BASICS

Timeline of events

Notation

- $k_1$: capital used for production in period 1 (decided upon in "period 0")
- $n_1$: labor used for production in period 1
- $w_1$: real wage rate for labor in period 1 ($w_1 = W_1/P_1$)
- $i$: nominal interest rate
- $P_1$: nominal price of output produced and sold by firm in period 1

AND nominal price of one unit of capital bought by the firm in period 1 for use in period 2 (recall time to build...)

Underlying assumption/view of world: capital goods are not necessarily "distinct" from consumption goods (i.e., computers purchased by both firms and individual consumers)
FIRM PROFIT MAXIMIZATION

- A dynamic profit maximization problem
  - Because firm exists for both periods
  - All analysis conducted from the perspective of the very beginning of period 1
  - Must consider present-discounted-value (PDV) of lifetime (i.e., two-period) profits

- Dynamic profit function
  - (specified in nominal terms – could specify in real terms...)
  - Dynamic profit function (PDV of) period-2 profits = 0

\[
P_1 f(k_1, n_1) + P_1 k_1 - P_1 W_1 n_1 - P_1 k_2 + \left( \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{P_2 W_2 n_2}{1+i} \right) = 0
\]

- Two-period model: \( k_3 = 0 \) (no machines needed in “period 3”)

Model Structure

FIRM PROFIT MAXIMIZATION

- FOCs with respect to \( n_1, n_2, k_2 \)

Identical except for time subscripts

- with respect to \( n_1 \):
  \[ P_1 f(k_1, n_1) - P_1 W_1 = 0 \]  
  Equation 1

- with respect to \( n_2 \):
  \[ \frac{P_2 f(k_2, n_2)}{1+i} \cdot \frac{P_2 W_2 n_2}{1+i} = 0 \]  
  Equation 2

- with respect to \( k_2 \):
Re-express equation 3

\[-P_i + \frac{P_f f_i(k_i, n_i)}{1+i} + \frac{P_i}{1+i} = 0\]

Divide by \(P_i\)

\[\frac{P_f f_i(k_i, n_i)}{P_i(1+i)} + \frac{P_i}{P_i(1+i)} = 1\]

Group terms informatively

\[\left(\frac{P_f}{P_i}\right)\left(\frac{1}{1+i}\right) f_i(k_i, n_i) + \left(\frac{P_i}{P_i}\right)\left(\frac{1}{1+i}\right) = 1\]

\[P_f/P_i = 1 + \pi_2\]

Fisher equation

\[f_i(k_i, n_i) + \frac{1}{1+r} = 1\]

Multiply by \(1+r\)

\[f_i(k_i, n_i) + 1 = 1+r\]

\[f_i(k_i, n_i) = r\]

Equivalent/alternative representation of firm profit-maximizing condition for capital
FIRM PROFIT MAXIMIZATION

\[
P_f(k_1, n_1) + P_k - P_w n_1 - P_k r + \frac{P_f(k_2, n_2)}{1+i} + P_k - P_w n_2 - \frac{P_{kr}^*}{1+i} = 0
\]

- FOCs with respect to \( n_1, n_2, k_2 \)

  - with respect to \( n_1 \):
    \[
    P_f f_n(k_1, n_1) - P_{w1} = 0
    \]  
    
    \text{Equation 1}

  - with respect to \( n_2 \):
    \[
    P_f f_n(k_2, n_2) - P_{w2} = 0
    \]  
    
    \text{Equation 2}

  - with respect to \( k_2 \):
    \[
    -P + \frac{P_f f_n(k_2, n_2)}{1+i} + P_k = 0
    \]  
    
    \text{equivalent to}  
    \[
    f_s(k_2, n_2) = r
    \]  
    
    \text{Equation 3}

Identical except for time subscripts

Profit-maximizing labor hiring implies \( f_s(k_1, n_1) = w_1 \)

Profit-maximizing capital purchases (for the future...) implies \( f_s(k_2, n_2) = w_2 \)
FIRM PROFIT MAXIMIZATION

\[ P_f(k_1, n_1) + P_f(k_2, n_2) - P_k^1 - P_k^2 = 0 \]

- FOCs with respect to \( n_1, n_2, k_2 \)
  - with respect to \( n_1 \):
    \[ \frac{P_f(k_1, n_1)}{1+i} - P_W^1 = 0 \]
  - with respect to \( n_2 \):
    \[ \frac{P_f(k_2, n_2)}{1+i} - P_W^2 = 0 \]
  - with respect to \( k_2 \):
    \[ -P + \frac{P_f(k_2, n_2)}{1+i} + \frac{P_f(k_2, n_2)}{1+i} = 0 \]

Marginal product of labor
- \( f_n(k_t, n_t) \)
- Sometimes denote by \( mpn_t \)

Marginal product of capital
- \( f_k(k_t, n_t) \)
- Sometimes denote by \( mpp_k_t \)

Cobb-Douglas Production Function

- A commonly-used functional form in modern quantitative macroeconomic analysis
  \[ f(k_t, n_t) = k_t^{\alpha} n_t^{1-\alpha} \]  
  (see Cobb-Douglas utility function on Practice Problem Set 1)

- Describes the empirical relationship between aggregate GDP, aggregate capital, and aggregate labor quite well

- \( \alpha \in (0,1) \) measures capital’s share of output
- Hence \( (1-\alpha) \in (0,1) \) measures labor’s share of output

