

# THE REPRESENTATIVE-AGENT APPROACH TO MACROECONOMICS

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Introduction

## REPRESENTATIVE-AGENT MACROECONOMICS

- Consumer A: Consumed \$50 in Year X No other consumers in the economy
- Consumer B: Consumed \$75 in Year X
- Consumer C: Consumed \$100 in Year X** THE REPRESENTATIVE CONSUMER
- Consumer D: Consumed \$125 in Year X
- Consumer E: Consumed \$150 in Year X
  
- Aggregate** (i.e., economy-wide) consumption = \$500
- Average** consumption = \$100
  
- Macroeconomics is most concerned with **aggregate** outcomes
  
- If we want to take a micro-based approach to modeling aggregate outcomes...
- ...**model Consumer C's behavior/decision-making**
  
- A simplistic approach – turns out to yield surprisingly rich results, insights, and predictions**

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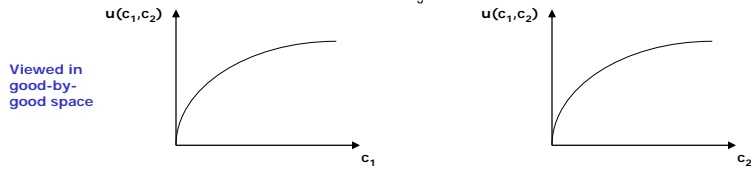
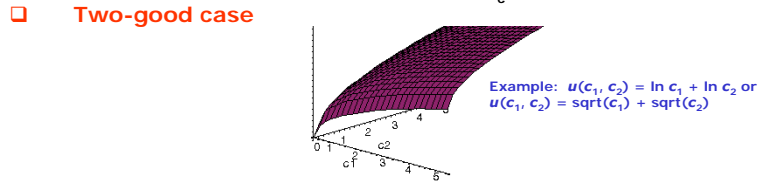
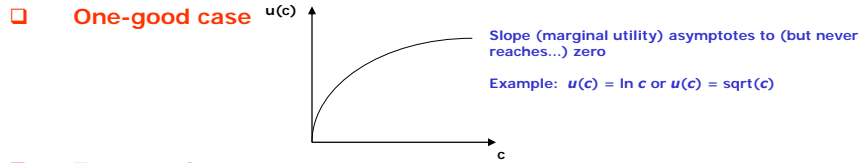
# REVIEW OF CONSUMER THEORY

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## UTILITY FUNCTIONS

- Describe how much “happiness” or “satisfaction” an individual experiences from “consuming” goods – the **benefit** of consumption
- **Marginal Utility**
  - The extra total utility resulting from consumption of a small/incremental extra unit of a good
  - Mathematically, the (partial) slope of utility with respect to that good  
Alternative notation:  $du/dc$  OR  $u'(c)$  OR  $u_c(c)$  OR  $u_1(c)$
- **One-good case:  $u(c)$ , with  $du/dc > 0$  and  $d^2u/dc^2 < 0$** 
  - Recall interpretation: strictly increasing at a strictly decreasing rate
  - Diminishing marginal utility
- **Two-good case:  $u(c_1, c_2)$ , with  $u_i(c_1, c_2) > 0$  and  $u_{ii}(c_1, c_2) < 0$  for each of  $i = 1, 2$** 
  - Utility strictly increasing in **each good** individually (partial)
  - Diminishing marginal utility in **each good** individually
- **Readily extendable to  $N$ -good case:  $u(c_1, c_2, c_3, c_4, \dots, c_N)$**

