Department of Applied Economics Johns Hopkins University

Economics 602
Macroeconomic Theory and Policy
Problem Set 7 Suggested Solutions
Professor Sanjay Chugh
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1. **Deriving a Money Demand Function.** Denote by $\phi(c_t, i_t)$ the **real** money demand function. Here you will generate particular functional forms for $\phi(\cdot)$ using the MIU model we have studied.

   In an MIU model, recall that the consumption-money optimality condition can be expressed as
$$\frac{u_{m_t}}{u_{c_t}} = \frac{i_t}{1+i_t},$$
   where u_{m_t} denotes marginal utility with respect to **real** money balances (what was named u_2 in our look at the MIU model) and u_{c_t} denotes marginal utility with respect to consumption (what was named u_1 in our look at the MIU model). In each of the following, you are given a utility function and its associated marginal utility functions. For each case, construct the consumption-money optimality condition and use it to generate the function $\phi(\cdot)$. In each case, your money demand function should end up being an increasing function of c_t and a decreasing function of i_t. (**Note:** Be careful to make the distinction between real money holdings and nominal money holdings. The marginal utility function u_{m_t} is marginal utility with respect to **real** money holdings.)

Solution: For each utility function, we have now written the marginal utility functions u_{c_t} and u_{m_t}. Also note that you are, in each question, being asked to solve for $\frac{M_t}{P_t}$ as a function of c_t and i_t, which is the consumer's real money demand.

a. $u\left(c_t, \frac{M_t}{P_t}\right) = \ln c_t + \ln\left(\frac{M_t}{P_t}\right)$, with $u_{c_t} = \frac{1}{c_t}$ and $u_{m_t} = \frac{1}{M_t / P_t}$.

Solution: Constructing the consumption-money optimality condition with the given functions, we have
$$\frac{u_{m_t}}{u_{c_t}} = \frac{1/(M_t / P_t)}{1/c_t} = \frac{P_t c_t}{M_t} = \frac{i_i}{1+i_t}.$$
Solving for M_t / P_t, we have

$$\frac{M_t}{P_t} = \frac{c_t(1+i_t)}{i_t}.$$

Thus, the function $\phi(\cdot)$ function is $\phi(c_t, i_t) = \frac{c_t(1+i_t)}{i_t}$, which is increasing in consumption and decreasing in the nominal interest rate, as expected.

b. $u\left(c_t, \frac{M_t}{P_t}\right) = 2\sqrt{c_t} + 2\sqrt{\frac{M_t}{P_t}}$, with $u_{c_t} = \frac{1}{\sqrt{c_t}}$ and $u_{m_t} = \frac{1}{\sqrt{M_t / P_t}}$.

Solution: Proceeding as above, the consumption-money optimality condition is

$$\frac{u_m}{u_{c_t}} = \frac{1/\sqrt{M_t / P_t}}{1/\sqrt{c_t}} = \frac{\sqrt{P_t}\sqrt{c_t}}{\sqrt{M_t}} = \frac{i_i}{1+i_t}.$$

Solving for M_t / P_t, we have

$$\frac{M_t}{P_t} = \frac{c_t(1+i_t)^2}{i_t^2}$$

(be careful with the algebra here – notice the squared terms in the solution). Thus, the function $\phi(\cdot)$ function is $\phi(c_t, i_t) = \frac{c_t(1+i_t)^2}{i_t^2}$, which is increasing in consumption and decreasing in the nominal interest rate, again as expected.

c. $u\left(c_t, \frac{M_t}{P_t}\right) = c_t^{\sigma} \cdot \left(\frac{M_t}{P_t}\right)^{1-\sigma}$, with $u_{c_t} = \sigma c_t^{\sigma-1}\left(\frac{M_t}{P_t}\right)^{1-\sigma}$ and $u_{m_t} = (1-\sigma)c_t^{\sigma}\left(\frac{M_t}{P_t}\right)^{-\sigma}$.

Solution: The consumption-money optimality condition is

$$\frac{u_m}{u_{c_t}} = \frac{1-\sigma}{\sigma} \cdot \frac{c_t^{\sigma}(M_t / P_t)^{-\sigma}}{c_t^{\sigma-1}(M_t / P_t)^{1-\sigma}} = \frac{i_t}{1+i_t}.$$

After combining exponents, we can write this as

$$\frac{1-\sigma}{\sigma} \cdot c_t \cdot \frac{P_t}{M_t} = \frac{i_i}{1+i_t}.$$

Solving for M_t / P_t, we have

$$\frac{M_t}{P_t} = \frac{1-\sigma}{\sigma} \cdot \frac{c_t(1+i_t)}{i_t}.$$

Thus, the function $\phi(\cdot)$ is $\phi(c_t, i_t) = \frac{1-\sigma}{\sigma} \cdot \frac{c_t(1+i_t)}{i_t}$.

2. **The Keynesian-RBC-New Keynesian Evolution.** Here you will briefly analyze aspects of the evolution in macroeconomic theory over the past 25 years.

a. Describe **briefly** what the Lucas critique is and how/why it led to the demise of (old) Keynesian models.

Solution: The old Keynesian models were large estimated systems of equations, and the estimated coefficients could not (because they were just based on historical observations) take into account how behavior might change if policy changed. In the 1970's, this led to the downfall of such models as policy-makers tried more and more to exploit these relationships, but the "coefficients" began to vary a lot (for some reason…) with policy, eventually causing the profession (through the Lucas critique) to understand that such models really were not all that useful for policy advice after all.

b. Briefly define and describe the neutrality vs. nonneutrality debate surrounding monetary policy today. Which type of shock does this debate concern?

