corr(x, z)
Perfect Competition						
τ^n	0.2218	0.0014	0.7692	-0.9515	—	-0.9227
π^P	-4.0001	4.0031	-0.0691	-0.1788	—	-0.2385
π^w	-4.0001	3.2989	-0.0625	-0.1630	—	-0.2227
R	0	—	—	—	—	—
w	0.9999	0.0119	0.8051	0.9957	—	1
y	0.2995	0.0036	0.8327	1	—	0.9957
n	0.2995	0.0002	-0.1473	0.1994	—	0.1078
Flexible prices, flexible wages						
τ^n	0.2771	0.0037	0.6769	-0.9732	—	-0.9361
π^P	-2.9192	2.1684	-0.0177	-0.2120	—	-0.1758
π^w	-2.9195	1.4692	0.0299	-0.1477	—	-0.1079
R	1.1253	0.4641	-0.3171	0.3097	—	0.3186
w	0.9090	0.0108	0.8051	0.9906	—	1
y	0.2627	0.0015	0.0061	1	—	0.9979
n	0.2627	0.0002	-0.0345	-0.9546	—	-0.9719
Sticky prices, flexible wages						
τ^n	0.2759	0.0149	0.1554	-0.9004	—	-0.6424
π^P	-0.1132	0.7033	-0.0743	0.1018	—	0.3301
π^w	-0.1127	1.4991	-0.2979	0.0995	—	0.1901
R	4.0585	0.6079	-0.1563	0.1669	—	-0.1231
w	0.9090	0.0125	0.4932	0.3363	—	0.6451
y	0.2626	0.0071	0.5206	1	—	0.8915
n	0.2626	0.0022	0.3558	0.9511	—	0.7082
Flexible prices, sticky wages						
τ^n	0.2759	0.0133	0.5353	-0.9578	—	-0.8192
π^P	-0.1126	0.9479	-0.0390	-0.2561	—	-0.1199
π^w	-0.1129	0.3657	0.2716	0.1324	—	0.2822
R	4.0616	0.8133	-0.0281	0.0124	—	-0.0766
w	0.9090	0.0108	0.8051	0.9344	—	1
y	0.2626	0.0076	0.6922	1	—	0.9344
n	0.2626	0.0024	0.6114	0.9727	—	0.8263
Sticky prices, sticky wages						
τ^n	0.2759	0.0091	0.5077	-0.9023	—	-0.8194
π^P	-0.0579	0.4332	0.0019	-0.2220	—	-0.1699
π^w	-0.0590	0.3011	0.6860	0.8828	—	0.8152
R	4.1067	0.5755	-0.1117	0.1741	—	0.1667
w	0.9090	0.0128	0.9136	0.8958	—	0.9512
y	0.2626	0.0072	0.6636	1	—	0.9336
n	0.2626	0.0022	0.5641	0.9685	—	0.9365

Table 10: Simulation-based moments with z_t as the only driving process. π^P , π^w , and R reported in annualized percentage points. Markup parameters are $\varepsilon^w = 1.05$ and $\varepsilon^P = 1.1$. For either sticky wages or sticky prices, ψ^w and/or ψ^P are set so that the respective nominal rigidity lasts three quarters on average.

References

- AIYAGARI, S. RAO, ALBERT MARCET, THOMAS J. SARGENT, AND JUHA SEPPALA. 2002. "Optimal Taxation without State-Contingent Debt." *Journal of Political Economy*, Vol. 110, pp. 1220-1254.
- ALBANESI, STEFANIA. 2004. "Comment on Benigno and Woodford: Optimal Monetary and Fiscal Policy." NBER Macroeconomics Annual 2003.
- CHARI, V.V., LAWRENCE CHRISTIANO, AND PATRICK KEHOE. 1991. "Optimal Monetary and Fiscal Policy: Some Recent Results." *Journal of Money, Credit, and Banking*, Vol. 23, pp. 519-539.
- CHRISTIANO, LAWRENCE J., MARTIN EICHENBAUM, AND CHARLES L. EVANS. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy*, Vol. 113, pp. 1-45.
- ERCEG, CHRISTOPHER J., DALE W. HENDERSON, AND ANDREW LEVIN. 2000. "Optimal Monetary Policy with Staggered Wage and Price Contracts." *Journal of Monetary Economics*, Vol. 46, pp. 281-313.
- FERNÁNDEZ-VILLAVERDE, JESÚS. 2005. "Comment on Schmitt-Grohe and Uribe: Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model." NBER Macroeconomics Annual 2005.
- KHAN, AUBHIK, ROBERT G. KING AND ALEXANDER L. WOLMAN. 2003. "Optimal Monetary Policy." *Review of Economic Studies*, Vol. 70, pp. 825-860.
- KOCHERLAKOTA, NARAYANA R. 2005. "Optimal Monetary Policy: What We Know and What We Don't Know." *Federal Reserve Bank of Minneapolis Quarterly Review*, Vol. 29, pp. 10-19.
- LEVIN, ANDREW T., ALEXEI ONATSKI, JOHN C. WILLIAMS, AND NOAH WILLIAMS. 2005. "Monetary Policy Under Uncertainty in Micro-Founded Macroeconomic Models." NBER Macroeconomics Annual 2005.
- LUCAS, ROBERT E. JR. AND NANCY L. STOKEY. 1983. "Optimal Fiscal and Monetary Policy in an Economy Without Capital." *Journal of Monetary Economics*, Vol. 12, pp. 55-93.
- LJUNGQVIST, LARS AND THOMAS J. SARGENT. 2004. *Recursive Macroeconomic Theory*. MIT Press.
- SCHMITT-GROHE, STEPHANIE AND MARTIN URIBE. 2004a. "Optimal Fiscal and Monetary Policy Under Imperfect Competition." *Journal of Macroeconomics*, Vol. 26, pp. 183-209.
- SCHMITT-GROHE, STEPHANIE AND MARTIN URIBE. 2004b. "Optimal Fiscal and Monetary Policy Under Sticky Prices." *Journal of Economic Theory*, Vol. 114, pp. 198-230.
- SCHMITT-GROHE, STEPHANIE AND MARTIN URIBE. 2004c. "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function." *Journal of Eco-*

conomic Dynamics and Control, Vol. 28, pp. 755-775.

SCHMITT-GROHE, STEPHANIE AND MARTIN URIBE. 2005. “Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model.” NBER Macroeconomics Annual 2005.

SIU, HENRY E. 2004. “Optimal Fiscal and Monetary Policy with Sticky Prices.” *Journal of Monetary Economics*, Vol. 51, pp. 576-607.

SMETS, FRANK AND RAF WOUTERS. 2005. “Comparing Shocks and Frictions and U.S. and Euro Area Business Cycles: A Bayesian DSGE Approach.” *Journal of Applied Econometrics*, Vol. 20, pp. 161-183.