Department of Economics | Boston College

Economics 202 (Section 05)
Macroeconomic Theory
Midterm Exam – Suggested Solutions
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Fall 2013

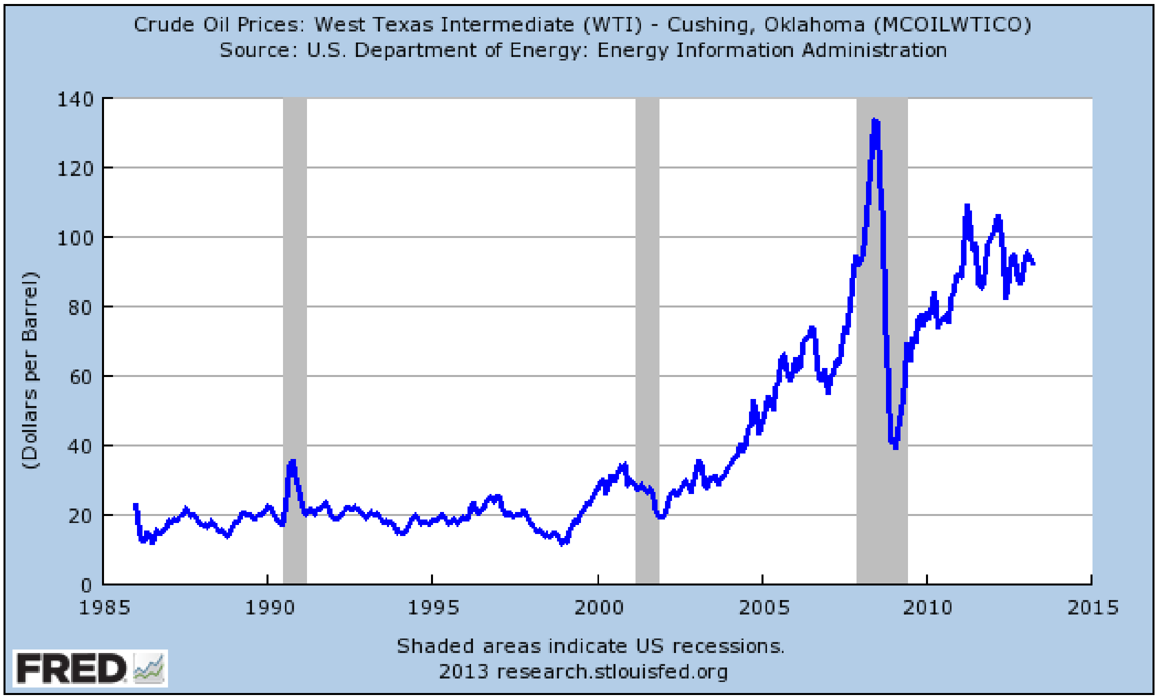
NAME:

The Exam has a total of four (4) problems and pages numbered (including this cover page) one (1) through sixteen (16). Each problem's total number of points is shown below. Your solutions should consist of some appropriate combination of mathematical analysis, graphical analysis, logical analysis, and economic intuition, but in no case should solutions be exceptionally long. Your solutions should get straight to the point – **solutions with irrelevant discussions and derivations will be penalized.** You are to answer all questions in the spaces provided.

You may use one page (double-sided) of hand-written notes. You may **not** use a calculator.

Problem 1	**/ 25**
Problem 2	**/ 25**
Problem 3	**/ 25**
Problem 4	**/ 25**
TOTAL	**/ 100**

Problem 1: Oil Markets (25 points). Displayed below is the price of one barrel of (WTI) oil over the past few decades. A barrel of oil can be thought of as an **asset** because it is **storable.**



For the sake of simplicity, suppose that this is the **only** asset via which the infinitely-lived representative consumer can accumulate wealth.