## UTILITY FUNCTIONS

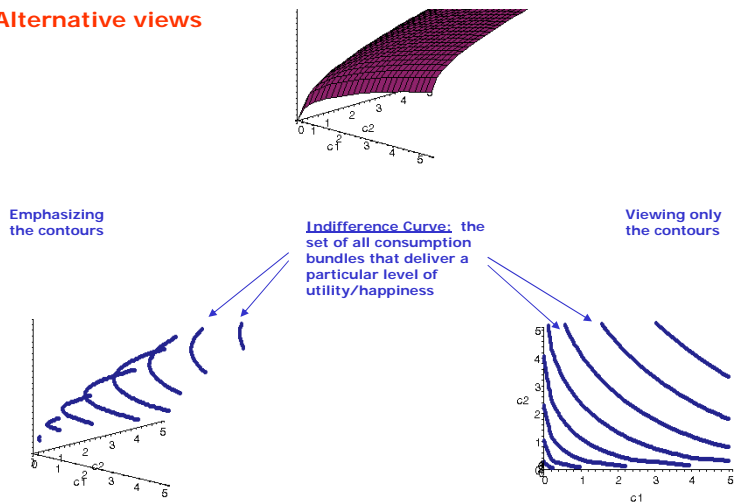


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## UTILITY FUNCTIONS

**Alternative views**

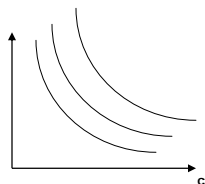


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## UTILITY FUNCTIONS

- **Marginal Rate of Substitution (MRS)**
  - **Maximum** quantity of one good consumer is **willing** to give up to obtain **one** extra unit of the other good
  - Graphically, the (negative of the) slope of  $c_2$  an indifference curve
  - MRS is itself a **function** of  $c_1$  and  $c_2$  (i.e.,  $MRS(c_1, c_2)$ )
  - **MRS equals ratio of marginal utilities**
    - $$MRS(c_1, c_2) = \frac{u_1(c_1, c_2)}{u_2(c_1, c_2)}$$
    - Using Implicit Function Theorem (see Problem Set 1)
- **Summary: whether graphically- or mathematically-formulated, utility functions describe the benefit side of consumer optimization**



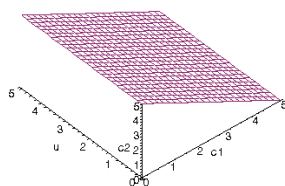
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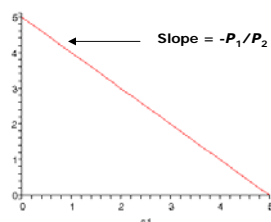
## BUDGET CONSTRAINTS

- Describe the **cost** side of consumption
- **One-good case (trivial):  $Pc = Y$** 
  - Assume income  $Y$  is taken as given by consumer for now...
- **Two-good case (interesting):  $P_1c_1 + P_2c_2 = Y$** 
  - Assume income  $Y$  is taken as given by consumer for now...

Plotted in 3D-consumption-space



Plotted in 2D-consumption-space



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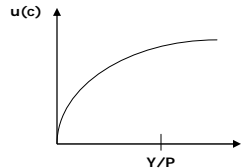
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## CONSUMER OPTIMIZATION

❑ **Consumer's decision problem:** maximize utility subject to budget constraint – bring together both **cost** side and **benefit** side

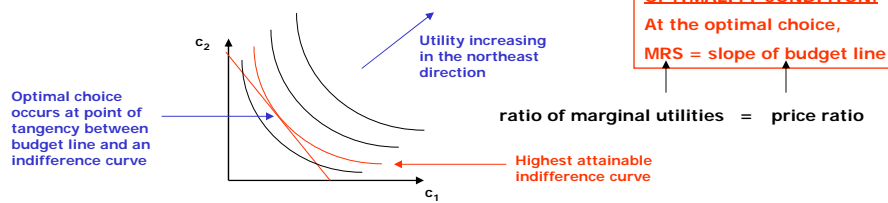
❑ **One-good case**

- ❑ Trivially, choose  $c = Y/P$
- ❑ No **decision** to make here...



❑ **Two-good case**

- ❑ How to optimally **allocate Y** across the two goods  $c_1$  and  $c_2$ ?
- ❑ A non-trivial **decision** problem...



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## LAGRANGE ANALYSIS

❑ Consumer optimization a **constrained optimization** problem

- ❑ Maximize some function (utility function)...
- ❑ ...taking into account some restriction on the objects to be maximized over (budget constraint)

❑ **Lagrange Method:** mathematical tool to solve constrained optimization problems

❑ **General mathematical formulation**

- ❑ Choose  $(x, y)$  to maximize a given objective function  $f(x, y)$ ...
- ❑ ...subject to the constraint  $g(x, y) = 0$  (**Note formulation of constraint**)

❑ **Step 1:** Construct Lagrange function ↗ Lagrange multiplier

$$L(x, y, \lambda) = f(x, y) + \lambda g(x, y)$$

❑ **Step 2:** Compute first-order conditions with respect to  $x$ ,  $y$ , and  $\lambda$

$$f_x(x, y) + \lambda g_x(x, y) = 0$$

$$f_y(x, y) + \lambda g_y(x, y) = 0$$

$$g(x, y) = 0$$

**Rationale:** setting first derivatives to zero isolates maxima (or minima...technically, need to check second-order condition...)

