Department of Economics University of Maryland

Economics 701
Advanced Macroeconomics I
Project 2
Professor Sanjay Chugh
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Objective

The basic objectives of this project are to learn how to work conceptually and computationally with search-theoretic DSGE models, which have recently become a very popular class of models. One particular aspect of these models that you will explore in this project is the extent to which the cyclical volatility of labor-market variables depends on the "value of households' outside option" relative to employment. And, more novel, issue you will explore is how growth shocks affect cyclical volatility of labor markets. **These are the two main conceptual issues around which you should revolve the analysis and discussion in your paper.**

Below is sketched a minimum description of the economic environment. **As part of your paper, you must fill in the necessary details of the model so that a reader would be able to fully understand the economic environment.**

As in Project 1, you are to conduct simulations around the deterministic steady state (along with any other experiments, either long-run or short-run, you deem relevant) using a first-order approximation to the dynamic decision rules of the model economy using the Schmitt-Grohe and Uribe (2004 *Journal of Economic Dynamics and Control*) algorithm. Also as in Project 1, because the primary objective here is **to learn how to implement** such solutions yourself, you are not permitted to use "off-the-shelf" programs provided by Schmitt-Grohe and Uribe or programs such as Dynare.

The Economy

As in Project 1, consider first an economy experiencing exogenous long-run growth, in which the deterministic component of (labor-augmenting) productivity growth is stochastic. Define the **level** of the deterministic component of productivity in period t as X_t. Define $\gamma_t \equiv \frac{X_{t+1}}{X_t}$ as the one-period growth rate of the deterministic component of productivity. The growth rate γ_t is assumed to be in the information set of period t (that is, it is in the state vector of period t), which captures the idea that, even though it may be variable over time, this is a **deterministic** component of growth. In the growing economy, let $b \in (0,1)$ denote the one-period subjective discount factor of the representative household.

Described below is the decentralized economy of the **stationary (i.e., transformed) version of the model.**

Households

There is an infinitely-lived representative household that has an "infinite" number of family members. The number of family members in the infinite-sized household is normalized to measure one. All household members are assumed to participate in the labor force.

Denoting by n_t the measure of household members that work in period t and by s_t the measure that do not work (and hence search for work), the representative household maximizes

$$\max_{c_t, k_{t+1}} E_0 \sum_{t=0}^{\infty} b^t X_t^{1-\sigma} u(c_t)$$

(note the discount factor $b^t X_t^{1-\sigma}$ between period zero and period t, with CRRA utility parameter σ introduced below) subject to the sequence of flow budget constraints

$$c_t + \gamma_t k_{t+1} - (1-\delta)k_t + T_t = w_t n_t + r_t k_t + s_t \chi + d_t, \; t = 0,1,2...$$

The household takes as given the wage w_t that each of its employed members earns. Each unemployed family member receives a government-provided transfer χ. The household is levied a lump-sum tax T_t, which, in equilibrium, finances the aggregate of such transfers. Note that γ_t affects capital accumulation by households.

The representative household is assumed to own the firms in the economy, hence it receives any flows of profits that the production sector earns. The household receives the flow of firms' total operating profits d_t in lump-sum fashion every period. For use in constructing the representative firm's profit-maximization problem below, define

$$\Xi_{t|s} \equiv \frac{b^t X_t^{1-\sigma} u'(c_t)}{b^s X_s^{1-\sigma} u'(c_s)}$$

as the one-period ahead stochastic discount factor.

TO DO: Characterize the solution to the household optimization problem.

Firms

There is a representative “large” firm that solves the dynamic profit-maximization problem

$$\max_{n_{t+1},v_t} E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left(z_t f(k_t, n_t) - w_t n_t - r_t k_t - \psi_t v_t \right)$$

subject to the sequence of perceived laws of motion

$$n_{t+1} = (1-\rho) n_t + v_t q_t \text{ , } t = 0,1,2...$$

The notation is standard: v_t denotes the number of vacancies the firm posts in period *t*, each of which has a probability q_t of attracting a searching individual. Due to Cobb-Douglas aggregate matching, the matching probability depends only on aggregate market tightness (defined as the ratio of aggregate vacancies to the number of unemployed individuals); thus, the matching probability is determined in equilibrium. Each vacancy the firm opens carries a stochastic cost ψ_t.

The number (measure) of employees that produce output in period *t* is n_t, which is pre-determined at the start of period *t*. Separation of employees occurs at the exogenous rate ρ per period (separation shocks occurs after period-*t* production). In making its period-*t* optimal decisions with respect to vacancy-postings and its future employment stock, the firm takes as given the wage rate w_t which applies to all workers regardless of their tenure at the firm.

TO DO: Characterize the solution to firm’s profit maximization problem; in particular, show clearly how you construct the firm’s job creation condition, carefully explaining its derivation and the intuition behind it.