Denote by $\boldsymbol{wti_{t-1}}$ the **quantity** of barrels owned by the consumer at the start of period t; denote by S_t^{wti} the nominal price during period t of one barrel of oil; as usual, let $0 < \beta < 1$ stand for the consumer's one-period-ahead subjective discount factor, P_t stands for the period-t nominal price of consumption, and Y_t stands for period-t nominal income. Finally, in each period t, the utility function of the consumer is

$$u(c_t, d \cdot wti_t)$$

in which $0 < d < 1$ is a **constant number** that stands for how much "utility" the consumers enjoys from wti_{t-1} (you can think of this as heat for the oven or for the house, etc.). (**Note: it is $\boldsymbol{wti_t}$ that appears in the period-t utility function, NOT $\boldsymbol{wti_{t-1}}$.)**

Thus, the consumer's lifetime utility function starting from the beginning of period t is

$$u(c_t, d \cdot wti_t) + \beta u(c_{t+1}, d \cdot wti_{t+1}) + \beta^2 u(c_{t+2}, d \cdot wti_{t+2}) + \beta^3 u(c_{t+3}, d \cdot wti_{t+3}) + \ldots$$

(OVER)

Problem 1 continued

a. **(5 points)** Using the notation above, construct the period-t budget constraint of the consumer, **and provide a ONE-SENTENCE economic comparison of this budget constraint to that in our study of stock-market pricing.**

Solution: The period-t budget constraint is

$$P_t c_t + S_t^{wti} wti_t = S_t^{wti} wti_{t-1} + Y_t.$$

The economic difference compared to our study of how stock market holdings affect the consumer's budget is that there is **no "dividend" payment** of holdings of *wti* that appears **directly** in the budget constraint.

b. **(6 points)** Based on your budget constraint in part a above and the lifetime utility function, construct the Lagrangian for the consumer's optimization **AND** compute the FOCs for c_t c_{t+1}, and wti_t.

Solution: The Lagrangian is

$$\begin{aligned} & u(c_t, d \cdot wti_t) + \beta u(c_{t+1}, d \cdot wti_{t+1}) + \beta^2 u(c_{t+2}, d \cdot wti_{t+2}) + \ldots \\ & + \lambda_t \left[Y_t + S_t^{wti} wti_{t-1} - P_t c_t - S_t^{wti} wti_t \right] \\ & + \beta \lambda_{t+1} \left[Y_{t+1} + S_{t+1}^{wti} wti_t - P_{t+1} c_{t+1} - S_{t+1}^{wti} wti_{t+1} \right] \\ & + \ldots. \end{aligned}$$

The three FOCs are, respectively, $\dfrac{\partial u(c_t, d \cdot wti_t)}{\partial c_t} - \lambda_t P_t = 0$, $\dfrac{\beta \cdot \partial u(c_{t+1}, d \cdot wti_{t+1})}{\partial c_{t+1}} - \beta \lambda_{t+1} P_{t+1} = 0$, and $\dfrac{d \cdot \partial u(c_t, d \cdot wti_t)}{\partial (d \cdot wti_t)} - \lambda_t S_t^{wti} + \beta \lambda_{t+1} S_{t+1}^{wti} = 0$. In the last FOC, two things to note. First, the *d* term (highlighted in red) in the numerator – it appears because of the chain rule of calculus. (Note that the *d* term appears in the consumer's utility function, as part of the second argument.) Second, the marginal utility function is with respect to wti_t.

Problem 1 continued

c. **(7 points)** Based on the FOCs you obtained in part b above, compute the period-t oil price. That is, construct the expression

$$S_t^{wti} = \ldots$$

(the "…" that appears on the right-hand-side is for you to construct).

Solution: Using the same algebraic steps (which are omitted here) as in solving for stock prices, we get

$$S_t^{wti} = \frac{\beta\lambda_{t+1}S_{t+1}^{wti}}{\lambda_t} + \frac{u_{2t}}{\lambda_t}$$

in which we are using u_{2t} as shorthand: $u_{2t} = \frac{d \cdot \partial u(c_t, d \cdot wti_t)}{\partial(d \cdot wti_t)}$.