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## LAGRANGE ANALYSIS

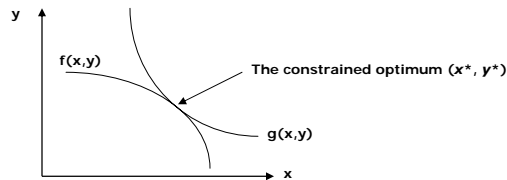
- **Step 3:** Solve the system of first-order conditions for  $x$ ,  $y$ , and  $\lambda$ 
  - Often most interested in simply eliminating the multiplier...

- ...
- ...
- ...

- **Optimality condition:** at the optimum  $(x^*, y^*)$

$$\frac{f_x(x^*, y^*)}{f_y(x^*, y^*)} = \frac{g_x(x^*, y^*)}{g_y(x^*, y^*)}$$

Graphical interpretation: at the constrained optimum, the function  $f(\cdot)$  is tangent to the function  $g(\cdot)$



## LAGRANGE ANALYSIS

- Apply Lagrange tools to consumer optimization
- Objective function:  $u(c_1, c_2)$
- Constraint:  $g(c_1, c_2) = Y - P_1c_1 - P_2c_2 = 0$

- **Step 1:** Construct Lagrange function

$$L(c_1, c_2, \lambda) = u(c_1, c_2) + \lambda [Y - P_1c_1 - P_2c_2]$$

- **Step 2:** Compute first-order conditions with respect to  $c_1$ ,  $c_2$ ,  $\lambda$

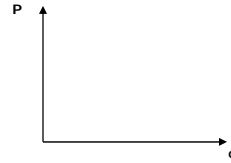
- **Step 3:** Solve (with focus on eliminating multiplier)

$$\frac{u_1(c_1^*, c_2^*)}{u_2(c_1^*, c_2^*)} = \frac{P_1}{P_2}$$

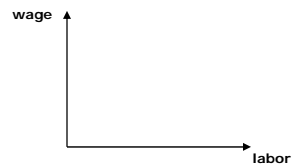
OPTIMALITY CONDITION  
i.e., MRS = price ratio

## THE THREE MACRO (AGGREGATE) MARKETS

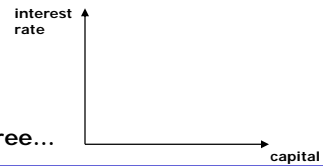
☐ Goods Markets



☐ Labor Markets



☐ Capital/Savings/Funds/Asset Markets



☐ Will put micro-foundations under all three...

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## CONSUMPTION-LEISURE MODEL

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## BASICS

- ❑ Consumption-Leisure Model – provides foundation for
  - ❑ Labor-market supply function
  - ❑ Goods-market demand function
  - ❑ An application of the basic consumer theory model...
  - ❑ ...we will put a macro interpretation on it
  - ❑ Only one time period – no “future” for which to save
  
- ❑ Notation
  - ❑  $c$ : consumption (“all stuff”)
  - ❑  $n$ : number of hours spent working per week
  - ❑  $l$ : number of hours leisure per week (time spent not working)
  - ❑  $P$ : dollar price of one unit of consumption (a nominal variable)
  - ❑  $W$ : hourly wage rate in terms of dollars (a nominal variable)
  - ❑  $t$ : tax rate on labor income
  
- ❑ “Weekly” model a detail
  - ❑ Could have called it a daily model, a monthly model, a yearly model, ...
  - ❑ Just need to take SOME stand on the length of a “period”

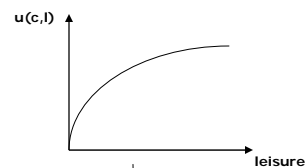
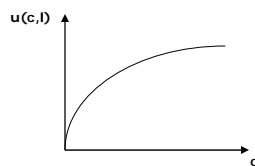
$n + l = 168$

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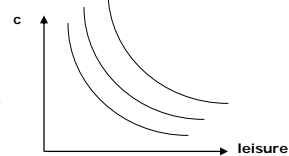
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## UTILITY