Wage Bargaining

As is standard in DSGE search models, assume that (re-)bargaining over the real wage occurs every period between all worker-firm pairs. For the purpose of the bargaining between any given worker and the firm, the outcomes of all other bargains is taken as given (hence there are no inter-bargaining strategic considerations). Moreover, all employed individuals receive the same wage and are treated symmetrically in bargaining, meaning wages have nothing to do with a given individual’s employment tenure. (Indeed, because we are focusing only on the symmetric equilibrium, we are not even tracking the “identities” of any given individual.) The value equations relevant for bargaining are presented below.

Denote by p_t the probability that an unemployed individual searching for work is matched with a vacancy. As with q_t, the matching probability depends only on aggregate market tightness and is taken parametrically by households.

For the household, the value (measured in terms of goods) in period t of the marginal employed individual (whether newly-matched or previously-matched) is

$$W_t = w_t + E_t\left\{\Xi_{t+1|t}\left[(1-\rho)W_{t+1} + \rho U_{t+1}\right]\right\},$$

whereas the value of the marginal unemployed individual is

$$U_t = \chi + E_t\left\{\Xi_{t+1|t}\left[p_t W_{t+1} + (1-p_t)U_{t+1}\right]\right\}.$$

For the firm, the value (again measured in terms of goods) in period t of having a marginal employee enter into production is

$$J_t = z_t f_n(k_t, n_t) - w_t + (1-\rho)E_t\left\{\Xi_{t+1|t} J_{t+1}\right\}.$$

The Nash bargaining problem is

$$\max_{w_t}\left(W_t - U_t\right)^{\nu} J_t^{1-\nu},$$

where $\nu \in (0,1)$ is the Nash bargaining power of individuals.

TO DO: Characterize the Nash wage outcome, carefully presenting/explaining its derivation and the intuition behind it. (Note: in constructing the Nash wage outcome, you must determine how the value J_t defined above relates to the "asset value of an employee" that appears in the job creation condition.)

Aggregation

The equilibrium employment stock evolves according to the aggregate law of motion

$$n_{t+1} = (1-\rho)n_t + m(s_t, v_t),$$

where $m(s_t, v_t)$ is an aggregate constant-returns-to-scale (CRS) matching function. Because matching is CRS, the matching rates p_t and q_t described above depend only on aggregate labor market tightness, $\theta_t \equiv v_t / s_t$.

In equilibrium, the size of the labor force is

$$n_t + s_t = 1.$$

Finally, the aggregate goods resource constraint of the economy is

$$c_t + \gamma_t k_{t+1} - (1-\delta)k_t + \psi_t v_t = z_t f(k_t, n_t) .$$

(Note: constructing this goods resource constraint relies on the trivial government budget constraint, $T_t = (1-n_t)\chi$.)

Equilibrium

TO DO: Construct a precise definition of (general) equilibrium for the model economy. Clearly listed as part of this description of equilibrium should be the aggregate endogenous and exogenous processes (the exogenous processes are given below)**, along with the conditions that characterize the evolution of the equilibrium.**

Parameterization

Assume the functional forms

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} ,$$

$$f(k_t, n_t) = k_t^{\alpha} n_t^{1-\alpha} ,$$

and

$$m(s_t, v_t) = \kappa s_t^{\zeta} v_t^{1-\zeta} .$$

There are three exogenous processes that affect the economy, each governed by an AR(1) process: (high-frequency) TFP fluctuations z_t ,

$$\ln z_{t+1} = (1-\rho_z)\ln \bar{z} + \rho_z \ln z_t + \varepsilon_{t+1}^z ,$$

(low-frequency) trend growth fluctuations,

$$\ln \gamma_{t+1} = (1-\rho_\gamma)\ln \bar{\gamma} + \rho_\gamma \ln \gamma_t + \varepsilon_{t+1}^\gamma ,$$

and fluctuations in the (per-vacancy) posting cost,

$$\ln \psi_{t+1} = (1-\rho_\psi)\ln \bar{\psi} + \rho_\psi \ln \psi_t + \varepsilon_{t+1}^\psi .$$

The steady-state level of TFP is $\ln \bar{z} = 1$, with innovations to TFP $\varepsilon_t^z \sim \text{iid } N(0, \sigma_z^2)$. The steady-state level of long-run productivity **growth** (in gross terms) is $\ln \bar{\gamma}$ (i.e., the "very-long-run growth rate"), with innovations to long-run growth $\varepsilon_t^\gamma \sim \text{iid } N(0, \sigma_\gamma^2)$. The steady-state level of vacancy posting costs is $\ln\bar{\psi}$, with innovations to posting costs $\varepsilon_t^{\psi} \sim \text{iid } N(0,\sigma_{\psi}^2)$.