Thus, there is a positive relationship between oil prices and *d*. More generally, there is a positive relationship between oil prices and the **marginal** utility of using "oil" for individuals.

Problem 1 continued

d. **(7 points)** You can see in the graph displayed above that oil prices have overall been much higher over the past decade than before. Provide a **brief** explanation **that is just based *only* (that is, *ceteris paribus)* on how the *d* term in the utility function may have changed over the years.** Your explanation should be stated in both mathematical terms **AND** in economic terms (that is, the economic explanation should not simply be a verbal restatement of the mathematics).

Solution: The main economic idea conveyed by the d term is that individuals receive direct happiness ("utility") from their stock of wti_t (again, think in terms of heating, cooking, etc).

Using the oil pricing expression in part c and in *ceteris paribus* terms, oil prices rose because individuals "enjoyed" using oil more. This would be interpreted as d during the last decade was larger than d during the earlier years.

We could offer a huge number of underlying explanations for this (a couple of prominent examples are lack of efficiency of energy use by individuals and growing demand for heating in developing economies). No matter what precise underlying explanation might carry the most weight, from the view of the framework in this question, it is **increased demand** for oil usage that caused the increase in oil prices. Technically, it is an increase in d that led to an increase in S^{wti}.

Finally, a note of caution in the economic interpretation here: this consumer-oriented problem only articulates a **demand-side** notion of oil prices. It does not speak much, if at all, to the **supply side** of oil markets.

Problem 2: Government Debt Ceilings (25 points). Just like we extended our two-period analysis of consumer behavior to an infinite number of periods, we can extend our two-period analysis of fiscal policy to an infinite number of periods.

The government's budget constraints (expressed in real terms) for the fiscal years 2013 and 2014 are

$$g_{2013} + b_{2013} = t_{2013} + (1+r)b_{2012} \qquad \text{(Fiscal Year 2013)}$$
$$g_{2014} + b_{2014} = t_{2014} + (1+r)b_{2013} \qquad \text{(Fiscal Year 2014)}$$

and analogous conditions describe the government's budget constraints in the fiscal years 2015, 2016, 2017, etc. The notation is as in Chapter 7: g denotes real government spending during a given time period, t denotes real tax revenue during a given time period (all taxes are assumed to be lump-sum here), r denotes the real interest rate, and b denotes the government's asset position at the end of any given period. For example, b_{2012} is the government's asset position at the end of the fiscal year 2012, b_{2013} is the government's asset position at the end of the fiscal year 2013, and so on.

At the end of fiscal year 2012, the government's asset position was roughly a **debt** of \$15 trillion (that is, b_{2012} = -\$15 trillion).

The current fiscal policy plans/projections call for: g_{2013} = \$4 trillion, t_{2013} = \$2 trillion, g_{2014} = \$4 trillion, and t_{2014} = \$2 trillion.

Finally, given how low interest rates are right now and how low they are projected to remain for at least the next couple of years, suppose that the real interest rate is always zero (i.e., $r = 0$ always).

a. **(6 points)** Respond to the following in no more than a ***total*** **of 20 words:** Assuming the projections above prove correct, what will be the numerical value of the federal government's asset position at the end of 2013? **Explain/justify as needed.**

Solution: Using the given numerical values and using the fiscal year 2013 government budget constraint given above, it is straightforward to calculate b_{2013} = -\$17 trillion.

Problem 2 continued

b. **(6 points)** Respond to the following in no more than a ***total*** **of 20 words:** Assuming the projections above prove correct, what will be the numerical value of the federal government's asset position at the end of 2014? **Explain/justify as needed.**

Solution: Using the given numerical values, the value for b_{2013} found in part a, and using the fiscal year 2014 government budget constraint given above, it is straightforward to calculate b_{2014} = -$19 trillion.

Under current federal law, the U.S. government's debt cannot be larger than $17 trillion at any point in time. This limit is known as the "debt ceiling."

c. **(6 points)** Respond to the following in no more than a ***total*** **of 20 words: Based on your answer in part a above,** does the debt ceiling pose a problem for the government's fiscal policy plans during the course of the fiscal year 2013? If it poses a problem, briefly describe the problem; if it poses no problem, briefly describe why it poses no problem.