- ❑ Preferences  $u(c, l)$  with all the “usual properties”
  - ❑ Strictly increasing in  $c$
  - ❑ Strictly increasing in  $l$
  - ❑ Diminishing marginal utility in  $c$
  - ❑ Diminishing marginal utility in  $l$
  - ❑ Plotted in good-by-good spaces:



- ❑ Plotted as indifference curves



- ❑ Utility side of consumption-leisure model no different than Chapter 1 model

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## BUDGET CONSTRAINT

- Consumer must **work** for his income
  - **Y** no longer “falls from the sky”

$$Pc = Y$$

$Y = (1-t)Wn$  (all income is after-tax labor income)

$$Pc = (1-t)Wn$$

$n = 168 - l$

$$Pc = (1-t)W(168 - l)$$

Rearrange

$$Pc + (1-t)Wl = 168(1-t)W$$

Simply an application/re-interpretation of our basic consumer theory model

Spending on consumption    “Spending” on leisure    A constant from the point of view of the individual (price-taker)

(After-tax) wage is *opportunity cost* of leisure, hence the “price” of leisure  
- opportunity costs are *real economic costs/prices*

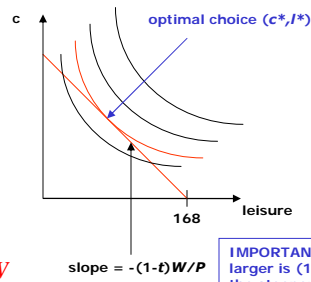
$$P_1c_1 + P_2c_2 = Y$$

Spending on  $c_1$     Spending on  $c_2$     A constant from the point of view of the individual

Chapter 1 budget constraint

## CONSUMER OPTIMIZATION

- **Consumer's decision problem:** maximize utility subject to budget constraint – bring together both **cost** side and **benefit** side
  - Choose  $c$  and  $l$  subject to  $Pc + (1-t)Wl = 168(1-t)W$
  - Plot budget line
  - Superimpose indifference map



- **At the optimal choice**

**CONSUMPTION-LEISURE OPTIMALITY CONDITION**  
- A key building block of modern macro models

$$\frac{u_l(c^*, l^*)}{u_c(c^*, l^*)} = \frac{(1-t)W}{P}$$

MRS (between consumption and leisure)

price ratio (inclusive of taxes)

**IMPORTANT:** the larger is  $(1-t)W/P$ , the steeper is the budget line

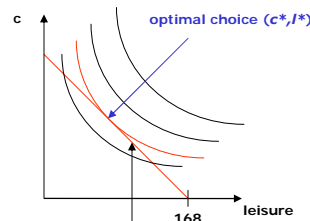
## REAL WAGE

- ❑  $W/P$  a key variable for macroeconomic analysis
  - ❑ Unit Analysis (i.e., analyze algebraic units of variables)
    - ❑ Units( $W$ ) = \$/hour of work
    - ❑ Units( $P$ ) = \$/unit of consumption
    - ❑ Units( $W/P$ ) =  $\frac{\frac{\$}{\text{hour of work}}}{\frac{\$}{\text{unit of consumption}}} = \frac{\cancel{\$}}{\text{hour of work}} \cdot \frac{\text{unit of consumption}}{\cancel{\$}}$
- $$= \frac{\text{unit of consumption}}{\text{hour of work}}$$
- Will sometimes denote using  $w$  (lower-case...)
- ❑ Economic decisions depend on **real wages ( $W/P$ )**, not nominal wages ( $W$ )
    - ❑ Measures the purchasing power of (nominal) wage earnings...
    - ❑ ...which is presumably what people most care about

## CONSUMER OPTIMIZATION

- ❑ **Consumer's decision problem:** maximize utility subject to budget constraint – bring together both **cost** side and **benefit** side
  - ❑ Choose  $c$  and  $l$  subject to  $Pc + (1-t)Wl = 168(1-t)W$

- ❑ Plot budget line
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CONSUMPTION-LEISURE OPTIMALITY CONDITION  
- key building block of modern macro models

$$\frac{u_l(c^*, l^*)}{u_c(c^*, l^*)} = \frac{(1-t)W}{P}$$

MRS (between consumption and leisure)
After-tax real wage

IMPORTANT: the larger is  $(1-t)W/P$ , the steeper is the budget line

Derive consumption-leisure optimality condition using Lagrange analysis

## LAGRANGE ANALYSIS

- ❑ Apply Lagrange tools to consumption-leisure optimization
- ❑ Objective function:  $u(c, l)$
- ❑ Constraint:  $g(c, l) = 168(1-t)W - Pc - (1-t)Wl = 0$