- Interpretation
  - The relative importance of either capital (or labor) in the production process
  - Estimates for U.S. economy: \( \alpha = 0.3 \)
  - Estimates for Chinese economy: \( \alpha = 0.15 \) (not yet a very capital-rich economy)

- Cobb-Douglas form useful for illustrating factor demands
  - \( mpn = f'_n(k_t, n_t) = (1-\alpha)k^{\alpha-1}n^{-\alpha} \)
  - \( mpp_k = f'_k(k_t, n_t) = \alpha k^{-\alpha}n^{1-\alpha} \)
Firm-level demand for labor defined by the relation

\[ w_t = (1 - \alpha) K^\alpha n_t^{\alpha - 1} (\text{mnp}) \]

for both \( t = 1 \) and \( t = 2 \)

Because exponent (-\( \alpha \)) is a negative number, can move to denominator

\[ w_t = (1 - \alpha) \left( \frac{k_t}{n_t} \right) \]

A NEGATIVE RELATIONSHIP BETWEEN \( w_t \) and \( n_t \)

Because exponent (-\( \alpha \)) is a negative number, can move to denominator

\[ w_t = (1 - \alpha) \left( \frac{k_t}{n_t} \right) \]

A RELATIONSHIP BETWEEN \( w_t \) and \( n_t \)
**Labor Demand in the Micro and the Macro**

- **Micro-Level Labor Demand**
  - Firm-level demand for labor defined by the relation
    \[ w_t = (1 - \alpha) K_t^\alpha n_tw_{n,m} \]
    for both \( t = 1 \) and \( t = 2 \)
    - Because exponent (-\( \alpha \)) is a negative number, can move to denominator
    \[ w_t = (1 - \alpha) \left( \frac{k_t}{n_t} \right)^{\frac{1}{\alpha}} \]
    
    Completes picture of the aggregate labor market

- **Micro-Level Capital Demand**
  - Firm-level demand for capital defined by the relation
    \[ r_t = \alpha k_t^{\alpha-1} w_{n,m} \]
    for \( \alpha \) is a negative number, can move to denominator
    \[ r_t = \alpha \left( \frac{n_t}{k_t} \right)^{\frac{1}{\alpha-1}} \]
    
    Follows from Equation 3 (will see it soon...)

**Labor Demand**

**Micro-Level Capital Demand**
**MICRO-LEVEL CAPITAL DEMAND**

- **Firm-level demand for capital** defined by the relation

  
  
  \[ r_i = \alpha k_i^{\alpha-1} n_i^{-\alpha} = mpk_i \]

  
  Because exponent \( (\alpha - 1) \) is a negative number, can move to denominator

  
  \[ r_i = \alpha \left( \frac{n_i}{k_i} \right)^{-\alpha} \]

  
  A **RELATIONSHIP BETWEEN** \( r_i \) and \( k_i \)

**CAPITAL DEMAND**

- **Firm-level demand for capital** defined by the relation

  
  
  \[ r_i = \alpha k_i^{\alpha-1} n_i^{-\alpha} = mpk_i \]

  
  Because exponent \( (\alpha - 1) \) is a negative number, can move to denominator

  
  \[ r_i = \alpha \left( \frac{n_i}{k_i} \right)^{-\alpha} \]

  A **RELATIONSHIP BETWEEN** \( r_i \) and \( k_i \)

- **Sum over all firms**

  (No tension between the micro and macro)

- **(Almost...)** completes picture of the aggregate capital market
**FROM CAPITAL DEMAND TO INVESTMENT DEMAND**

- Capital is a **stock variable**

  - Capital demand function

- Investment is a **flow variable**

  - Investment demand function

  \[ \text{inv}_1 = \text{k}_2 - \text{k}_1 \]

  - At start of period 1, \( \text{k}_1 \) cannot be changed. Thus any rise in demand for \( \text{k}_2 \) is reflected one-for-one in a rise in \( \text{inv}_1 \).

  - Capital demand and investment demand functions have same shape.

**THE THREE MACRO (AGGREGATE) MARKETS**

- **Goods Markets**
  - Demand derived from C-L framework
  
    (For \( S \), have to consider how aggregate NOMINAL \( P \) is determined...Chapter 14)

- **Labor Markets**
  - Supply derived from C-L framework
  - Demand derived from firm theory in C-S framework

- **Capital/Savings/Funds/Asset Markets** (aka Financial Markets)
  - Supply derived from C-S framework
  - Demand derived from firm theory in C-S framework
**REAL INTEREST RATE**

- *r* a key variable for macroeconomic analysis
- Chapter 4: *r* measures the price of period-1 consumption in terms of period-2 consumption
- Chapter 8: *r* reflects degree of impatience (in the long run)
- *r* often reflects rate of consumption growth between periods
- **Now:** *r* measures the price of capital (machine and equipment) purchases by firms
  - Reflects (real) opportunity cost of sinking funds into capital *today* that won’t bear fruit (i.e., help produce output) until the *future*
  - Regardless of whether firm actually has to "borrow" to purchase capital

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**Equation 3 (FOC on *k*)**

\[
-P + \frac{P_2 f_i(k_2, n_2)}{1+i} + \frac{P_1}{1+i} = 0
\]

When firms make optimal investment decisions \( r = mpk \)