Solution: No, the debt ceiling poses no problem for fiscal policy for the year 2013. This is because the t and g plans call for a debt at the end of 2013 of $17 trillion, which (just barely…) does not exceed the ceiling.

Problem 2 continued

d. **(7 points)** Respond to the following in no more than a ***total*** **of 20 words: Based on your answer in part b above,** does the debt ceiling pose a problem for the government's fiscal policy plans during the course of the fiscal year 2014? If it poses a problem, briefly describe the problem; if it poses no problem, briefly describe why it poses no problem.

Solution: Yes, the debt ceiling poses a problem for the fiscal policy plans for the fiscal year 2014. This is because the t and g plans call for a debt at the end of 2014 of $19 trillion, which violates the ceiling

Problem 3: Two-Period Consumption-Savings Framework (25 points). Consider the two-period economy (with zero government spending and zero taxation), in which the representative consumer has no control over his real income (y_1 in period 1 and y_2 in period 2). The lifetime utility function of the representative consumer is

$$u(c_1, c_2) = \ln c_1 + \ln c_2 .$$

The lifetime budget constraint (in real terms) of the consumer is, as usual,

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} + (1+r)a_0 .$$

Suppose the consumer begins period 1 with zero net assets ($a_0 = 0$), and as per the notation in Chapters 3 and 4, r denotes the real interest rate.

a. **(5 points)** Set up a **lifetime** Lagrangian formulation for the representative consumer's lifetime utility maximization problem. Define any new notation you introduce.

Solution: The lifetime Lagrangian is $\ln c_1 + \ln c_2 + \lambda\left[y_1 + \frac{y_2}{1+r} + (1+r)a_0 - c_1 - \frac{c_2}{1+r} \right]$, which contains Lagrange multiplier λ.

Problem 3 continued

b. **(5 points)** Based on the Lagrangian from part a, compute the first-order conditions with respect to c_1 and c_2.

Solution: The first-order conditions with respect to c_1 and c_2 are

$$\frac{1}{c_1} - \lambda = 0$$
$$\frac{1}{c_2} - \frac{\lambda}{1+r} = 0$$

c. **(5 points)** Using the first-order conditions computed in part b, provide the algebra to arrive at the consumption-savings optimality condition for the **given** utility function. **NOTE:** Your final expression of the consumption-savings optimality condition should be presented in terms of the **ratio** $\frac{c_2}{c_1}$**.** Thus, the final form of the condition to present is

$$\frac{c_2}{c_1} = \ldots$$

in which the right hand side is for you to determine. **Your final expression may NOT include any Lagrange multipliers in it.** Clearly present the important steps and logic of your analysis.

Solution: The second expression in part b tells us $\lambda = \frac{1+r}{c_2}$. Inserting this expression for the multiplier into the first expression gives $\frac{1}{c_1} = \frac{1+r}{c_2}$. Rearranging this expression gives the consumption-savings optimality condition in the requested form

$$\frac{c_2}{c_1} = 1+r.$$

Problem 3c continued (more work space)

d. **(5 points)** In no more than two brief sentences, qualitatively (that is, in economic terms, not mathematical terms) describe what the "marginal rate of substitution between c_1 and c_2" is, not **just** at the optimal choices, but for **any** pair (c_1, c_2).

Solution: The marginal rate of substitution (MRS) between c_1 and c_2 is the number of units of c_2 the individual would be **willing** to give up in order to get one more unit of c_1, starting from **any** pair (c_1, c_2) (whether optimal or not).