- ❑ **Step 1:** Construct Lagrange function

$$L(c, l, \lambda) = u(c, l) + \lambda [168(1-t)W - Pc - (1-t)Wl]$$

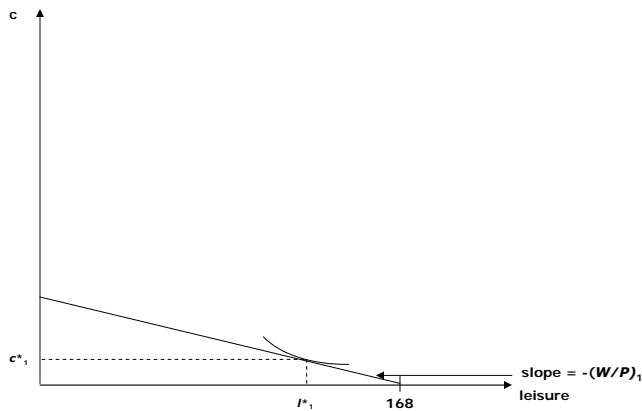
- ❑ **Step 2:** Compute first-order conditions with respect to  $c, l, \lambda$

- ❑ **Step 3:** Solve (with focus on eliminating multiplier)

$$\underbrace{\frac{u_l(c^*, l^*)}{u_c(c^*, l^*)}}_{\text{MRS (between consumption and leisure)}} = \underbrace{\frac{(1-t)W}{P}}_{\text{After-tax real wage}} \quad \text{CONSUMPTION-LEISURE OPTIMALITY CONDITION}$$

## MICRO-LEVEL LABOR SUPPLY

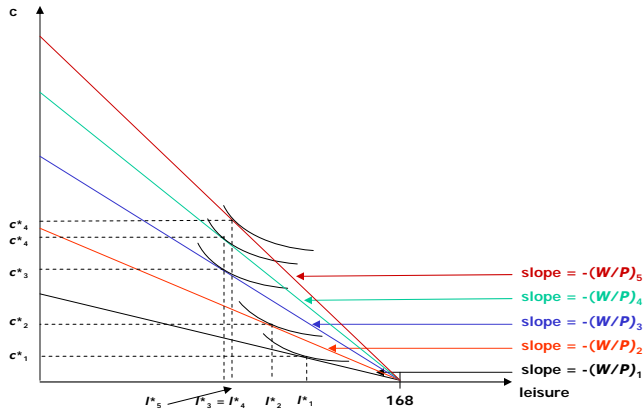
**An experiment:** how do micro-level consumption/leisure choices change as the real wage changes (assume  $t = 0$  here for simplicity)



## MICRO-LEVEL LABOR SUPPLY

**An experiment:** how do micro-level consumption/leisure choices change as the real wage changes (assume  $t = 0$  here for simplicity)

REAL WAGES:  $(W/P)_1 < (W/P)_2 < (W/P)_3 < (W/P)_4 < (W/P)_5$



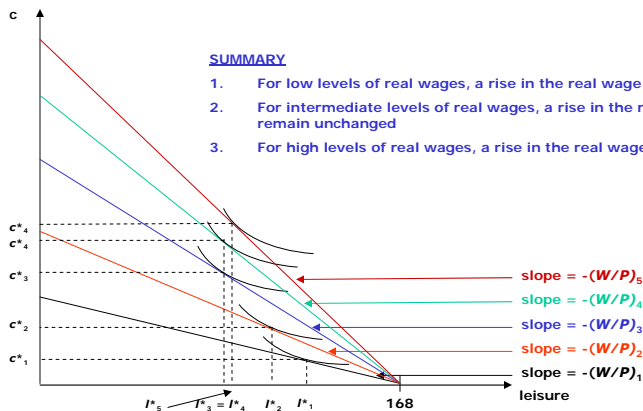
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## MICRO-LEVEL LABOR SUPPLY

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REAL WAGES:  $(W/P)_1 < (W/P)_2 < (W/P)_3 < (W/P)_4 < (W/P)_5$