Solution: The RBC view holds that money shocks do not affect real variables (i.e., consumption or GDP) in the economy (neutrality), while the New Keynesian view holds that they do (nonneutrality) because prices take time to adjust (are "sticky").

3. **Portfolio Adjustment Costs.** In the infinite-period MIU model with three assets (i.e., stocks, nominal money, and nominal bonds), suppose that bond purchases are subject to a transactions cost (portfolio adjustment cost). Specifically, suppose the transactions cost in period t depends on by how much nominal bond holdings are changed during period t, and this transactions cost is increasing in the change in bond holdings (we'll assume it's a quadratic adjustment cost). The period-t budget constraint of the representative consumer is thus

$$P_t c_t + P_t^b B_t + \frac{\psi^B P_t}{2}(B_t - B_{t-1})^2 + M_t + S_t a_t = Y_t + M_{t-1} + B_{t-1} + (S_t + D_t)a_{t-1},$$

where ψ^B is simply a scaling parameter.

a. In this extended version of the MIU model, express the asset-pricing kernel (i.e., the term $\frac{\beta\lambda_{t+1}}{\lambda_t}$) using just the first-order-condition on bond-holdings B_t. That is, isolate $\frac{\beta\lambda_{t+1}}{\lambda_t}$ from the consumer's FOC on bonds. Does the presence of the transactions cost affect the kernel?

Solution: The infinite-horizon Lagrangian for this problem is

$$u\left(c_t, \frac{M_t}{P_t}\right) + \lambda_t \left[Y_t + M_{t-1} + B_{t-1} + (S_t + D_t)a_{t-1} - P_t c_t - P_t^b B_t - \frac{\psi^B P_t}{2}(B_t - B_{t-1})^2 - M_t - S_t a_t \right] +$$
$$\beta u\left(c_{t+1}, \frac{M_{t+1}}{P_{t+1}}\right) + \beta\lambda_{t+1} \left[Y_{t+1} + M_t + B_t + (S_{t+1} + D_{t+1})a_t - P_{t+1} c_{t+1} - P_{t+1}^b B_{t+1} - \frac{\psi^B P_{t+1}}{2}(B_{t+1} - B_t)^2 - M_{t+1} - S_{t+1} a_{t+1} \right] + ...$$

which by now you should have no trouble formulating. Note that the Lagrangian includes the transactions cost terms. In period t, the first-order condition with respect to bond-holdings at the end of period t, B_t, is

$$-\lambda_t P_t^b - \lambda_t \psi^B P_t (B_t - B_{t-1}) + \beta\lambda_{t+1} + \beta\lambda_{t+1}\psi^B P_{t+1}(B_{t+1} - B_t) = 0 .$$

(Note you have to use the chain rule when differentiating inside the transactions cost term.) Rearranging this expression to isolate $\frac{\beta\lambda_{t+1}}{\lambda_t}$, we have, after a few steps of algebra (which of course you should be able to carry out),

$$\frac{\beta\lambda_{t+1}}{\lambda_t} = \left[\frac{P_t^b + \psi^B P_t (B_t - B_{t-1})}{1 + \psi^B P_{t+1}(B_{t+1} - B_t)} \right].$$

Clearly, the presence of the transactions cost affects the kernel since the kernel depends on ψ^B.

b. Does the presence of the transactions cost on bond holdings affect the period-t **stock** price? Show algebraically why or why not, and qualitatively explain why or why not?

Solution: The first-order condition on stocks as usual can be re-arranged to give our usual stock price equation.

$$S_t = \frac{\beta\lambda_{t+1}}{\lambda_t}(S_{t+1} + D_{t+1}) .$$

If we insert the expression we found in part a for the pricing kernel into our stock price expression, clearly we would see that ψ^B, which governs the transactions cost on bond holdings, **does** affect the stock price.

The intuition for why this is the case is the following: the transactions cost on bonds directly affects the demand for bonds. But consumers' decisions about bond holdings are made in the context of a decision about the composition of their entire portfolio of assets, which are made up of bonds, stocks, and money. Reduction in the demand for bonds affects the demand for other assets, including stock, and hence the price of those other assets.

c. Does the presence of the transactions cost on bond holdings affect the consumption-money optimality condition? Show algebraically why or why not, and qualitatively explain why or why not?

Solution: The answer again is "yes," and for a similar reason: money is one of the alternative assets to bonds, so if the price of bonds is affected, the price of money is affected as well – in turn, the consumption-money optimality condition is affected. Formally, computing the first-order-condition with respect to money exactly as we've done in class (and which you should be able to reproduce), we end up with

$$\frac{u_{m_t}}{P_t} - \lambda_t = -\beta\lambda_{t+1}.$$

Using the FOC on bond holdings to substitute for $\beta\lambda_{t+1}$, we have

$$\frac{u_{m_t}}{P_t} - \lambda_t = -\lambda_t P_t^b \left[\frac{1+\psi^B P_t (B_t - B_{t-1})}{1+\psi^B P_{t+1}(B_{t+1} - B_t)} \right].$$

Using the FOC on consumption (which is also the same as usual), and making a few algebraic simplifications, we arrive at the consumption-money optimality condition,

$$\frac{u_{m_t}}{u_{c_t}} = \frac{1+i_t - \left[\frac{1+\psi^B P_t (B_t - B_{t-1})}{1+\psi^B P_{t+1}(B_{t+1} - B_t)} \right]}{1+i_t}$$

which clearly shows that the bond transactions cost affects the consumption-money optimality condition.