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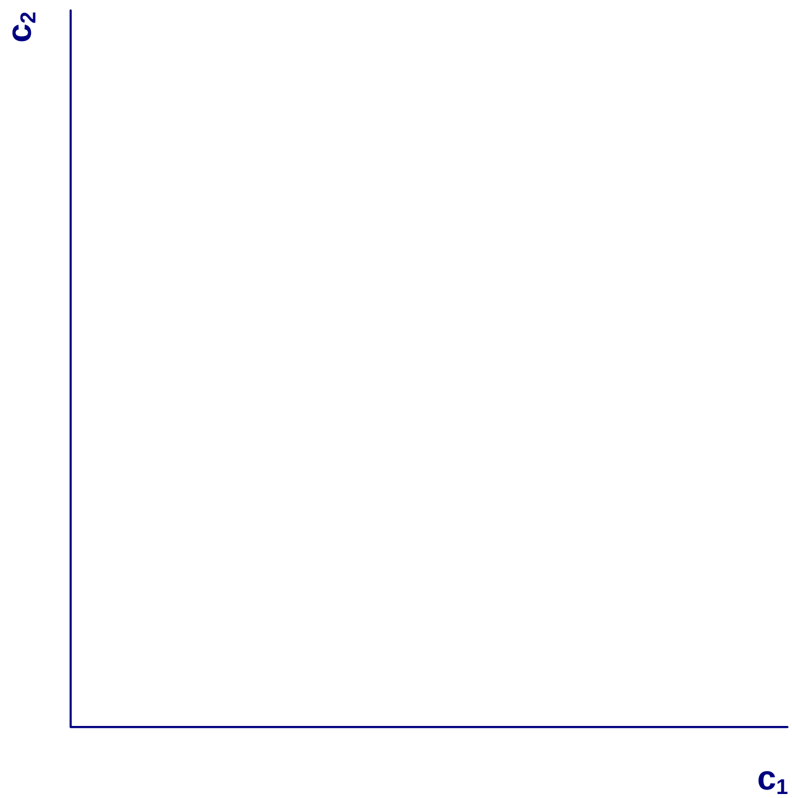
Problem 3 continued

e. **(5 points)** In the axes below, plot an indifference-curve/budget constraint diagram for the consumption. **Clearly indicate what the slope of the budget line is, as well as the consumption-savings optimality condition.**

Solution: The usual diagram we have seen many times in class. The slope of budget line is $-(1 + r)$, and the consumption-savings optimality condition is the point at which one indifference curve is tangent to the budget line.



Problem 4: The Consumption-Leisure Framework (25 points). In this question, you will use the basic (one period) consumption-leisure framework to consider some labor market issues.

Suppose the representative consumer has the following utility function over consumption and labor,

$$u(c,l) = \ln c - \frac{A}{1+\phi} n^{1+\phi},$$

where, as usual, c denotes consumption and n denotes the number of hours of labor (i.e., the number of hours the consumer chooses to work). The constants A and ϕ are outside the control of the individual, but each is strictly positive. (As usual, $\ln(\cdot)$ is the natural log function.)

Suppose the budget constraint (expressed in real, rather than in nominal, terms) the individual faces is $c = (1-t)\cdot w \cdot n$, where t is the labor tax rate, w is the **real** hourly wage rate, and n is the number of hours the individual works.

Recall that in one week there are 168 hours, hence $n + l = 168$ must always be true.

The Lagrangian for this problem is

$$\ln c - \frac{A}{1+\phi} n^{1+\phi} + \lambda\left[(1-t)wn - c\right],$$

in which λ denotes the Lagrange multiplier on the budget constraint.

a. **(4 points)** Based on the **given Lagrangian,** compute the representative consumer's first-order conditions with respect to consumption and with respect to labor. Clearly present the important steps and logic of your analysis.