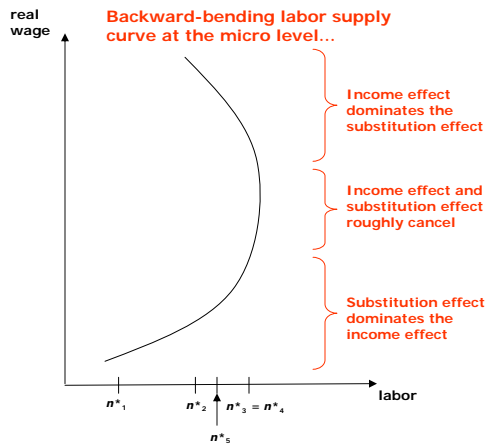


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## MICRO-LEVEL LABOR SUPPLY

Using the relation  $n = 168 - l$

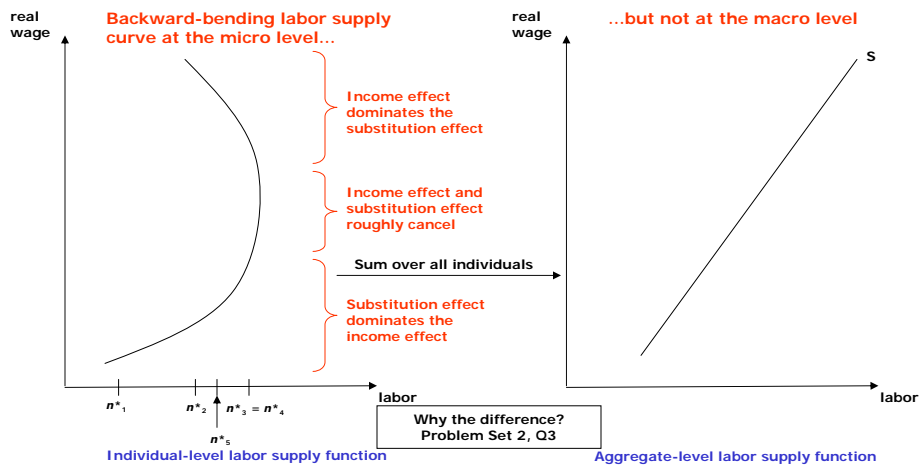


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## LABOR SUPPLY

Using the relation  $n = 168 - l$



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## CONSUMPTION DEMAND

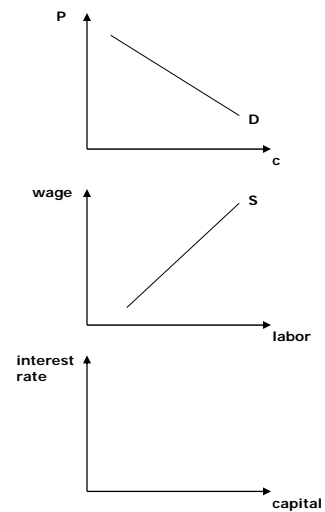
- ❑ Optimal choice of consumption was always rising as real wage was rising
- ❑ Could have conducted the entire analysis assuming nominal  $W$  was held fixed and nominal  $P$  was falling
  - ❑ Which means real wage  $W/P$  is rising
- ❑ Result: Fall in  $P \rightarrow$  rise in optimal  $c$  always
  - ❑ Implies downward-sloping consumption demand function at the micro level...
  - ❑ ...and at the aggregate level
- ❑ Consumption demand over two-thirds of aggregate demand in developed countries

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## THE THREE MACRO (AGGREGATE) MARKETS

- ❑ **Goods Markets**
  - ❑ Demand derived from C-L model
- ❑ **Labor Markets**
  - ❑ Supply derived from C-L model
- ❑ **Capital/Savings/Funds/Asset Markets (aka Financial Markets)**



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## THE MACROECONOMICS OF TIME

- ❑ Consumption-leisure model a **static** (i.e., one time period) model
- ❑ **Dynamic** models the core of modern macroeconomic theory
- ❑ Explicit consideration of how economic decisions/behaviors/outcomes unfold over multiple time periods
- ❑ Two-period framework (Chapters 3 and 4) the simplest possible multi-period framework
  - ❑ Will allow us to begin analyzing issues regarding interest rates and inflation (phenomena that occur **across time**)
- ❑ Infinite-period framework (Chapter 8)
  - ❑ Allows a richer quantitative description of the macroeconomy
  - ❑ Highlights the role of assets and the intersection between finance and macroeconomics