The parameters of the economy that are fixed are:

	Fixed Parameters
β	0.99
δ	0.02
α	0.36
σ	2
ξ	0.40
κ	0.70
$\bar{\psi}$	0.40
ν	0.40
ρ	0.10
$\bar{z}$	1
ρ_z	0.95
σ_z	0.007

<u>TO DO:</u>

- **For the long-run growth rate of the economy, consider the two values** $\bar{\gamma}=1$ and $\bar{\gamma}=1.03^{1/4}$.
- **Given the parameters above, for *<u>each</u>* of these two long-run growth rates, you should implement a Shimer (2005) style calibration and a Hagedorn and Manovskii (2008) style calibration. Specifically, in the Shimer-style calibration, set χ so that it constitutes 40 percent of steady-state real wages; in the HM-style calibration, set χ so that it constitutes 95 percent of steady-state real wages.**
- **In the parameterization section of your paper, you should describe the calibration strategy and report your numerical values of χ for the *<u>four</u>* different model economies you are solving.**
- **When allowing for shocks to the long-run growth rate, experiment with a range of values of ρ_{γ} and σ_{γ}.**
- **When allowing for shocks to posting costs, experiment with a range of values of ρ_{ψ} and σ_{ψ}.**

Analysis/Discussion

With the basic guidelines immediately above, the range of quantitative (both steady state and simulation) experiments you conduct is left up to you. The goal is to learn something about the model vis-à-vis the empirics. With this goal in mind, you should compare your results with appropriate empirical data, either collected and summarized yourself or referencing some existing study/studies (more on this immediately below). These comparisons should be made by using some appropriate combination of tables and/or graphs and/or text. The exact data to which you compare your model to are left up to you – this should also be reflected in how you motivate your paper in the introduction and abstract.

A range of papers you could/may/should compare your results against is: Shimer (2005), Hagedorn and Manovskii (2008), Andolfatto (1996), Merz (1995), Arseneau and Chugh (2010, Table 1).

Given that you are simulating a general-equilibrium business cycle model, it also seems any analysis that doesn't at least document other "basic" aggregate properties of the model (i.e., GDP, consumption) would be incomplete.

The discussion of results is in many ways the most important part of your paper. Here, you should provide interesting and relevant analysis for the reader, describing the successes as well as shortcomings of your model. Describe the intuition/economic mechanism for any major successes; discuss the intuition/economic mechanism for any important shortcomings. Your discussion need not describe every nitty-gritty detail of the results you obtain (and should certainly NOT be just a verbal description of what a reader could find in tables, etc.), but should provide a fair and scientific view of your results and how they do or do not meet the study's basic objectives.

Data

If you are interested in doing some data work, you can collect the relevant labor-market data for the U.S. economy yourself (it is available through Shimer's website: http://robert.shimer.googlepages.com/flows), detrend it (using, say, the H-P filter), and compute relevant standard deviations and correlations/cross-correlations. If you do this, you should describe in your paper the data sources you use along with how you construct the summary statistics to which you compare your model's results.

Alternatively, you are permitted to simply refer to the empirical evidence in the papers described above (and related papers).

What To Submit

Your submission should be a stand-alone, complete paper – i.e., one should be able to read it independent of knowing what the "description" of "Project 2" was. As before, your submission **must be typed, not hand-written,** and the general structure of the paper should include:

- Abstract
- Introduction/Motivation
- Precise empirical methodology (if constructing your own empirical statistics)
- Model details, with important derivations/conditions, along with important intuition/discussion behind the structure of the model; a complete model section is ***not*** simply a list of equations, but rather should present the "story" of the model; also required is a definition/description of the decentralized equilibrium
- Solution method for the model, including
  - A clear description of how you solve for χ in the alternative parameter sets, with its numerical values reported
  - Presenting your computed g_x and h_x matrices (i.e., report the numerical values of these matrices)
- Description of the experiments you perform
- Results from relevant experiments, including comparison with data and appropriate discussion and/or interpretation
- Conclusions, summarizing your results, the importance of the contribution, and any thoughts on future work and extensions

As for Project 0 and Project 1, attach a print-out of your code to your submission.

References

Andolfatto, David. 1996. "Business Cycles and Labor-Market Search." *American Economic Review,* Vol. 86, p. 112-132.

Arseneau, David M. and Sanjay K. Chugh. "Tax Smoothing in Frictional Labor Markets." Mimeo, University of Maryland.

Shimer, Robert. 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review,* Vol. 95, pp. 25-49.

Hagedorn, Marcus, and Iourii Manovskii. 2008. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited." *American Economic Review,* Vol. 98, p. 1692-1706.

Merz, Monika. 1995. "Search in the Labor Market and the Real Business Cycle." *Journal of Monetary Economics,* Vol. 36, p. 269-300.