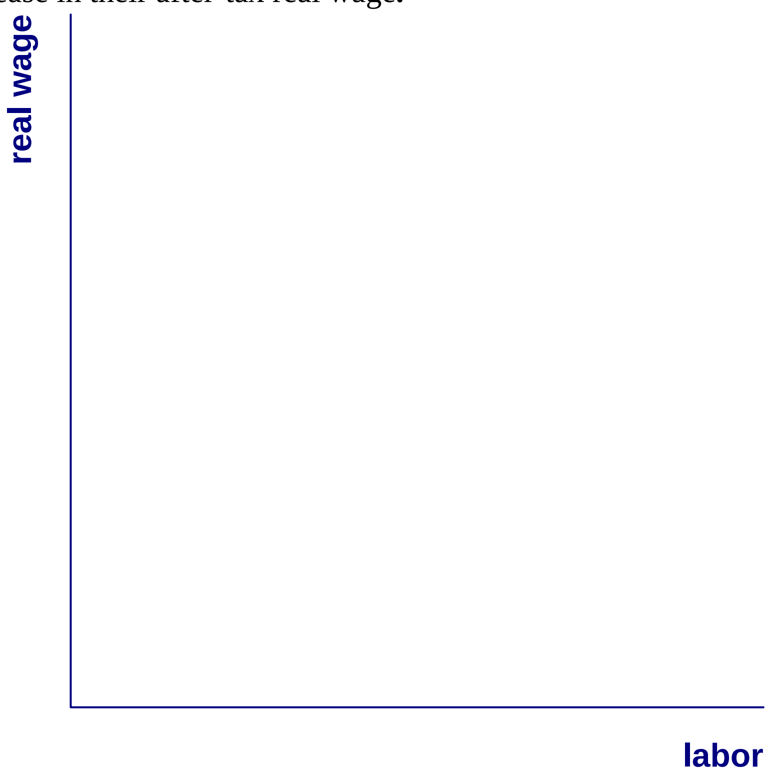
Solution: The first-order conditions with respect to consumption and labor are

$$\frac{1}{c} - \lambda = 0$$
$$-An^{\phi} + \lambda(1-t)w = 0$$

Problem 4 continued

b. **(6 points)** Based on **ONLY** the first-order condition with respect to labor computed in part a, **qualitatively** sketch two things in the diagram below. **First,** the general shape of the relationship between w and n (perfectly vertical, perfectly horizontal, upward-sloping, downward-sloping, or impossible to tell). **Second,** how changes in t affect the relationship (shift it outwards, shift it in inwards, or impossible to determine). **Briefly** describe the **economics** of how you obtained your conclusions. (**REMINDER: use ONLY the first-order condition with respect to labor.**)

Solution: The first-order condition with respect to labor can be rearranged to (if we want to put it in vertical axis/horizontal axis form) $w = \frac{1}{\lambda}\frac{An^{\phi}}{(1-t)}$. Given that $\phi > 0$, there is clearly an upward sloping relationship between w and n. Plotting this below (and ignoring convexity/concavity issues, which are governed by the particular magnitude of ϕ) gives an upward sloping relationship holding A, t, and λ constant. This is the labor supply function. Then, starting from this upward-sloping relationship, a rise in the tax rate t (holding A, λ, and n constant) causes the entire function to shift inwards. The latter effect is due to individuals working fewer hours when the tax rate rises, all else equal, due to the decrease in their after-tax real wage**.**



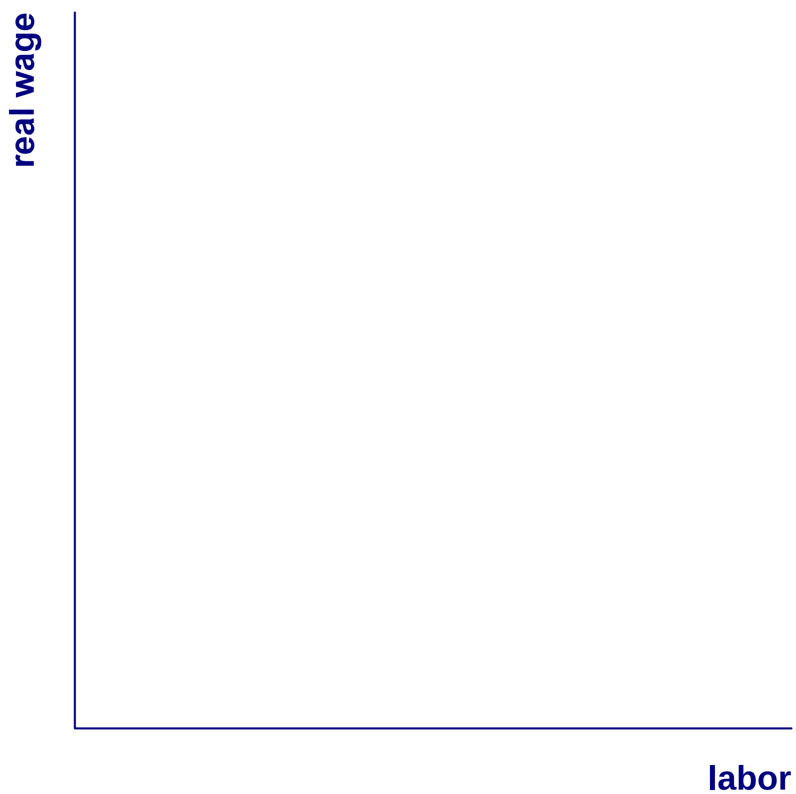
Problem 4 continued

c. **(4 points)** Now based on **both** of the two first-order conditions computed in part a, construct the consumption-leisure optimality condition. Clearly present the important steps and logic of your analysis.

Solution: As usual, this requires eliminating the Lagrange multiplier across the two expressions. The first-order condition on consumption gives lambda = 1/c. Inserting this into the first-order condition on labor gives $An^{\phi} = \frac{(1-t)w}{c}$. Or, multiplying through by *c*, the consumption-leisure optimality condition can be expressed as

$$\frac{An^{\phi}}{1/c} = (1-t)w.$$

d. **(6 points)** Based on both the consumption-leisure optimality condition obtained in part c and on the budget constraint, **qualitatively** sketch two things in the diagram below. **First,** the general shape of the relationship between w and n (perfectly vertical, perfectly horizontal, upward-sloping, downward-sloping, or impossible to tell). **Second,** how changes in t affect the relationship (shift it outwards, shift it in inwards, or impossible to determine). **Briefly** describe the **economics** of how you obtained your conclusions.



Problem 4d continued (more work space)

Solution: The budget constraint says that $c = (1-t)wn$. Substituting this into the consumption-leisure optimality condition from part c, we have $(1-t)wn \cdot An^{\phi} = (1-t)w$. The (1-*t*)*w* terms on the left-hand and right-hand sides obviously cancel, leaving $n \cdot An^{\phi} = 1$, or, combing the powers in *n*, $An^{1+\phi} = 1$. Plotting this in the space above, we will have

$$n = \left(\frac{1}{A}\right)^{\frac{1}{1+\phi}}$$

which clearly does not depend on the real wage *w* at all. Hence, this is a vertical line at the value $\left(\frac{1}{A}\right)^{\frac{1}{1+\phi}}$.

e. **(5 points)** How do the conclusions in part d compare with those in part b? Are they broadly similar? Are they very different? Is it impossible to compare them? **In no more than 60 words,** describe as much as you can about the **economics (do *not* simply restate the mathematics you computed above)** when comparing the pair of diagrams.

Solution: Broadly, the difference between part b and part d is that part b is a "microeconomic" analysis, while part d is a "macroeconomic" analysis. More precisely, part b is, intuitively, a purely "slope" argument, rather than both a "slope" and a "level" argument in part d. The analysis in part b is tantamount to analyzing the effects of policy on **just** the labor market (why? – because the analysis there treats consumption as a constant). The analysis in part d instead is tantamount to analyzing **jointly** the effects of policy on labor markets **and** goods markets. To the extent that there are feedback effects between the two markets, there is no reason to think the answers from the analyses must be the same.

The latter is the basis for thinking of the analysis in part b as a "microeconomic" analysis and the analysis in part d as a "macroeconomic" analysis. What this implies is that one way (perhaps the most important way) to understand the difference between "microeconomic" analysis and "macroeconomic" analysis is that the latter routinely considers feedback effects across markets, whereas the former usually does not.

END OF EXAM